

Times Table Triangles

The investigation is an opportunity to explore arithmetic progressions. At KS2 & KS3 they may be represented as a staircase, using Multilink cubes or simply squared paper. The children will also need a multiplication square (preferably one going as far as 15 x 15, as in the examples) and a '45-45-90' set square.

We'll talk at KS4 level but all cases can be instanced with the above materials.

An arithmetic progression has a first term, a , a last term, l , and a mean, m , which is both the mean of the whole sequence and the mean of the first and last terms:

$$m = \frac{a+l}{2} . \tag{1}$$

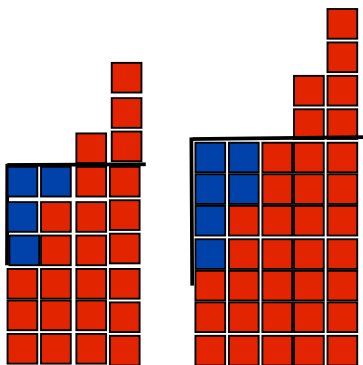
If there are n terms, and the difference between successive terms is d ,

$$l = a + (n-1)d . \tag{2}$$

This makes $m = \frac{a+[a+(n-1)d]}{2} = \frac{2a+(n-1)d}{2}$ (3)

and the sum of the whole sequence n times this, $mn = \frac{n[2a+(n-1)d]}{2}$. (4)

The children can break off the top of their Multilink staircase, give it a half turn and use it to complete a rectangle of height m and width n . If n is even, the break falls across the two middle values; if odd, the middle value is m :



What do the 6 cases presented have in common?

$d = 2$. So it's useful if the children make up sequences where d is *not* 2, and test them with their set squares. Take 2, 5, 8, 11, 14 (total 40). They find the total on the symmetry axis of the set square but above the right angle. Add the next term, 17, to give a total of 57. This time the sum is not to be found at all. The symmetry axis cuts the first row of the table between cells 9 and 10. If the table was graduated in halves, 57 would be there in row 6: $57 = 6 \times 9\frac{1}{2}$. For the mean to be a whole number, a and l must have the same parity, but here a is even, l odd. Notice from equation **(2)**, therefore, that $(n-1)d$ must be even so at least one of $(n-1)$ and d must be even.

Can we work out where on the set square's symmetry axis the sum will fall?

Yes. It must fall in row n because the symmetry axis passes through m and the total is mn . If d is 1, it will fall below the apex; if 2, exactly on it; if more than 2, above it.