3
Steinhaus' Sliced Cylinder

www.magicmathworks.org/geomlab3

A) Unroll the sliced paper towel roll.

B) Roll the sliced wooden cylinder, marking points of contact with the board.

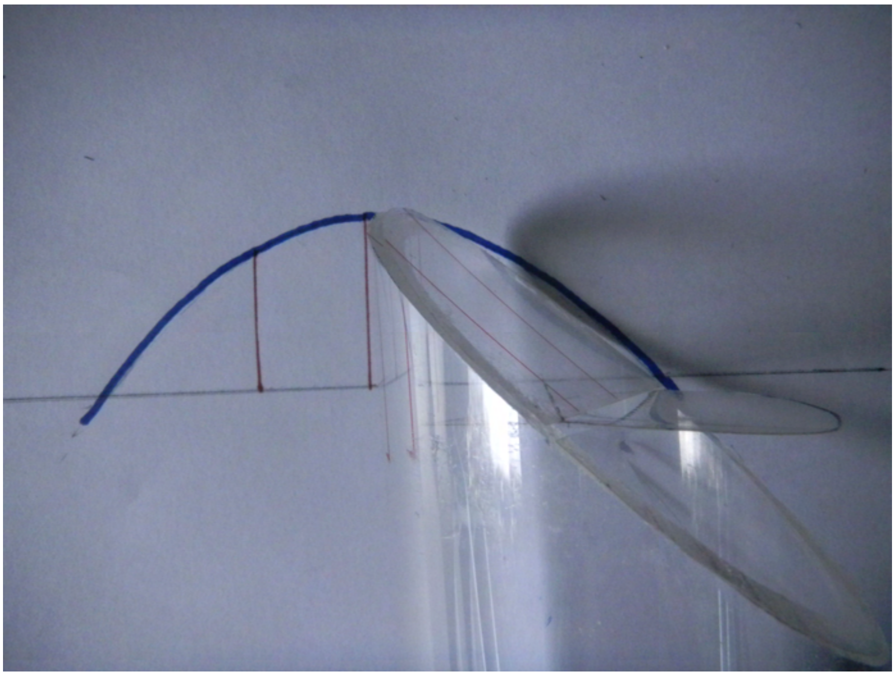
C) Roll the perspex cylinder.
Look where the altitudes of the red triangles fall.
Note from the disk that these are proportional to the *sine* of θ, whereas the distance rolled is proportional to θ itself.

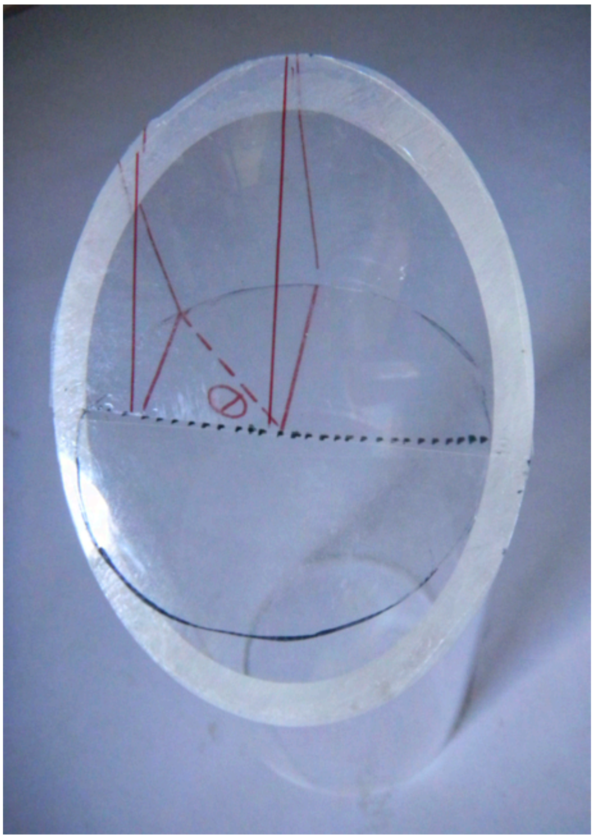
The locus is therefore a sine curve.

