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Parabola and Paraboloid

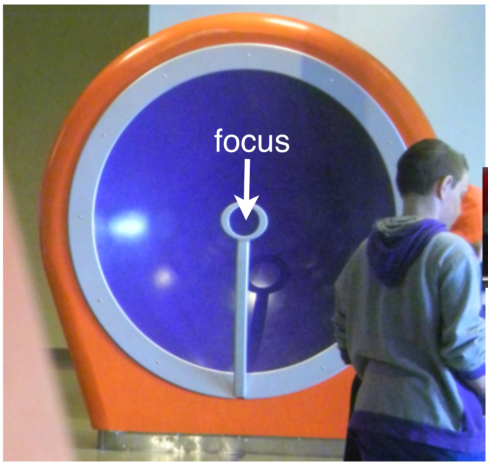
www.magicmathworks.org/geomlab12

When the sun shines in a cup of coffee the reflected rays don't meet but form an envelope curve. A cup is round. For the rays to meet, the cross-section must be parabolic.

Set the spotlight several metres away from the vertical card with multiple slits so that near-parallel rays fall on the card behind.

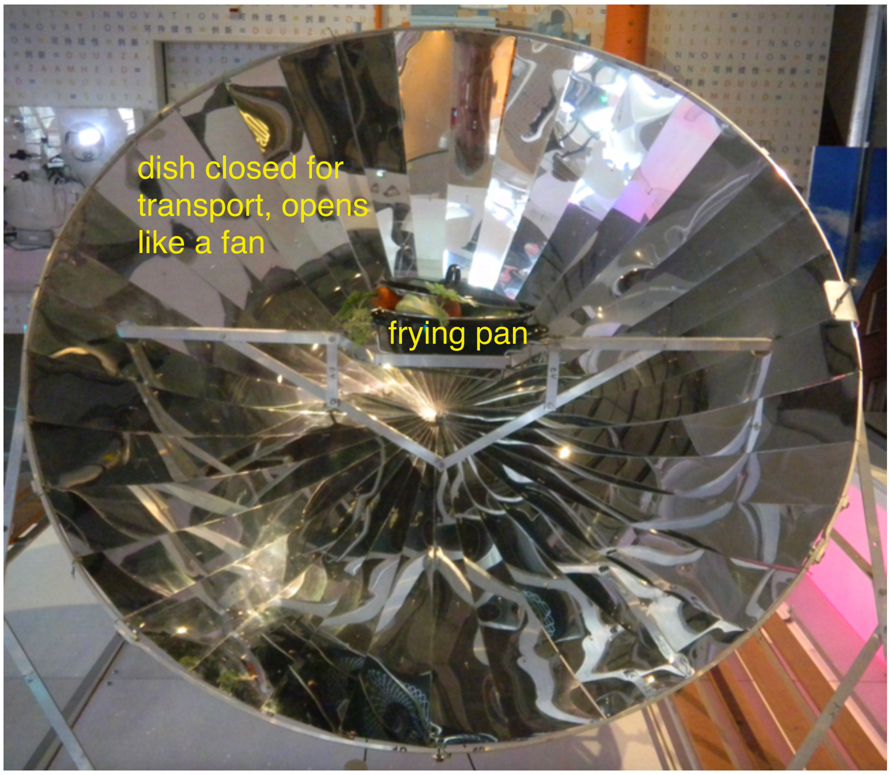
Bend the mirror strip along the purple curve (half a parabola). The more accurately you do so, the more accurately will the reflected rays converge on a single point on the symmetry axis.

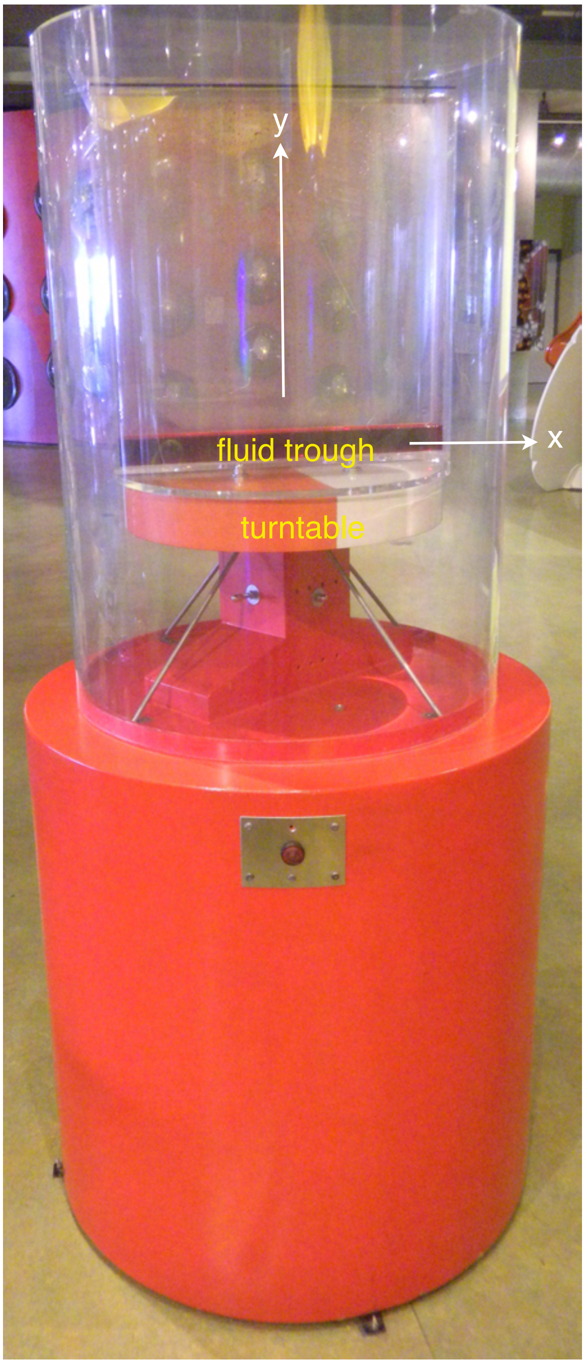
Mirrors focussing any sort of wave on a single point must therefore conform to the solid of revolution of the parabola, the parabol*oid*. The surface of a fluid in a cylindrical tank rotating at uniform speed takes this form. Some reflecting telescopes use mercury in that way.



‘Whisper’ dishes

Solar cooker





In this model a trough of fluid rotates at a uniform speed ω . Consider the forces on an element of fluid of unit cross-section, density ρ and length δx , and therefore mass $m = \rho\delta x$. Since pressure in a fluid acts in all directions, we can ignore the vertical components. However, the pressure at $(x+\delta x, 0)$ exceeds that at $(x, 0)$, resulting in an inward force on the element of $F_i = \rho g \delta y = \rho g(\frac{dy}{dx})\delta x = c\frac{dy}{dx}\delta x$.

This is opposed by the outward force due to rotation
$F_0 = m\omega^2 x = (\rho\delta x)\omega^2 x = kx\delta x$.

We therefore have:

$F_i = F_o$,

$c\frac{dy}{dx} = kx$,

$\int dy = \frac{k}{c}\int x\,dx$,

y = constant x x^2 .

This is the equation of a parabola.