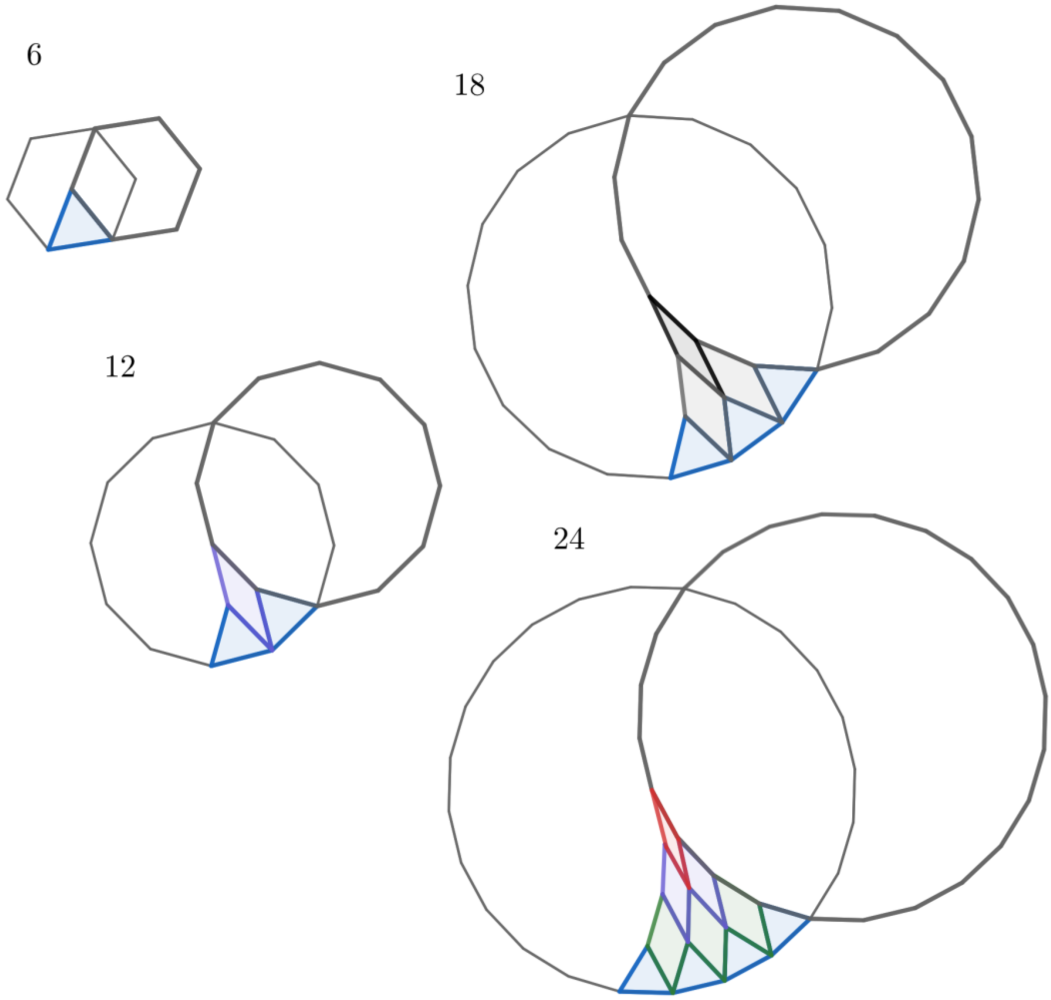
Seed crystal notes



As shown above, what happens is that:

(1) a rhombus vertex coincides with a vertex of the chosen polygon and
(2) the next generation of polygons are bisected by the perimeter to give equilateral triangles.

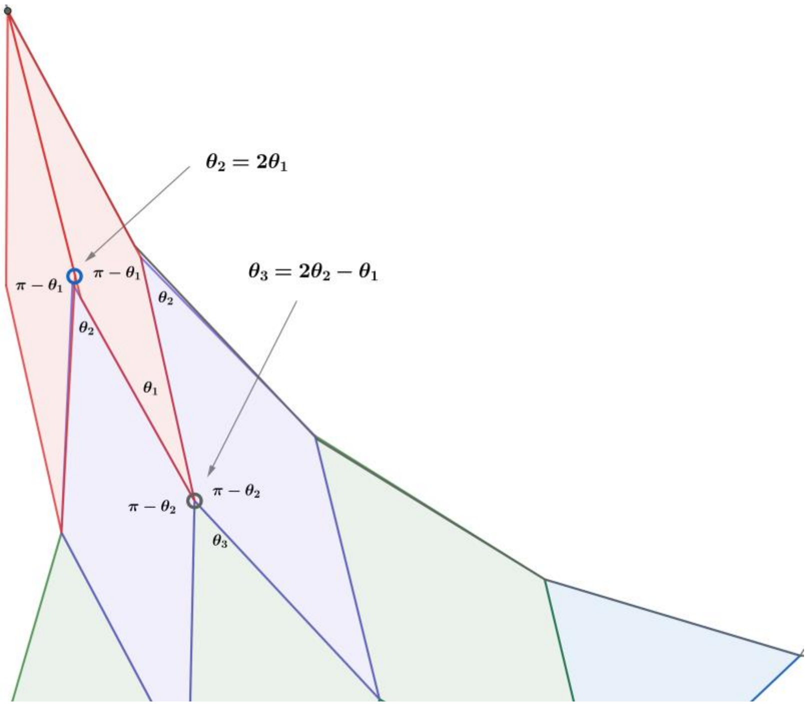
The figures have also been drawn to reveal another property:

