## The attempt to destroy the cube's symmetries

s.d.a. ... = symmetrically disposed about ... \# = single introduced cube

3 -axis $=$ axis of rotation symmetry order 3 , etc.
mirror $=$ plane of symmetry

| 2-COLOUR CUBE ARRANGEMENTS IN $2 \times 2 \times 2$ CUBE |  | How cubes arranged with respect to symmetry elements |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | s.d.a. a diagonal mirror | on a diagonal mirror | s.d.a. an edgebisecting mirror | $\begin{aligned} & \text { s.d.a. } \\ & \text { a 2- } \\ & \text { axis } \end{aligned}$ | $\begin{aligned} & \text { s.d.a. } \\ & \text { a 3- } \\ & \text { axis } \end{aligned}$ | on <br> a 3- <br> axis | $\begin{aligned} & \text { s.d.a. } \\ & \text { a 4- } \\ & \text { axis } \end{aligned}$ | distinguishing <br> feature: cubes share ... |
| Number of cubes of second colour and their coded arrangement <br> '3a' means a cube has been added to ' $2 a$ ', etc. | 1 |  | \# |  |  |  | \# |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  | 2a |  | \#\# |  |  |  | \#\# |  | a space diagonal |
|  | 2b |  | \#\# | \#\# | \#\# |  |  |  | an edge |
|  | 2c |  | \#\# |  |  |  |  | \#\# | a face diagonal |
|  |  |  |  |  |  |  |  |  |  |
|  | 3a |  | \#\#\# |  |  |  |  |  |  |
|  | 3 b | \#\# | \# |  |  |  |  |  | a face |
|  | 3c |  |  |  |  | \#\#\# |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| The resulting tree is helpful but not unique | 4a |  |  |  | \#\#\#\# |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  | 4b1 |  | \#\#\#\# |  | \#\#\#\# |  |  | \#\#\#\# |  |
|  | 4b2 | \#\# | \#\# |  |  | \#\#\# | \# |  |  |
|  | 4b3 | \#\# | \#\# |  |  |  |  | \#\#\#\# | a face |
|  |  |  |  |  |  |  |  |  |  |
|  | 4c | \#\# | \#\# |  |  | \#\#\# | \# |  | no edges |

Observations:

1. Every cube lies at a vertex, on an edge, on a face.
2. Every cube lies on a diagonal mirror and a 3 -axis.
3. No cube lies on a 2-axis, a 4 -axis or an edge-bisecting mirror.
4. There is no need to go beyond 4 introduced cubes because doing so simply reverses the colours.
