11

Mary Boole's Parabola

www.magicmathworks.org/geomlab11

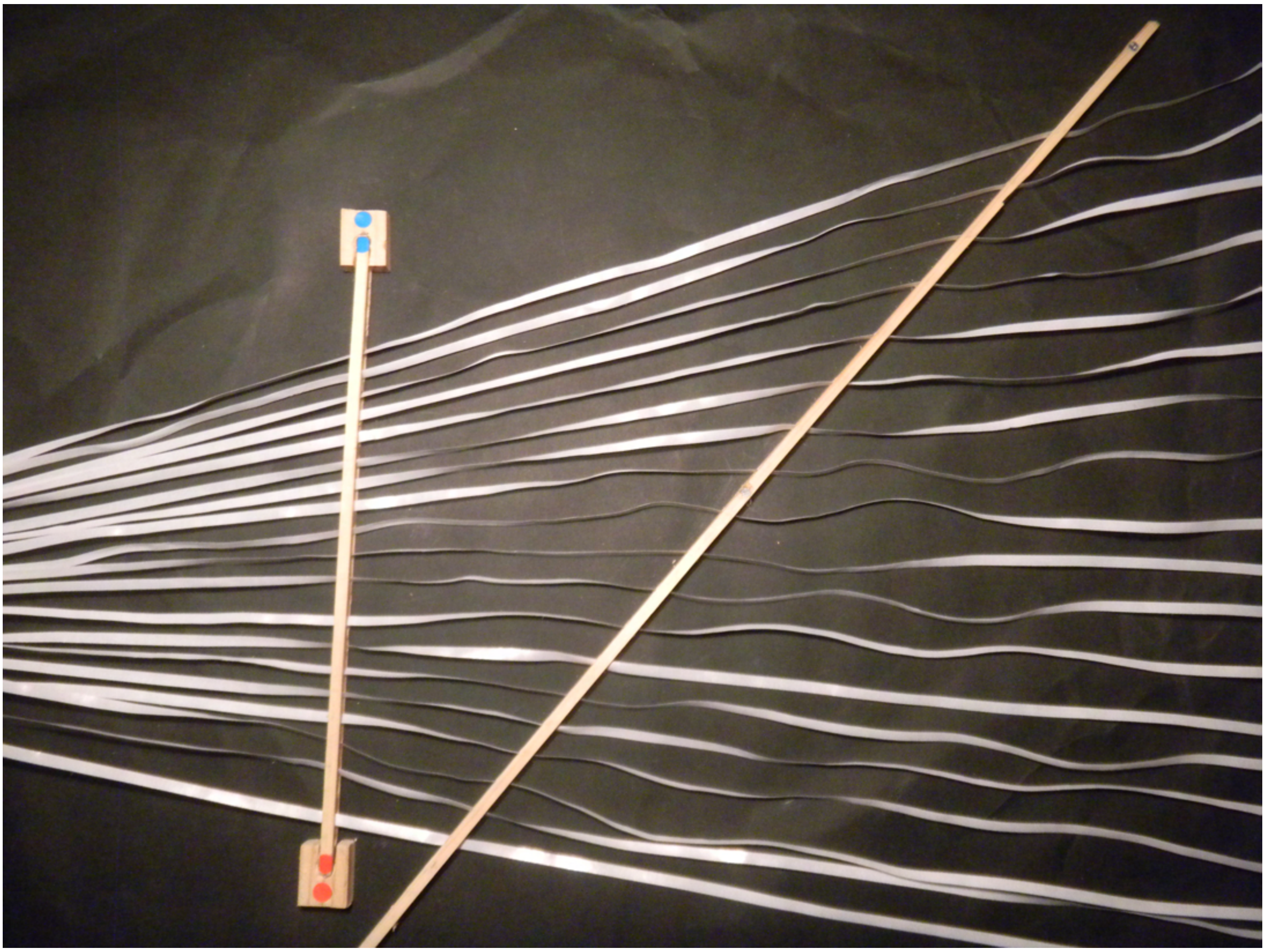
1. Set the stick in the slots with the red end in the red slot, the blue end in the blue slot.

