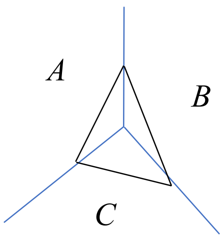
The World Paving Championships

A confidential report from the coach:

He was lucky to get 3 out of 10. You can see intuitively what the problem is. The cut with a red circle at each end signals a change of angle. But that would create a gap where the slab leaves the stretch of pavement above and to the right of the white line.

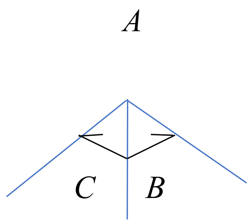
I always remind my pavers that a pavement is part of a polyhedron – not necessarily convex, but a polyhedron nonetheless. Consider these 3 vertices, where slabs *A, B, C* meet. Let their interior angles there be $\measuredangle A, \measuredangle B, \measuredangle C$. The black triangles are the vertex figures obtained by joining the points 1 unit from the vertex along each edge. Note the anomaly in **3**: the vertex figure would run through the vertex. The red lines in **3** show planes containing the respective dihedral angles.

1



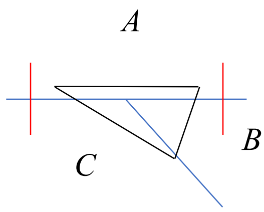
All angles $< 180°$

2



$\measuredangle A > 180°$

3



$\measuredangle A = 180°$

(We shall ignore the case where $\measuredangle A + \measuredangle B + \measuredangle C = 360°$ and the vertex is that of a tiling, i.e. where we are dealing with a plane figure.)

$\measuredangle A + \measuredangle B + \measuredangle C < 360°$

$\measuredangle A + \measuredangle B + \measuredangle C > 360°$

$\measuredangle A + \measuredangle B + \measuredangle C = 360°$
$\measuredangle B + \measuredangle C = 180°$

The size of the angle sum depends critically on whether the largest angle is less than, equal to, or greater than a straight angle.

Vertex part of a convex or concave solid according to what happens elsewhere

Vertex necessarily part of a concave solid: if *A* lies in the plane of the page, the edge *BC* either rises from the page or dips below it

The anomaly is removed by denying that there is a vertex where the 3 line segments meet. *AC* and *AB* are parts of the same edge. *BC* has no meaning. *B* and *C* lie in the same plane. Since the red planes are parallel, the dihedral angles are equal.