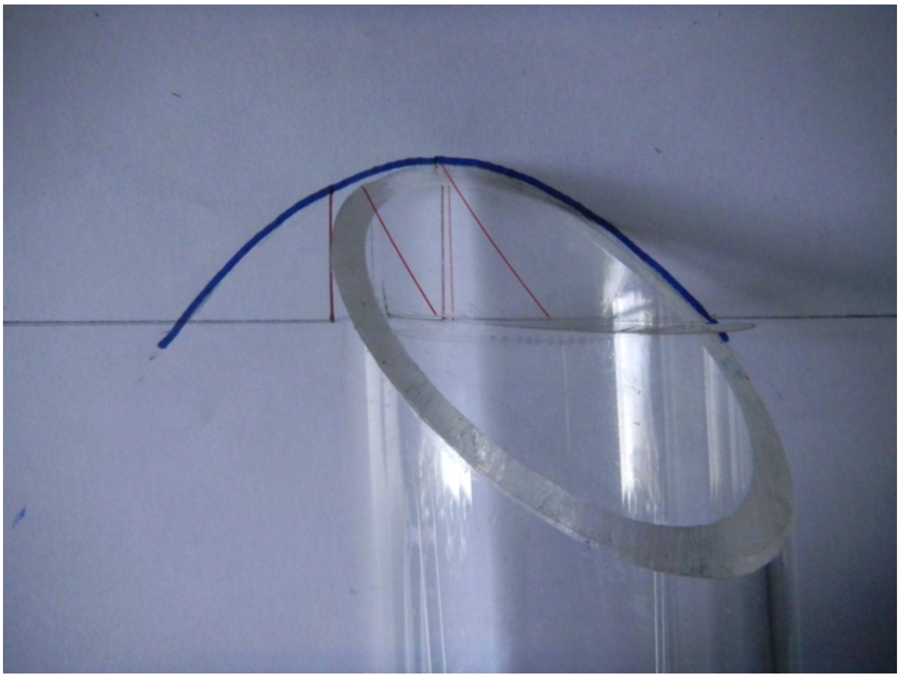
Paraboloid
Hyperbola
Hyperboloid
Hyperbolic paraboloid
Line pair
Sine curve
Tractrix
Exponential curve
Catenary
Catenoid
Helix
Helicoid
Plane
Polygon
Polyhedron
Tiling
Archimedean spiral
Equiangular spiral
Loxodrome
Line family
Circle
Cylinder
Cone
Sphere
Ellipse
Line
Parabola

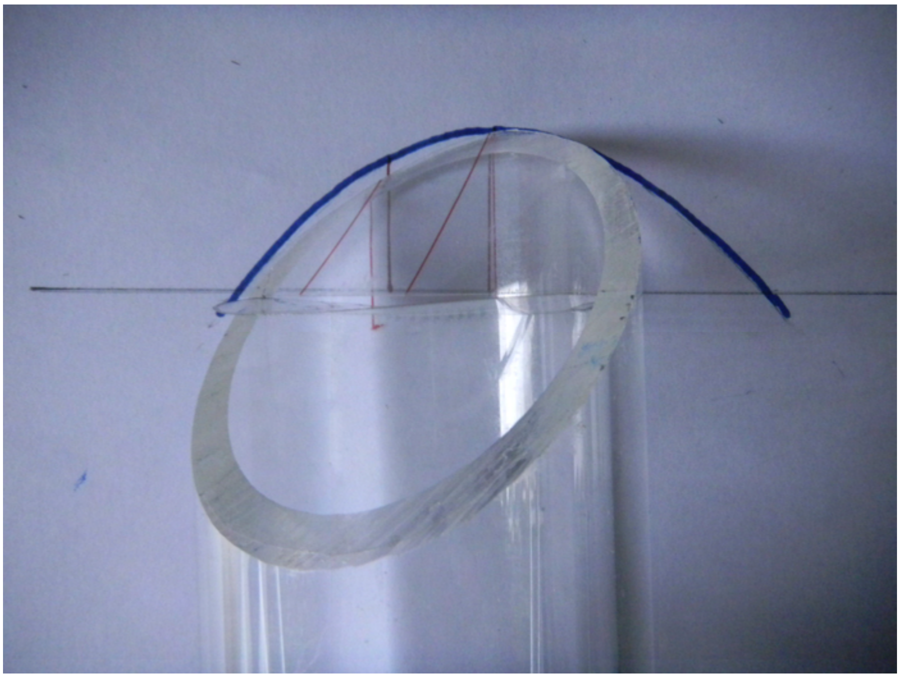
3 C

Here is the perspex cylinder, which we show in 3 positions as it rolls.

