

Maths in Hats

A 'People Maths' Workshop

The tasks are collaborative, involving, at the leader's discretion, either the participants – the audience staying silent, or the audience – the participants staying silent.

1a. Latin Squares (colour Sudoku)

Set-up: 9 chairs in 3 x 3 array; 3 children in blue, 3 in red, 3 in yellow hats; rest as audience in circle of chairs surrounding

Task: To obtain rows and columns in which all 3 colours are present

Term to learn: Latin square

1b. Graeco-Latin Squares (double colour Sudoku): orders 3 (Task 1), 4 (Task 2)

Question: If you wear a stack of two hats, one on top of the other (*demonstrate*), in any of 3 colours (*demonstrate red-on-red, red-on-blue, blue-on-red*), how many different hat-stacks can you wear?

(Invite children to use whiteboard - they are in fact forming the Cartesian product of the set of upper hats and the set of lower hats)

Set-up: 9 chairs in 3 x 3 array; 9 children, each wearing one of the above hat-stacks; rest as audience

Task 1: To obtain Latin squares with both the upper hats and the lower hats

Term to learn: Graeco-Latin square (*demonstrate on whiteboard how Euler used Greek letters for the upper hats and Latin letters for the lower hats*)

Task 2: As Task 1 but for 4-colour equivalent

Homework task: To arrange the 4 court cards in each of the 4 suits in a 4 x 4 array so that no court card and no suit repeat in a row or column (*demonstrate with acetates on OHP*)

Question: "Study this" (*show acetate on OHP with 3 orthogonal order-4 squares, shown as nested coloured pentagons, squares, triangles respectively*). "What have I succeeded in doing?"

2a. Sliding block puzzle: 'Sliding Sam'

Set-up: 4 chairs in 2 x 2 array; 1 child in red hat (top left), 2 children in yellow hats, empty chair (bottom right), rest of children as audience

Task 1: To get 'red-hat' from top left to bottom right in the fewest moves, allowing only horizontal and vertical moves

Task 2: As Task 1 but for 3 x 3 equivalent

Task 3: As Task 1 but for 4 x 4 equivalent

Discussion: (*Same sequence shown with acetates and counters on OHP*)

Observations: in optimal solution, only necessary for 2 'yellow-hats' to move to advance 'red-hat' one chair; number of moves goes up in 8s from Task 1 to 2, 2 to 3.

Term to learn: arithmetical progression (*number staircase*)

2b. Sliding and jumping puzzle: ‘Leapfrog’

Set-up: 3 chairs in a row; child in red hat (left), empty chair, child in yellow hat (right)

Task 1: To swap ‘red-hat’ and ‘yellow-hat’, allowing a slide on to a vacant chair or a horizontal jump past a single hat of the other colour into a vacant chair.

Task 2: As Task 1 but for 2 ‘red-hats’, 2 ‘yellow-hats’.

Task 3: As Task 1 but for 3 of each colour.

Task 4: As Task 1 but for 4 of each colour.

Task 5: As Task 1 but for 5 of each colour.

Discussion: *(The progression from Task 1 to Task 2 shown with acetates and counters on OHP)*

Difference table written out: children shown how to use it to deduce the number of moves in Task 5 from the previous results

3. Alignment problem: ‘Safe Queens’

Set-up: 16 chairs in 4 x 4 array (the chessboard); 4 children (standing) with red hats

Task 1: The ‘red-hats’ to so seat themselves that no two lie in the same row, column or diagonal

Task 2: As Task 1 but for 5 children on 5 x 5 chessboard

Discussion: *(Shown with acetates and counters on OHP how one solution can be transformed into another – by rotation, reflection and, in the 5 x 5 case, translation)*

Task 3: As Task 1 but for 6 children on 6 x 6 chessboard

Task 4: As Task 1 but for 7 children on 7 x 7 chessboard

Homework task: Google ‘problem of the non-attacking queens’

4a. Logic exercise: ‘Top hats’

Set-up: Row of 3 chairs facing left to right, a child on each; rest as audience

Task 1: Leader: “I have 2 red hats, 2 blue hats. Shut your eyes while I place 3 of those hats on your heads in an order I choose. Now open them. You can each see the hats on the heads in front of you, but not your own or those of the people behind. I am going to start at the right, working my way back and then forwards again. I shall always ask the same question: ‘Do you know the colour of your own hat?’.” *Does so, choosing the order red, red, blue. Invites the audience to comment on the logic of the answer given.*

Task 2: As Task 1 but with order: red, blue, red.

4b. Logic exercise: ‘Mexican hats’

As 4a but:

(i) a child’s hat hangs behind him (as a Mexican might wear a sombrero). Thus a child can see only the hat immediately in front of him.

(ii) only 3 hats are in play, not 4, i.e. 2 hats of one colour, 1 of the other is chosen and announced, e.g.: red, red, blue.