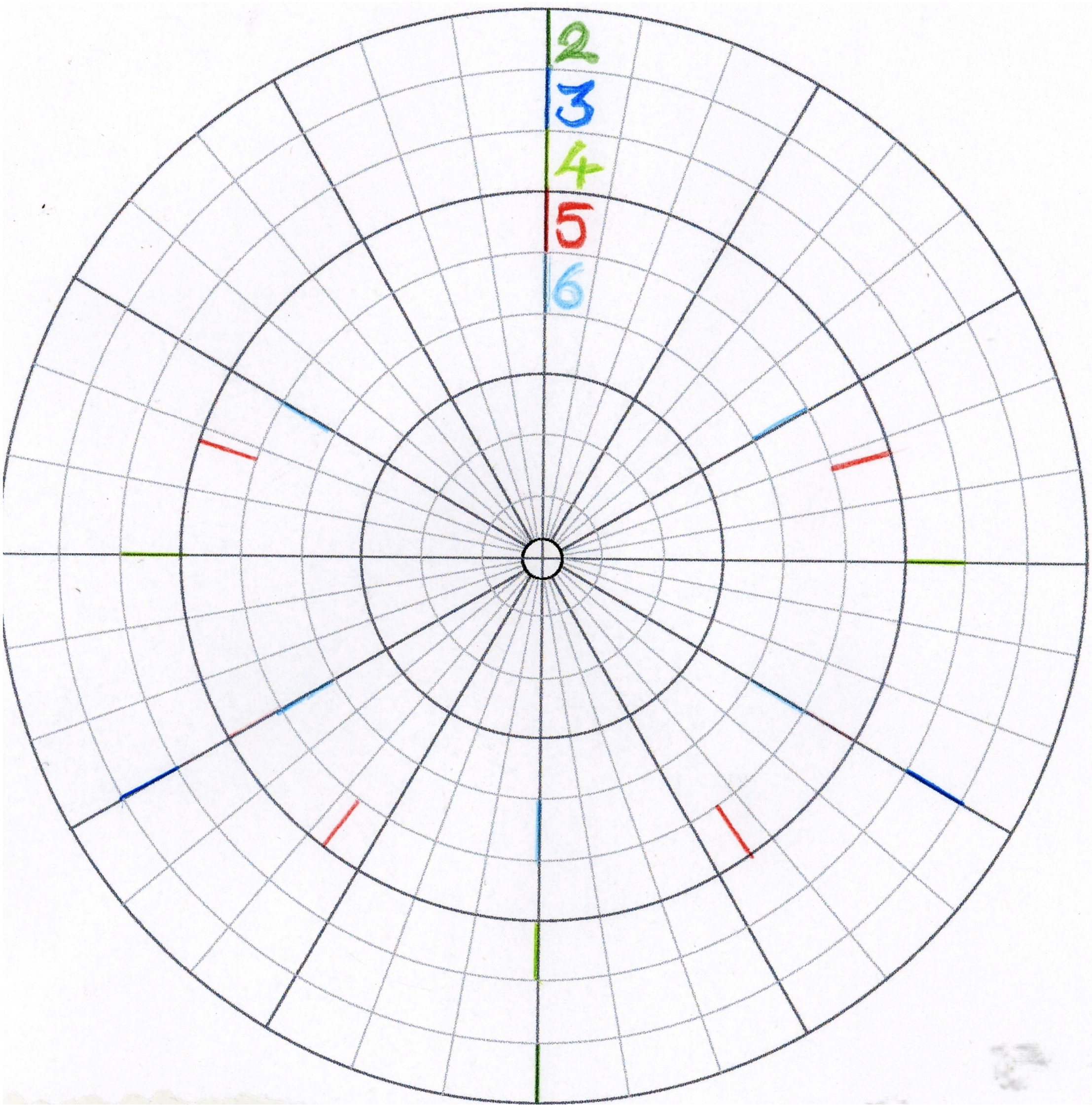
SYMMETRY

This workshop is suitable for Y5. If time allows, each member of the pair can complete the task. Take a half-time break at the end of **Part 2**.