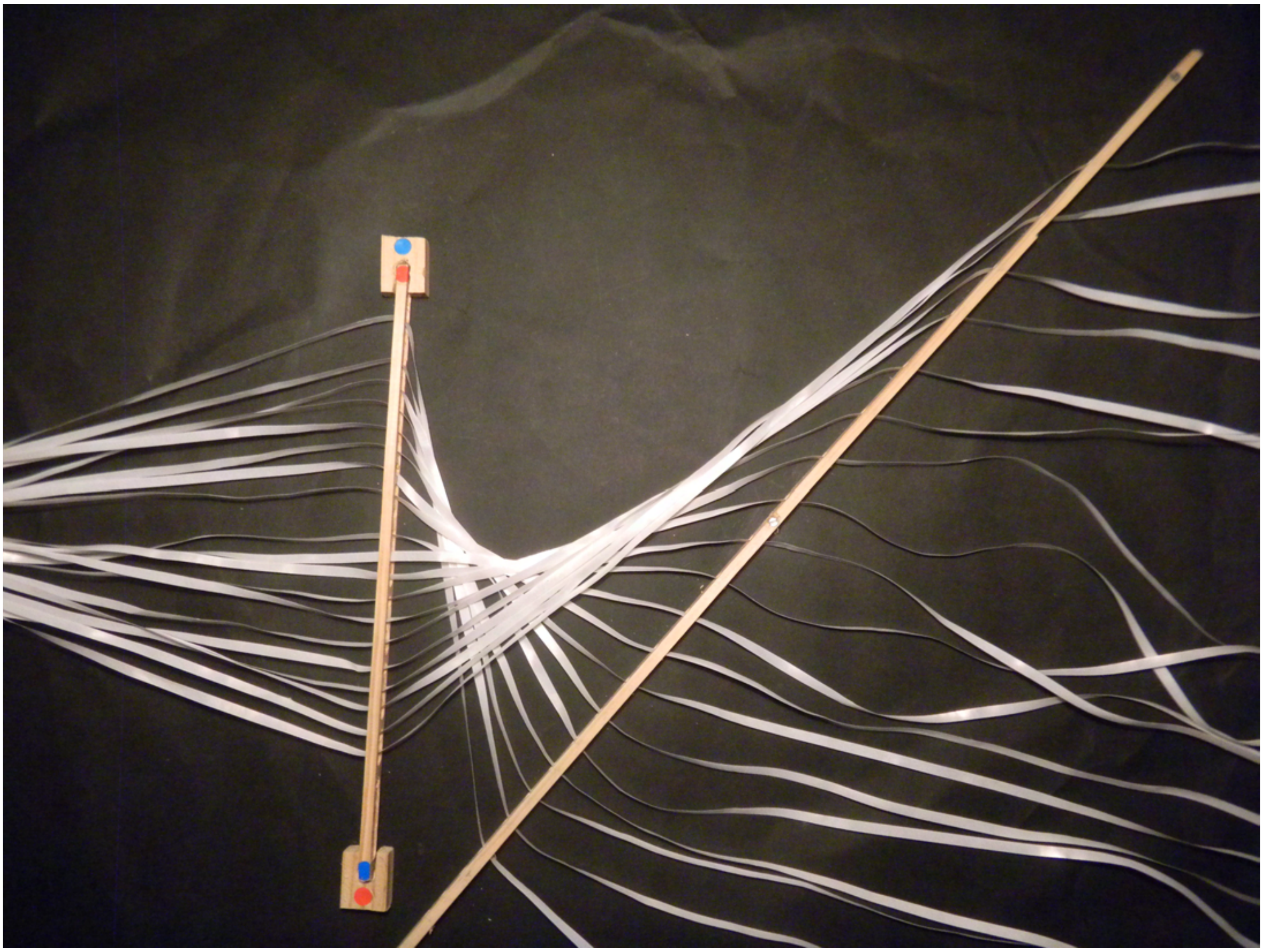
Stretch out the ribbons.

