What is a maths lab?

The Magic Mathworks Travelling Circus describes itself as a mobile ‘maths lab’. What do we mean by this?

A lab is a place where you do experiments. In mathematics you don’t do experiments in the sense used in science. In a university the word ‘experiment’ is used to mean testing a mathematical model of a situation against the real thing: you are ignorant of the science, not the mathematics. But the Circus is for learners, people who are ignorant of the mathematics at a level above their own. In it they do experiments in the scientific sense. The difference from what they do in the chemistry, biology or physics lab is that a station in a maths lab is designed to embody a mathematical concept. By doing the experiment they are led from something concrete to the abstraction it represents.

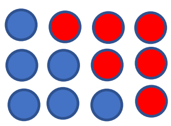
In mathematics there are levels of abstraction. An essential feature of a maths interactive, whether in our collection or that of a permanent hands-on maths museum, is that the experimenter can access the lowest of these levels and derive from the experience (a) a sense of achievement, (b) a feeling of what the psychologists call ‘ownership’.

For those with the necessary mathematics, the activity should suggest a route to the next level up. The term ‘low threshold, high ceiling’ describes activities with this property, whether or not they have a practical starting point.

We can best illustrate these qualities by describing a particular topic station. The topic is ‘Triangle numbers’, the station, ‘Handshakes’. This is a classic problem, which asks, ‘How many handshakes are made when n people meet?’ In the Circus, the people are pegs, the handshakes, rubber bands. By joining two pegs with a band you show one handshake. Sets of 3, 4 and 5 pegs are provided, in the shape of a triangle, square and pentagon respectively. The children can advance from this, totally structured, activity to one with slightly less structure: a dotted circle is provided with a dry-wipe surface. The children put dots on the circumference for a chosen number of people, and connect them with lines for the handshakes.

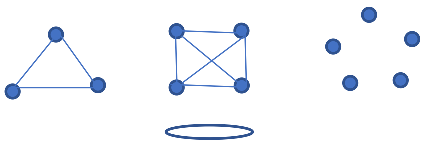
The schematic which follows begins at the top with the basic experiment. As you go down the page, the level of abstraction (and the age of child for which the activity is appropriate) rises. A line divides the page into two. To the left, you find a pattern, indeed a formula, but your findings are purely empirical. To the right, you are doing mathematics. The results you derive to the right explain and justify the findings to the left. To access these results the child must therefore move across the line from left to right.

However, for the child who stays to the left of the line, the fact that the numbers follow the triangle pattern is an important discovery. Armed with this procedural understanding, s/he is excited to realise that s/he can calculate the number of handshakes for any number of people, simply by adding the successive natural numbers. A corollary to the ‘handshakes number theorem’, $h_n = \frac{n(n-1)}{2}$, is indeed the formula for the $(n-1)th$ triangle number. Moving from right to left across the centre line, the child can make this model:



This summarises the whole activity in a single image.

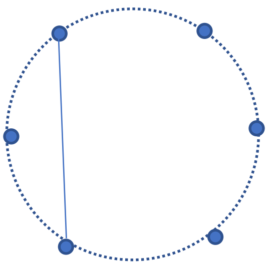
Year 5 (Grade 4)



Pegs and rubber bands

The empirical, inductive approach: maths as science

People	Handshakes	Differences
1	0	
		1
2	1	
		2
3	3	
		3
4	6	
		4
5	10	
		5
6	15	



Lines drawn on dry-wipe surface. Tally mark made for each one erased.

Predict . Test.

Explanation provided.

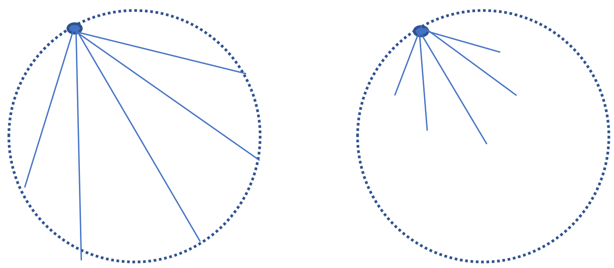
Finite differences:

n	h_n	Δ_0	Δ_1	Δ_2	$h_n = an^2 + bn + c$	
1	0				$a + b + c$	
		1			$3a + b$	
2	1		1		$4a + 2b + c$	$2a$
		2		0	$5a + b$	
3	3				$9a + 3b + c$	

$a = ½$, $b = -1/2$, $c = 0$ $\qquad h_n = \frac{n(n-1)}{2}$

Justification provided.

The theoretical, deductive approach



Physical model becomes mathematical model: the complete graph on n points. But two interpretations still possible:

Person 6 adds (6 - 1) lines.

Person n adds (n - 1) lines.

Recursive formula:

$h_n = h_{n-1} + (n-1)$

Each of the 6 people contributes (6 - 1) ½-lines.

Each of n people adds (n - 1) ½-lines.

$h_n = \frac{n(n-1)}{2}$

Convert recursive formula into explicit formula by generalising part of difference table:

$h_{n-1} = a(n-1)^2 + b(n-1) + c$

$\Delta = n - 1$

$h_n = an^2 + bn + c$

$h_n - h_{n-1} = n - 1$

Substitute, equate coefficients of like powers. Result follows.

Year 13 (Grade 12)