A word about the "write-on polar base", on which the kaleidoscope is used in experiments **E5** and following:

Though angular measure in degrees is implied by the 10 degree spacing of the radii, the board is marked only to indicate the first 6 exact divisors of a whole turn, whatever the unit of measurement. However, the choice to divide the whole angle into 360 parts is justified by the divisibility of that number and Y5 children like to answer the following question: "360 divides by every number from 1 to 10 except one: which is that?". With or without a calculator, the children will enjoy working out the aliquot parts of 360, in which case they will not require the 'navigation' at the head of the second column in the table for **E6**.



materials needed	*activity (Teacher demonstration/Pupil experiment)*

Part 1: What is symmetry?

Cuboidal box	**E1** *Teacher demonstration*

Proceed somewhat as follows.

"Look at this box. Shut your eyes." Give the box a half-turn about a vertical axis. "Open your eyes. What have I done to the box? You might answer 'Nothing' or 'You've turned it round'. But the best answer would be 'I don't know whether you've done nothing or turned it round'. (In fact I've turned it round.) If you can move an object so that it looks just the same, you say it has *symmetry*. We shall study two kinds of symmetry an object can have: *mirror symmetry* and *rotation symmetry*."

Part 2: Mirror symmetry

E2 *Teacher demonstration*

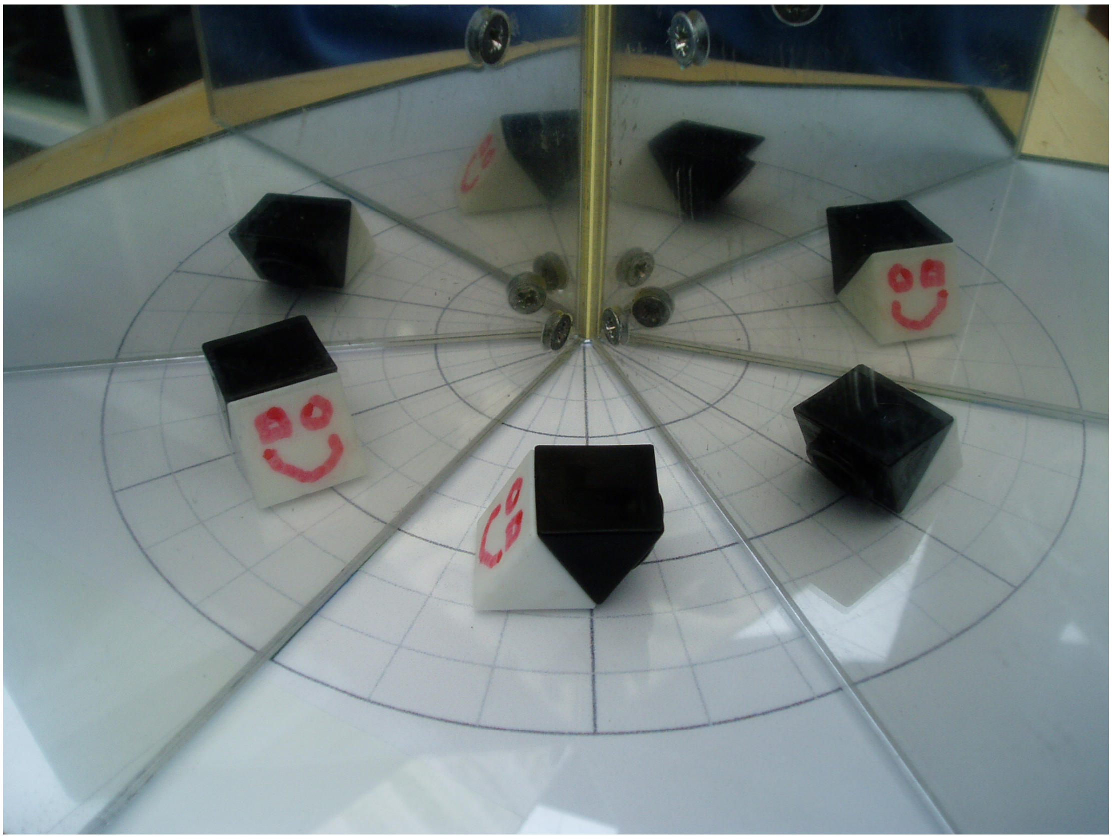
Stick your arm out and swing. Tell the children you measure how far you swing by the *angle* you turn through. (If a room or cupboard door is handy, demonstrate with that too.) Turn all the way round. Tell them that you've swung through a whole turn, which is 360°.