2. Turn the stick over in its own length so that the red end fits the blue slot, the blue end fits the red slot.

Stretch out the ribbons.

Paraboloid
Hyperbola
Hyperboloid
Hyperbolic paraboloid
Line pair
Sine curve
Tractrix
Exponential curve
Catenary
Catenoid
Helix
Helicoid
Plane
Polygon
Polyhedron
Tiling
Archimedean spiral
Equiangular spiral
Loxodrome
Line family
Circle
Cylinder
Cone
Sphere
Ellipse
Line
Parabola



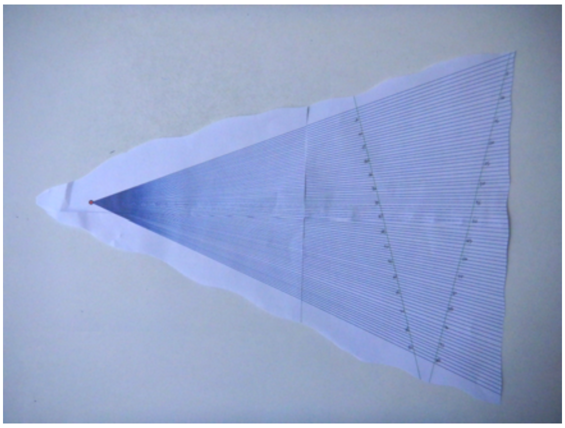


Here is a pencil-&-paper version of the flipped-stick experiment, in which we also study properties of the parabola:

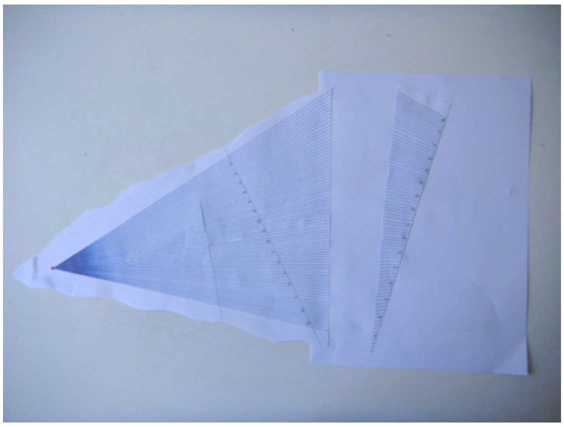
Draw lines from a point.

Rule two lines across them.

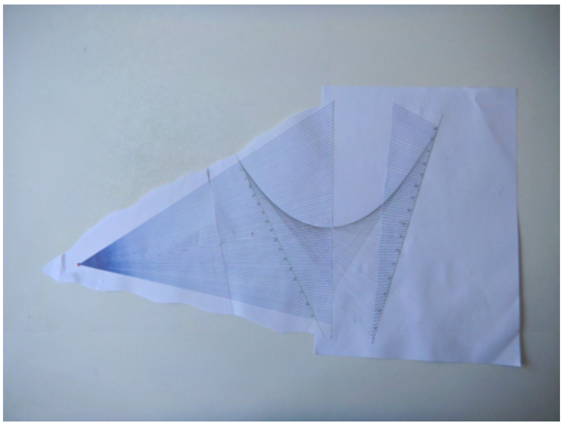
Number intersects.



Cut off the right-hand set and turn them round.



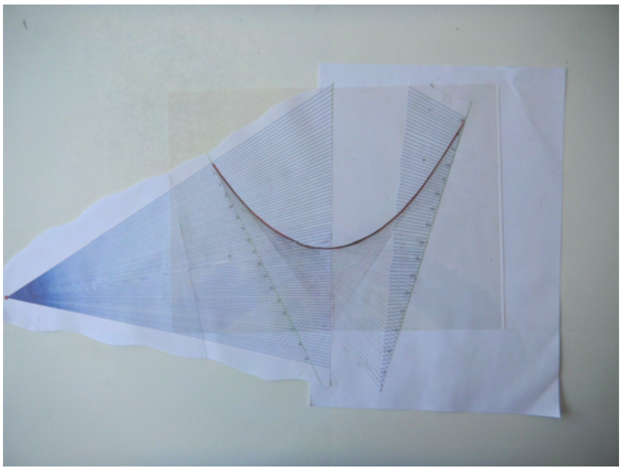
Join corresponding points.



