

numbercityprompts

Project: numbercityoblique, then numbercityplan.

This architect's model is of 'Number City', laid out on one side of a waterway. The 'number' in 'Number City' must mean something. Exactly what we'll find out as we go along.

What do you notice about the arrangement of the buildings?

- *In each row parallel to the river the blocks are equally spaced.*
- *The spaces go 1, 2, 4, 6, 10, 12, 16, 18.*

How does that sequence run if you include the buildings?

- *2, 3, 5, 7, 11, 13, 17, 19, ... : the prime numbers..*

Project: numbercityplanannotated.

Look at the **upper part**.

How do the powers of 2 correspond to the heights of the blocks?

- *The power is the height of the block.*

How will the powers of 3 run?

- *3, 3, 3², 3, 3, 3², 3, 3, 3³, 3, There are two numbers of the same power before you go to a higher one.*

Then how many will there be with 5?

- *Four.*

The patterns made by the powers are very interesting. Take, for example, the powers of 2 and see what you can discover.*

How does the model show the prime factorisation of a number?

- *The numbers run along the top. Choose your number, say 12, and read downwards:
 $12 = 2^2 \times 3$.*

What do the red arrows show?

- *The first time a given prime occurs. Because it is prime, there can be no numbers between it and the waterside.*

Apart from 2 and 3, you notice that the primes fall either side of a multiple of 6. Why is this?

- *Take for example 12 to 18. There are just three numbers between 13 and 17. Two of those are even and the one in the middle is half way between two multiples of 6, and must therefore divide by 3. That leaves only 13 and 17 as possible primes (which in this case they are). But what is certain is that no other numbers in the interval can be.*

You may have noticed that the blocks are single: there are no pairs. Why is that?

- *Two consecutive numbers cannot have the same divisor.*

Look at the **lower part**.

What can you say about the red template?

- *It falls on the same pattern as it goes along.*

The width of the rectangle covers the primes 2 and 3. What does the length cover?

- *The lowest common multiple of 2 and 3.*

How can you extend the idea?

- *Do the same with, say, 3 and 5, or 2, 3 and 5.*

What can you do with 2 and 5, which are separated by the '3' row?

- *Make a separate rectangle for each and the two rectangles will work as a parallel pair of length 10.*

Long-term projects

1. For younger children

Build the model.

The one shown uses 1 cm cubes.

With 2 cm cubes there's space to put more information on the model.

Go on-line for a source of wooden cubes.

*If one group of children find all they can about powers of 2, they can compare their findings with another group who have studied the 3s.

2. For expert programmers among your older students

Program a computer animation for the templates.