(3) the perimeter of an identical polygon can be traced through consecutive rhombus edges.

The first question students are likely to ask is, 'Why does the number of sides have to be a multiple of 6?'

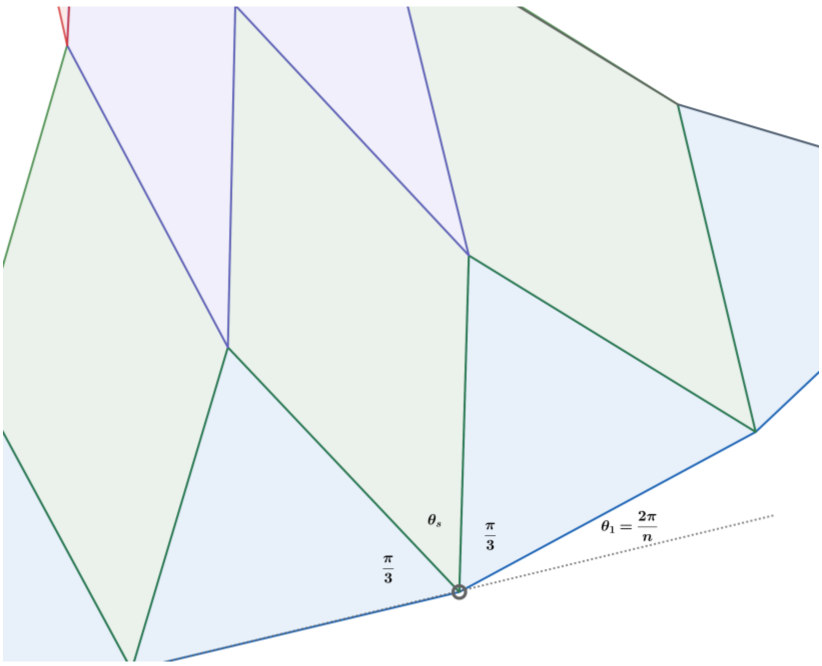
We'll answer this question in dealing with **(1)** and **(2)**.

Working from the centre outwards, we have, from the figure below:



$\theta_1 = \frac{2\pi}{n}$,
$\theta_2 = 2\theta_1$,
$\theta_3 = 2\theta_2 - \theta_1 = 3\theta_1$,
.
.
.
$\theta_m = 2\theta_{m-1} - \theta_{m-2} = m\theta_1$,
.
.
.
$\theta_s = s\theta_1 = \frac{2s}{n}\pi$ $(\boldsymbol{A})$,
where θ_s is the smaller angle of the last complete rhombus.

Working from the perimeter inwards, we have, from the figure below:



$\theta_s = \frac{n-6}{3n}\pi$ $(\boldsymbol{B})$.

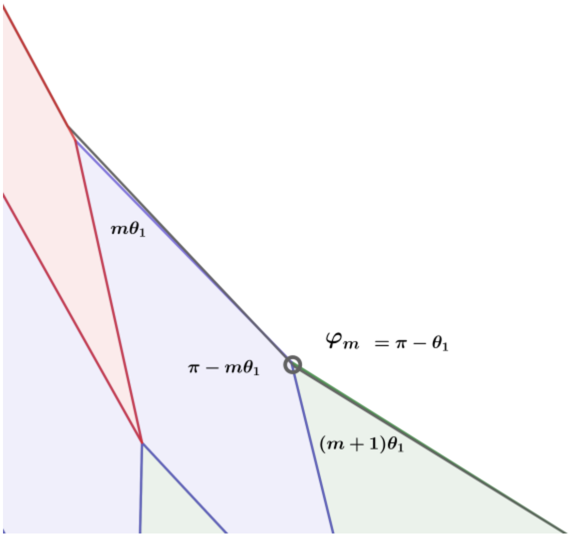
Where the two figures meet, we can equate **(*A*)** and **(*B*)**, leading to:

$s = \frac{n}{6} - 1$.

For *s* to be an integer, *n* must be a multiple of 6.

Writing $n = 6k$, we have
$s = k - 1$.
This is the number of complete rhombuses between the centre and the perimeter.