Draw in the curve (for visibility).

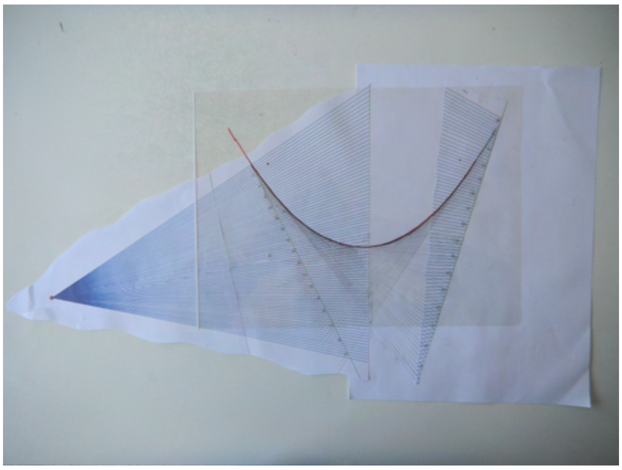
Mark a point.

Trace the curve and the point on to an acetate.

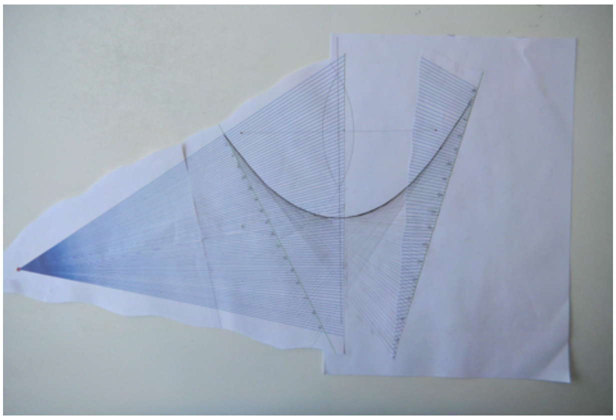


Flip over.

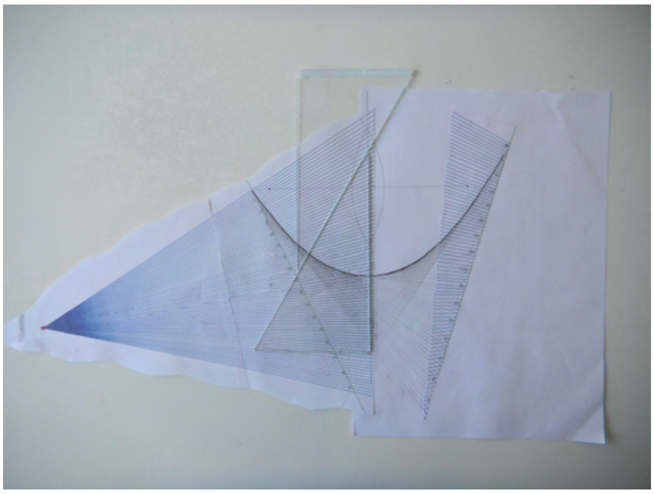
Turn back the acetate and mark the reflected point on the original.



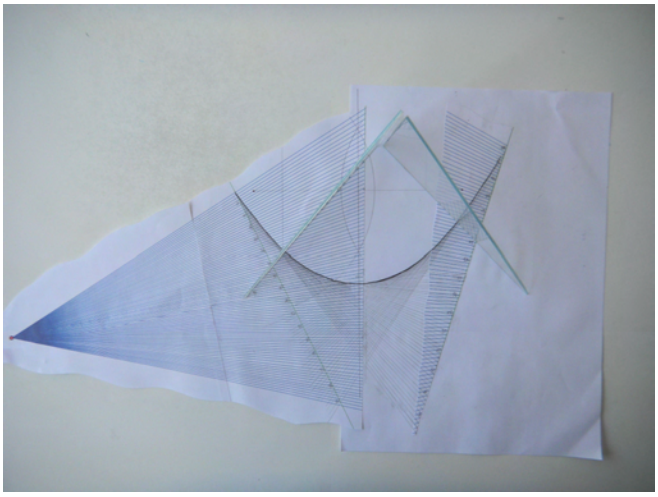
Construct the perpendicular bisector.