Paper disks in blue and in red, x 15, scissors, x 15 pairs	**E3** *Pupil experiment* Show the children how to fold one disk to reveal two intersecting *diameters*, thus revealing the *centre* and a *radius*. They must flatten out that disk again, set it over the other, and cut a radius through both. Show them how to interleave the disks to display an angle in one colour against the background of the other. Call it an 'angle-r'.
Demonstration 'angle-r' with centred polar grid	**E4** *Teacher demonstration and pupil experiment* Show different angles on the demonstration angle-r which the children are to match on their own. Point out the light grey *sectors*. Tell the children there are 36 of them and ask how many degrees each angle therefore is [360 divided by 36 = 10.] Point out that 3 light grey sectors make a dark one, [3 x $10^\circ = 30^\circ$ therefore].

2-mirror kaleidoscope, 15 cm square, on base as described x 15, model head as described, x 15, red dry-wipe pen, blue dry-wipe pen, x 15

E5 *Pupil experiment*

Give each pair a kaleidoscope and a write-on polar base on which to centre it, also a white and black Multilink equilateral prism stuck together to form a head. Ask them to draw a face in red on the white end. Show how the kaleidoscope opens and closes like a book, the two mirrors forming an *angle*. The children should write a blue 'R' for 'right' on the right-hand mirror near the hinge and a red 'L' for 'left' in the corresponding position on the left-hand mirror. Ask them to set the head between the mirrors and open and close the kaleidoscope slowly, observing carefully what happens, and telling the group.



Note down their observations on the whiteboard. Some qualitative examples:
1. "The smaller the angle, the more heads you see."
2. "If one mirror reflects a face, the other reflects the hair."
3. "If you move the head towards a mirror, its reflection in that mirror comes to meet it; the reflection in the other mirror moves away."
4. "If you turn the head clockwise, the reflections turn anticlockwise; the reflections-of-reflections clockwise again."

The next step is to try to turn **1.** and **3.** into quantitative statements. **3.** is clearer. Point out that the head seems to be as far in front of the mirror as the reflection is behind. **1.** is the subject of the next experiment.

As **E5** | **E6** *Pupil experiment*

Tell the children to set their polar base with the numbers upside down and the left-hand mirror along their *radius*.
Tell them always to sit the head roughly halfway between the mirrors.

Check that they've set their kaleidoscopes correctly and complete the third column of this table on the whiteboard.

Angle as *fraction* of a whole turn/in °	Where to find it	Number of heads altogether (the original and all its reflections)	Angle at which a head vanishes (**V**) as you open the kaleidoscope, angle at which two heads fuse (**F**), angle at which the lines of the mirrors are *axes of symmetry* (**S**)
$\frac{1}{9}$ (40)	4th light grey line	[9]	[**V**]
$\frac{1}{8}$ (45)	Halfway between 4th and 5th light grey line	[8]	[**F**] [**S**]
$\frac{1}{7}$ ($51\frac{3}{7}$)	About a quarter of the way from the 5th light grey line to the 6th	[7]	[**V**]
$\frac{1}{6}$ (60)	6th light grey line (2nd dark grey line, light blue mark)	[6]	[**F**] [**S**]
$\frac{1}{5}$ (72)	Just past 7th light grey line (red mark)	[5]	[**V**]
$\frac{1}{4}$ (90)	9th light grey line (3rd dark grey line, light green mark)	[4]	[**F**] [**S**]
$\frac{1}{3}$ (120)	12th light grey line (4th dark grey line, dark blue mark)	[3]	[**V**]
$\frac{1}{2}$ (180)	18th light grey line (6th dark grey line, dark green mark)	[2]	[**F**] [**S**]

Ask the children to repeat the experiment but to open the kaleidoscope continuously.
Ask them to note what happens when they hit one of the previous angles: does one of the heads vanish or do two fuse (combine to make one)? Complete the left side of the fourth column of the table.
Point out that when a real mirror edge forms a straight line with a reflected edge, i.e. it just seems to continue the whole way across, the picture is *symmetrical* wherever they place the head. Suggest that they also draw something. Tell them the mirror lines are then *axes of symmetry* of the picture. Ask them at what angles this happens and complete the right side of the fourth column of the table.
Ask them what they notice. [An **F** always goes with an **S** and the angle is an *even divisor* of a whole angle – which means it is an exact divisor of a *straight angle* (180°).]