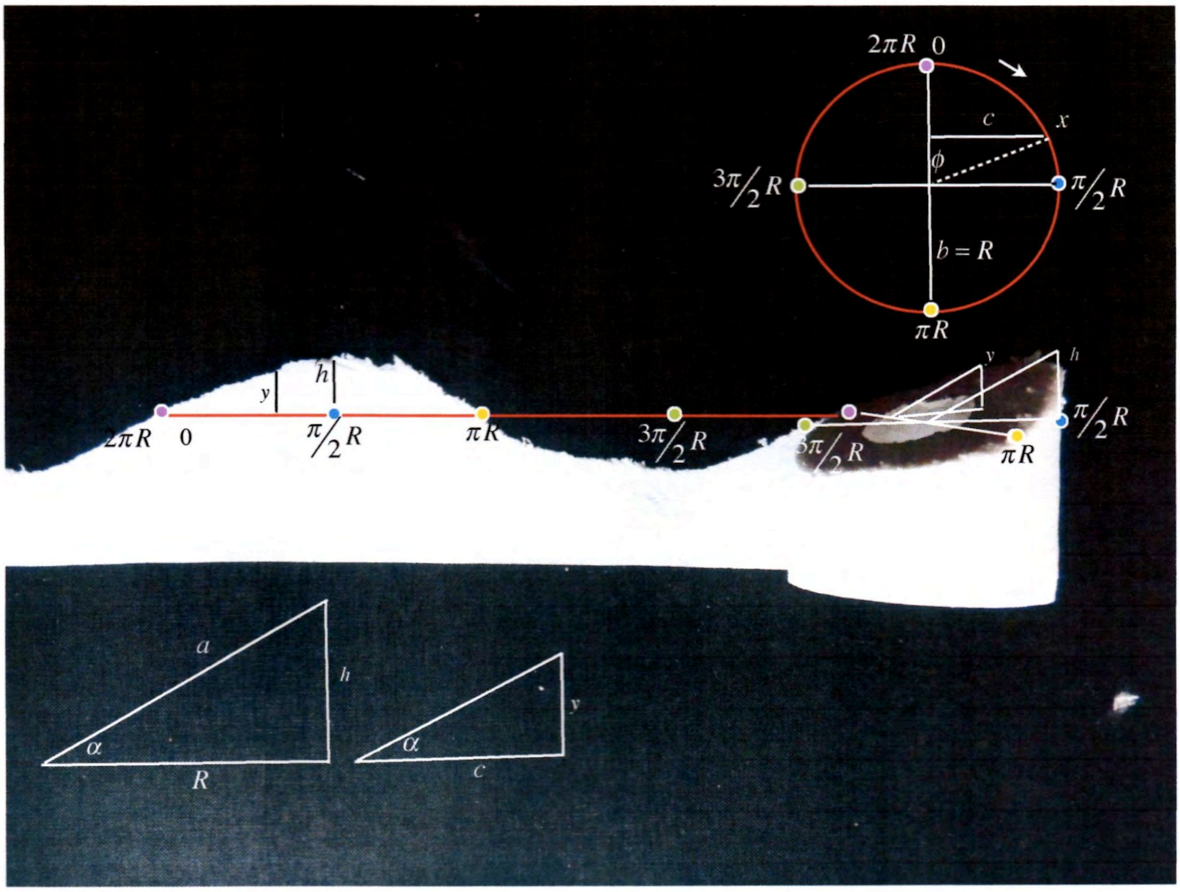








The diagram below superimposes the information from the perspex model (**3 C**) on the paper towel model (**3 A**).



The picture shows the points on one complete wave of the sine curve which correspond to those on the ellipse. The circle above is the ellipse in plan view, corresponding to a straight cut through the roll. Locate on the plan the horizontal sides of the two similar right triangles shown in the perspective view and also below. Their apices lie on the ellipse and the sine curve. Their vertical sides lie in the surface of the cylindrical roll, parallel to its axis, and in the plane of the sine curve.

x is measured around the circle. It is equal to the angle φ in radians multiplied by the radius, R. In order to show that the ragged edge of the towel roll is indeed a sine curve we need to find y in terms of x. To use our similar triangles we need these three equations:

$\varphi = \frac{x}{R}$. $c = R\sin\phi$. $\frac{y}{h} = \frac{c}{R}$ i.e. $y = \frac{hc}{R}$.   So …

$$y = \frac{hc}{R} = \frac{h}{R} R\sin\frac{x}{R} = h\sin\frac{x}{R}\text{, i.e. } y = h\sin\frac{x}{R}.$$