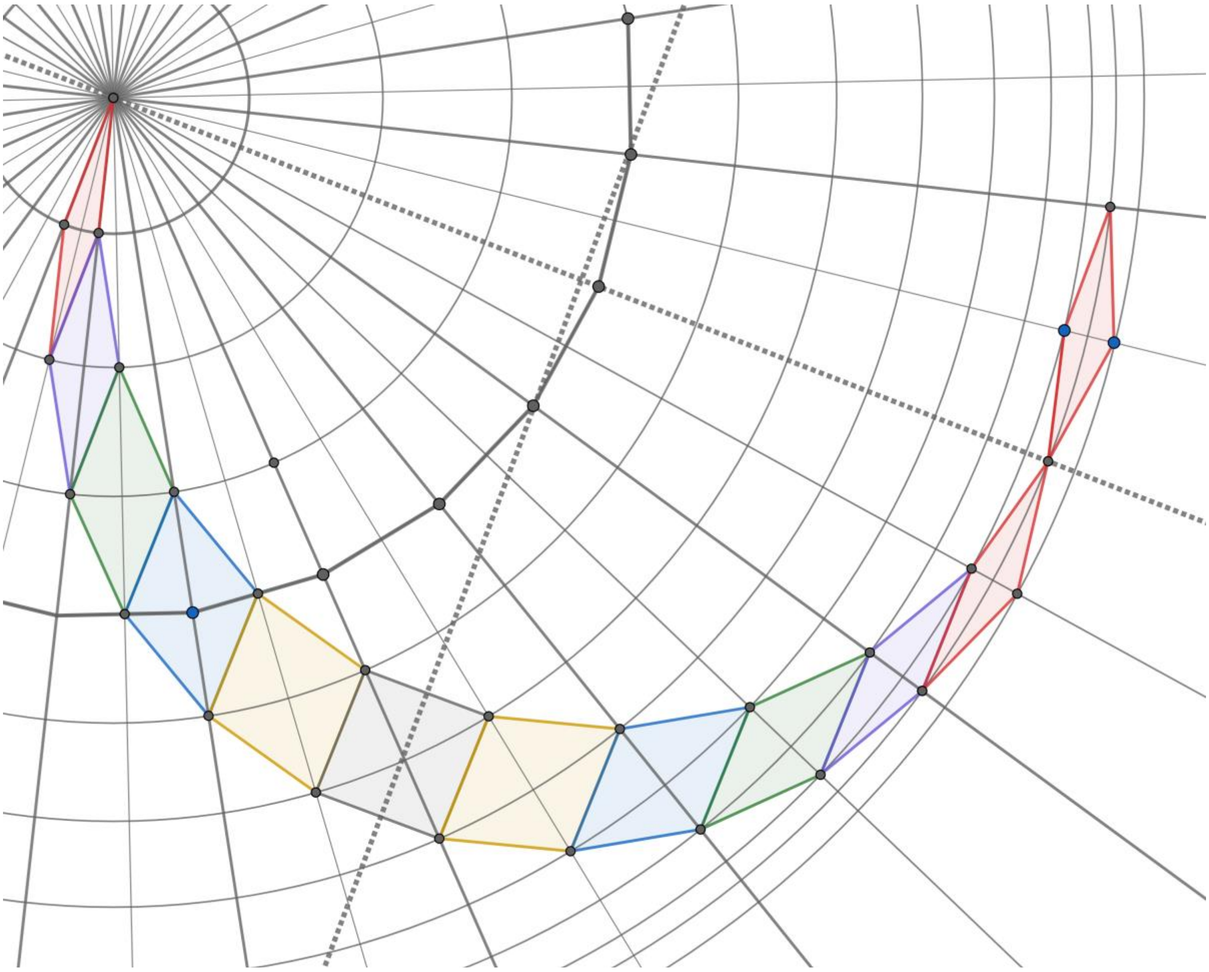
Now for **(3)**. From the figure below we have:



$\varphi_m = \pi - \theta_1$, an expression independent of m and, since $\theta_1 = \frac{2\pi}{n}$, this is the interior angle of the regular n-gon.

Further remarks

We can trace the sequence of rhombuses beyond the perimeter of the chosen polygon:



The dotted symmetry axis running bottom left to top right shows how the orientation of the rhombuses reverses, that is, the acute and obtuse vertices swap places. Beyond the dotted symmetry axis running top left to bottom right, the rhombuses pack on the inside to complete the perimeter of the ‘trace’ polygon.

In our dissection of the polygon, we end up with half-rhombuses (equilateral triangles) round the perimeter. Students may be interested to learn that all regular polygons with an even number of sides – including ours therefore – can be dissected into rhombuses. In such dissections, as in ours, the side length of the rhombus is the side length of the chosen polygon. They should choose the Wikipedia entry ‘Regular polygon’ and go down to ‘Dissections’.

Your students will readily seize on the artistic possibilities the maths opens up.