As **E5**,
coloured paper disks,
as required,
scissors, x 15 pairs,
ruler, x 15,
dry-wipe pen
and duster, x 15

E7 *Pupil experiment*

Ask the children to choose one of the colours marked on the polar base. For dark blue ('3') and red ('5') they should rule a line right across the polar disk. For dark green ('2'), pale green ('4') and pale blue ('6') not only should they do that but also add lines halfway between. They should then number the radii 'round the clock' 1, 2, 3, … Ask them to centre their paper disk on top, their ruler across that, flat side down, and crease fold lines along the ruler as far as the centre along radii with an *odd* number only. (They can locate the centre by pushing down in the middle. A shallow crater will appear where the hole for the mapping pin is.) They must then invert their disk so that the creased radii land back in the odd-numbered positions. Now they must repeat the creasing process for radii with an *even* number only. When they fold their disk, radii should be 'hill' and 'valley' folds alternately. (If they can bring alternate radii together, so that the disk forms a 'flower', they have succeeded.) They should now fold the 'petals' together so that the sectors lie on top of each other and the disk makes a fan. They must then use their scissors to cut out interestingly-shaped pieces from one or both edges. When they open their fans, they will have a design in which a diameter is an *axis of symmetry.* They can check this by sitting their kaleidoscope on top of it.

If time, they should cut a fan of one colour, lay a fan of a different colour behind, and cut to leave a border between the two colours.

As **E7**, but 'angle-r' from **E3** swapped for **E7** cut-out

E8 *Pupil experiment*

The children must set their 'angle-rs' at a straight angle and centre them on the polar base with red on the left and blue on the right. They should set the head on the boundary, facing left, and sit the kaleidoscope on top. They may then move either or both sides of the kaleidoscope and observe what happens.
Ask the children to state and explain what they see.

['Blue' mirrors and their reflections run down the middle of (*bisect)* 'blue' sectors; 'red' mirrors and their reflections, 'red' sectors. Reflected heads of the first order sit on the boundaries of complete red and blue sectors and are therefore separated by twice the angle between the mirrors; reflected heads of the second order are separated by 2 complete red and 2 complete blue sectors and are therefore separated by 4 times that angle; those of the third order by 6 times the angle, and so on.

An interesting rider to **E6**: though images fuse on *even* divisors of a whole angle, reflections of the mirrors themselves fuse on *odd* divisors.]

Part 3: Rotation symmetry

Polar base,
tracing paper sheet,
mapping pin,
dry-wipe pen
and duster,
x 15,
pencil crayon, x 15

E9 *Pupil experiment*

Ask the children to choose a set of coloured marks as in **E7**. They should then draw or write something on the base between a pair of marks. Next, they must lay a sheet of tracing paper over their drawing and pin it to the polar origin. They should then trace their drawing together with one coloured mark. They must then turn their sheet so that the traced mark lies over a second like mark on the polar base and trace their original again. Continuing in this way, they will find they have made a design with *rotation symmetry*. The *order of rotation symmetry* is the number of replicas of their original motif on the tracing.

Coloured paper
disks,
as required,
scissors, x 15 pairs,
pencil crayon, x 15,
ruler, x 15

E10 *Pupil experiment*

Whatever the *order of rotation symmetry* they chose for **E5**, the children should take that number of paper disks, each in a different colour. Putting them together, they should rule a radius and cut along it. Now they must cut out a motif through the pack of disks but only to one side of the cut. By slotting them together and rotating each disk the same amount, they will have produced a design with the same symmetry as in **E5.**

(Point out that the different colours are only for show, that for the symmetry to be perfect all the pieces should have the same colour. Exhibit such a model.)

Part 4: The two kinds alone and together

Polygonal tiles, as required

E11 *Pupil experiment*

Draw this chart on the whiteboard. Tell the children to make their own designs with the polygonal tiles. When they have done so, ask where they fall on the chart. Discuss whether the blank cells can be filled [**P** for 'possible'] or not.

Number of mirror lines / Order of rotation symmetry	0	1	2	3	4	5	6
1	[**P**]	[**P**]					
2	[**P**]		[**P**]				
3	[**P**]			[**P**]			
4	[**P**]				[**P**]		
5	[**P**]					[**P**]	
6	[**P**]						[**P**]

P.S. 20.5.11