Extension activities

These investigations are prompted by activities at the Circus stations named. Relevant items from other parts of this website are listed.

Station	Tables Race
Extension 1	Counting products on the multiplication square
Level	Upper KS 2/Grades 4-5

Task	How many numbers are there on a multiplication square
	
	1 x 1? 2 x 2? 3 x 3? …?
	
	x 1 x 1 2 x 1 2 3
	1 1 1 1 2 1 1 2 3
	   2 2 4 2 2 4 6
	      3 3 6 9
	
	1 3 6 …?

Comment	The ‘9 x 9’ case is interesting because of Extension 2.
Extension 2	Jon Millington’s Problem
Level	Accessible at KS 2-3/Grades 5-6 but several factors must be considered together, making this a hard problem for anybody.
Task	There are 36 products on the 9 x 9 multiplication square. Assign them to the faces of 9 tetrahedra so that the blocks can be arranged to spell out each times table from 1 to 9.
Help on this site	*Magic Manual*: 1.3.2 *Virtual Circus*: Multiplication: Tables Race: Information

Station	Squaring Up and Down, Cubing Up and Down
Extension	Scale factor, area factor, volume factor
Level	KS 4/Grades 9-10
Task	When you double the edge length of a similar 2-D shape, what happens to the area? What happens to the volume of a 3-D shape? Find formulae for the area factor and the volume factor when the scale factor is k .
Help on this site	*Magic Manual*: 2.3.1, 2.3.3

Station	Polyhedral Packing
Extension	Similar solids
Level	KS 4/Grades 9-11
Task	Pack the tetrahedral hopper 1 layer deep; contents: 1 tetrahedron. Pack it 2 layers deep; contents: 4 tetrahedra, 1 octahedron. When you scale up an edge of a solid x2, what happens to its volume? If the volume of a tetrahedron is t and that of an octahedron, o, How many t make an o?
Help on this site	*Magic Manual*: 4.3; 11.3, 11.4.1, 11.4.2 *Heuristics*: p.22

Station	Shadow-making, Perspective Drawing, Slide Show Tricks
Extension	Projections
Level	KS 3-4/Grades 8-9
Task	The shadow of a square traffic sign cast by the sun has both pairs of opposite sides parallel. Study the shadow of a square cast by an overhead projector on a screen: set the square at different angles to the edge of the projector; set the projector at different angles to the screen.
Help on this site	*Magic Manual*: 2.4, 2.6, 2.9 *Virtual Circus*: Transformations: Leonardo Screen, Summary *Heuristics*: p. 11

Station	Mirror Symmetry, The Magic Mirror Cube
Extension	Reflections in planes, lines and points
Level	KS 3/Grades 6-8
Task	Look into a plane mirror. What happens to your face? Look into the junction where 2 plane, perpendicular mirrors meet. What happens to your face? Look into the corner where 3 plane, mutually perpendicular mirrors meet. What happens to your face?
Help on this site	*Magic Manual*: 3.1.1, 3.1.3

Station	Squaring Up and Down, Symmetrical Dissections – 2D: regular hexagons, 3 from 1, Symmetrical Dissections – 2D: regular hexagrams, 3 from 1, Tangram Polygons: squares, 2 from 1
Extension 1	Irrational scale factors
Level	KS 4/Grades 9-10
Task	A square enlarged by scale factor $k = 2$ gives a square which can be dissected into $k^2 = 4$ squares the original size. Likewise some other polygon. But what must the scale factor be to yield 2 squares (say) the original size? What must the scale factor be to yield 3 regular hexagons or hexagrams (say) the original size?
Help on this site	*Magic Manual*: 2.3.1, 3.6.4B, 3.6.6, 5.1.1
Extension 2	Nested squares
Level	KS 4/Grades 9-10
Task	Draw the 3 Tangram triangles to scale on squared paper. How are their edge lengths related? How are their areas related?
Help on this site	*Magic Manual*: 5.1.1 *Heuristics*: p. 13

Station	Polyhedral Packing
Extension	Dihedral angle
Level	KS 3/Grades 6-8
Task	With an ordinary school protractor you can measure the angle between 2 lines. To measure the angle between 2 *planes* (the 'dihedral' angle), as a crystallographer does, you need a protractor which sets itself at right angles to the line joining them. Use a Polydron protractor to find this angle for the regular tetrahedron and octahedron. What do you notice about their sum? How could you have known this just from the way they pack?

Station	Brick Bonds
Extension 1	Brick neighbours
Level	KS 3-4/Grades 8-9
Task	For a wall to be strong, a brick needs to be cemented to as many other bricks as possible. See how different bonds compare.
Help on this site	*Magic Manual*: 4.4
Extension 2	Brick edge ratios
Level	KS 3-4/Grades 8-9
Task	The standard house brick has edges in the ratio 2:3:6. Find out why this makes so many different bonds possible. Experiment with cuboids which have edges in a different ratio, e.g. matchboxes.
Help on this site	*Magic Manual*: 4.4

Station	Pentominoes: The 3 x 20 Rectangle, The Soma Cube
Extension 1	Counting polyominoes
Level	KS 3/Grades 6-8
Task	In how many different ways can you set 2, 3, 4, … squares edge-to-edge? It's no problem to represent the figures: you just need squared paper. But to count the possibilities you need a system. Does your system allow you to distinguish figures which are the same but rotated or flipped?
Help on this site	*Magic Manual*: 5.1.3
Extension 2	Counting polycubes
Level	KS 3/Grades 6-8
Task	Moving into 3 dimensions, the problems are compounded.
Help on this site	*Magic Manual*: 5.2.1.1 *Heuristics*: p. 32

