**Bongard problem: analysis**

In all cases we have one figure with rotation symmetry (the red figure) centred on another (the blue/white figure).

To the **left**, the orders of rotation symmetry are either the same or that of the red figure is a divisor of that of the blue/white figure (but not vice versa).

To the **right**, the orders of symmetry are coprime.

In all cases the circle is divided equally into blue and white regions.

To the **left**, the red figure preserves this division; to the **right** it does not.

Even if the blue : white ratio was different, this ratio would be preserved.

We can formulate a little theorem, we’ll call it the ‘symmetrical division’ theorem:

*If we have a figure with rotation symmetry of order pq divided by two colours in the area ratio a : b, and, centred on this, we have a red figure with rotation symmetry of order p, then, in each red region, the area ratio of the two colours will also be a : b.*

Proof

By the definition of rotation symmetry, each of the *pq* symmetrical regions into which the blue/white figure is divided must be coloured blue/white in the ratio *a* : *b.* Each of the *p* regions into which the red figure is divided contains *q* blue/white regions. Since the

blue/white ratio in each is *a* : *b*, so must it be for each complete red region.