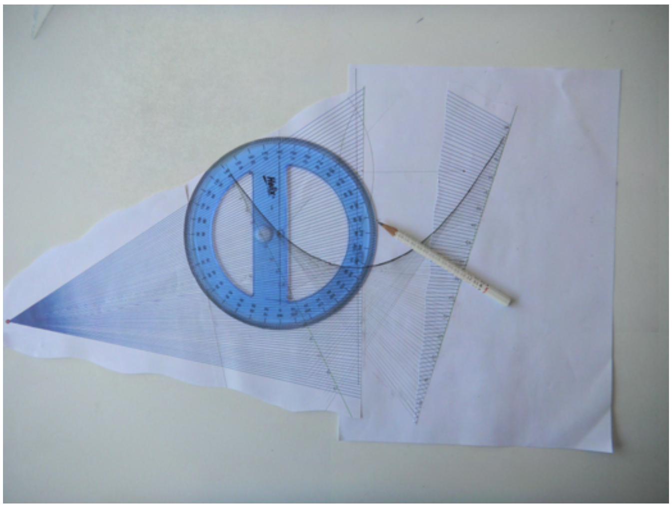
Construct a parallel.



Use a semi-silvered mirror
(a set square, attached by velcro
to a second set square to keep it
vertical) to locate and draw the
normal at the point of intersection of
the parallel with the curve.



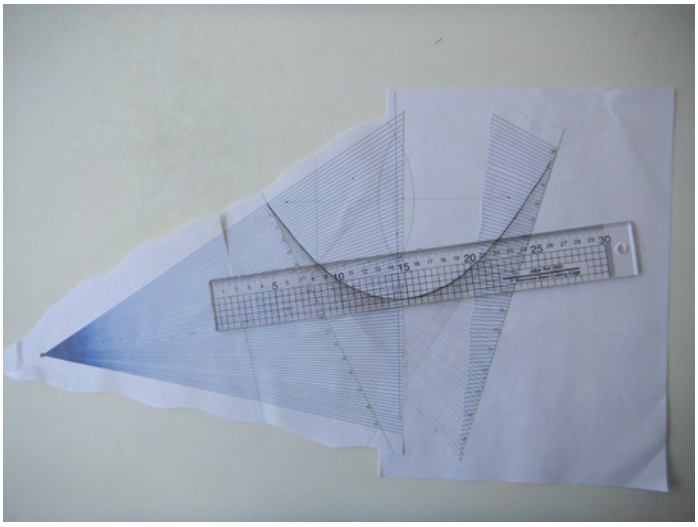
Use a protractor to construct a line
making an equal angle with the
normal on the other side.