Station	Left and Right
Extension	Chiral molecules
Level	KS 3/Grades 6-8
Task	Using the Molymod kit, taking black for C, white for H, build a model of the methane molecule, CH_4. Using a different colour for each atom of another element, swap them for Hs one at a time. How many did you have to swap before the molecule gained a handedness (became 'chiral')? The thing you put in place of the H ('substituent') can be more than a single atom. Design and build chiral molecules of your own.
Help on this site	*Magic Manual*: 2.1.1, 2.1.2

Station	The Soma Cube: Mirror Images
Extension	Building left- and right-handed Soma models
Level	KS 3/Grades 6-8
Task	The caption to the station talks you through the process.
Help on this site	*Magic Manual*: 2.1.1, 2.1.2, 3.1.1, 5.2.1.2 *Heuristics*: p. 25

Station	Polyhedron-Building; Corners, Faces and Edges
Extension	Counting corners
Level	KS 3-4/Grades 8-9
Task	The smaller the exterior angle, the more vertices a polygon has: what is the relation? What is the analogy if we substitute polyhedra for polygons? What do we have to know in order to apply Euler's formula to the problem?
Help on this site	*Magic Manual*: 6.1.1, 6.1.2, 6.1.3, 6.1.4; 6.3.1, 6.3.2 *Masterclasses*: *Polyhedra*

Station	Map-Colouring Polyhedra
Extension	Counting colours
Level	KS 3/Grades 6-8
Task	The 4-Colour Map Theorem states that no more than 4 colours are needed to colour the faces of a convex polyhedron so that every edge divides faces of different colours. Can you predict the smallest number needed in a particular case?
Help on this site	*Magic Manual*: 6.2.1, 6.2.2, 6.2.3, 6.2.4 *Heuristics*: p.23

Station	Sections
Extension 1	Sections of a regular tetrahedron
Level	KS 3/Grades 6-8
Task	How would you go about listing all the polygons possible when you slice a regular tetrahedron with a plane?
Extension 2	Sections of a cube
Level	KS 4/Grades 9-11
Task	Test the following conjecture empirically, then try to prove it: 'You can obtain 3-, 4-, 5- and 6-gons by slicing a cube with a plane. But: every 4-gon must have at least 1, every 5-gon must have 2, every 6-gon must have 3 pairs of parallel sides.'
Help on this site	*Magic Manual*: 6.5.1, 6.5.2 *Heuristics*: p.17

Station	Soap Films
Extension	Film rules
Level	KS 3/Grades 6-8
Task	Examine the soap films formed within skeleton polyhedra. You can change them by blowing gently along an edge or popping chosen walls. How many walls meet in an edge? What is special about the angles between the walls? How many edges meet in a vertex? What is special about the angles between the edges?
Help on this site	*Magic Manual*: 3.9.1 *Masterclasses*: Soap Films

Station	Triangle Numbers
Extension	The triangular number formula
Level	KS 4/Grades 9-11
Task	What is the n^{th} triangular number?
Help on this site	*Magic Manual*: 7.1.1, 7.1.2, 7.1.3, 7.1.4, 7.1.5 *Masterclasses*: Number Shapes

<table>
<tr><td>Station</td><td>Mirror Symmetry</td></tr>
<tr><td>Extension 1</td><td>Kaleidoscope images of a point</td></tr>
<tr><td>Level</td><td>KS 3/Grades 6-8</td></tr>
<tr><td>Task</td><td>Investigate how the number of points you see changes as you close up a kaleidoscope.
Use dynamic geometry software or constructions using a ‘mira’ to help you make predictions.

<table>
<tr><td>Angle $\theta°$</td><td></td></tr>
<tr><td>$360°/\theta°$</td><td></td></tr>
<tr><td>Number of reflections, n</td><td></td></tr>
<tr><td>Number of repetitions including the object, $n+1$</td><td></td></tr>
</table>

Then test them with a real kaleidoscope.

When the kaleidoscope is closed up, the mirrors are parallel. Use separate mirrors to see what happens in this situation.</td></tr>
<tr><td>Extension 2</td><td>Kaleidoscope images of a shape</td></tr>
<tr><td>Level</td><td>KS 3/Grades 6-8</td></tr>
<tr><td>Task</td><td>Use the kaleidoscope to study what happens to an asymmetric object of finite size: Are the images always complete? Is there a pattern to the way they face?</td></tr>
<tr><td>Help on this site</td><td>Magic Manual: 3.1.1, 3.1.2</td></tr>
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<table>
<tr><td>Station</td><td>The Fibonacci Slide Rule</td></tr>
<tr><td>Extension</td><td>Other Fibonacci sequences</td></tr>
<tr><td>Level</td><td>KS 3/Grades 6-8</td></tr>
<tr><td>Task</td><td>The classic Fibonacci sequence begins: 1, 1. You continue by adding the preceding pair of numbers to produce 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …

Find the effect of using different starting numbers and different rules.

Here are 3 examples:
• In Edouard Lucas’ sequence you begin: 3, 1, to produce 3, 1, 4, 5, 9, 14, 23, 37, 60, 97, …
• In Richard Padovan’s sequence you begin: 1, 1, 1, but continue by adding the last pair but one, to produce 1, 1, 1, 2, 2, 3, 4, 5, 7, 9, …
• In Mark Feinberg’s ‘Tribonacci sequence’ you begin: 1, 1, 1, and continue by adding the preceding 3 terms to produce 1, 1, 1, 3, 5, 9, 17, 31, 57, 105, …</td></tr>
<tr><td>Help on this site</td><td>Magic Manual: 7.4.1, 7.4.2</td></tr>
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