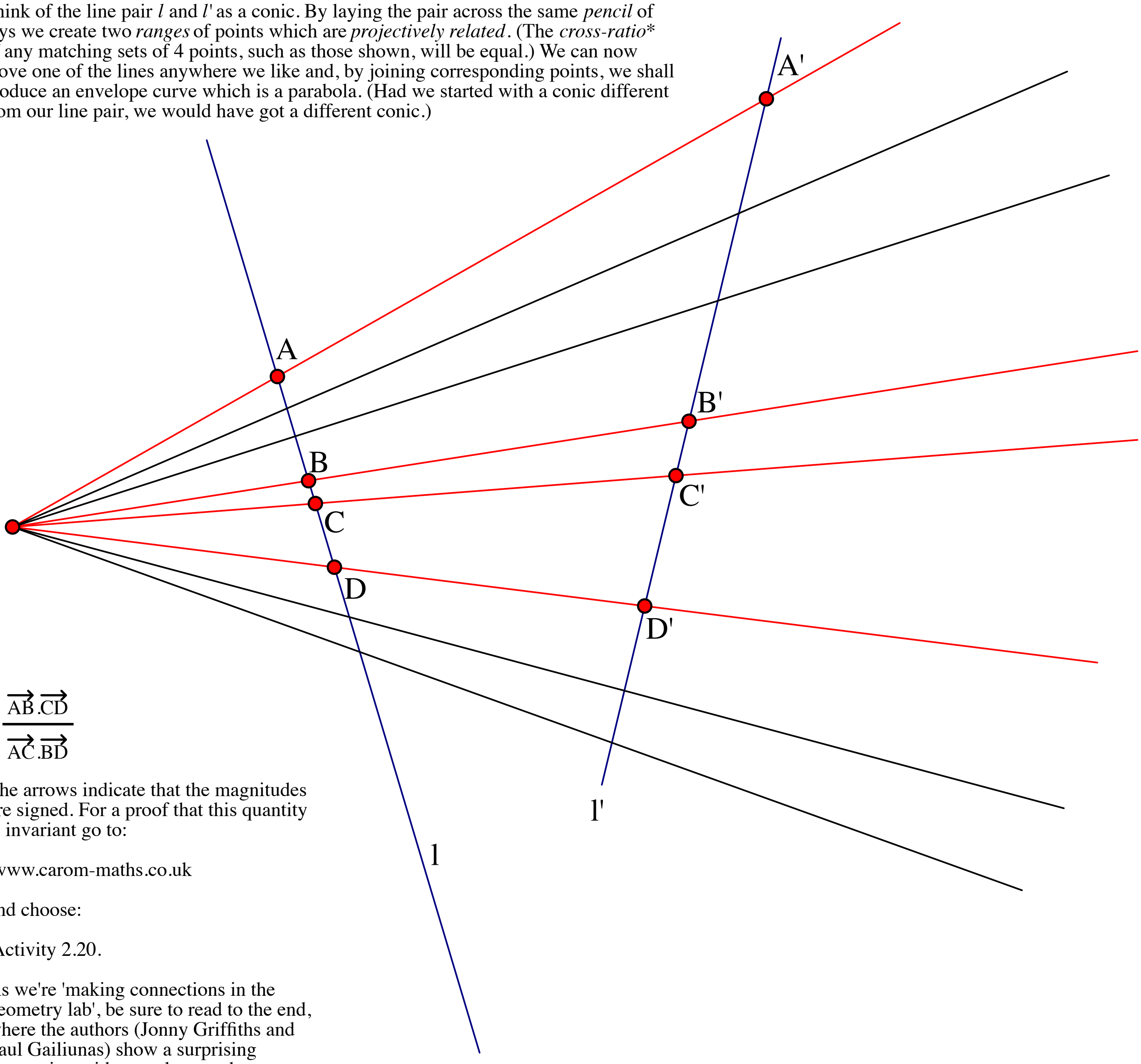
The intersection of this line with the axis is the focus.

For accuracy make many such constructions.

Check that the distance from the axis to the curve measured at the focus is twice the distance to the vertex (*a* in the standard formula).



Think of the line pair l and l' as a conic. By laying the pair across the same *pencil* of rays we create two *ranges* of points which are *projectively related*. (The *cross-ratio** of any matching sets of 4 points, such as those shown, will be equal.) We can now move one of the lines anywhere we like and, by joining corresponding points, we shall produce an envelope curve which is a parabola. (Had we started with a conic different from our line pair, we would have got a different conic.)



* $\dfrac{\overrightarrow{AB}.\overrightarrow{CD}}{\overrightarrow{AC}.\overrightarrow{BD}}$

The arrows indicate that the magnitudes are signed. For a proof that this quantity is invariant go to:

www.carom-maths.co.uk

and choose:

Activity 2.20.

As we're 'making connections in the geometry lab', be sure to read to the end, where the authors (Jonny Griffiths and Paul Gailiunas) show a surprising connection with complex numbers.