TOPPLE-PRINTING FOR BEGINNERS

Actually, we're all beginners, because I've only just made up the name. But I didn't make up the technique. This was the invention of a man who had half a dozen such ideas before breakfast. His book was last reprinted in 1983 but is out of print at the moment. Ask your librarian to request a copy from the National Book Bank:

Mathematical Snapshots ~ Hugo Steinhaus, published by Oxford University Press (3rd edition)

First: PRINTING. To PRINT you have an original and make copies from it. I cut a potato in half, carve a polar bear in relief, dip it in ink and reproduce as many polar bears as the planet will stand.

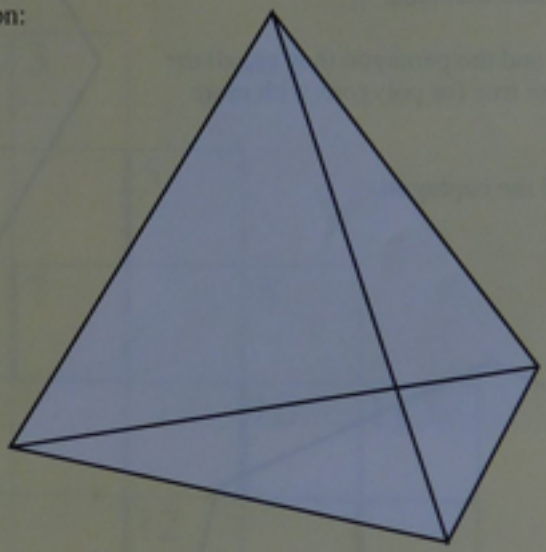
Second: TOPPLE. To TOPPLE is simply to fall over, but our definition is tighter. The thing which topples is a polyhedron. It topples when, having sat on one face it rotates about an edge on the table and lands on another face.

Finally: TOPPLE-PRINTING: To TOPPLE-PRINT you paint the faces of a polyhedron sitting on your paper and roll it around so that it keeps TOPPLING & PRINTING. Perhaps you've made a solid by folding a net; here you perform the 'inverse' operation – you make a net by unfolding the solid. But you don't stop there. A net is finite. It's as if you kept refolding and unfolding the solid, keeping a different face stuck to the paper each time, till you had an infinite collage of nets, matching up perfectly in shape and colour to produce a tessellation.

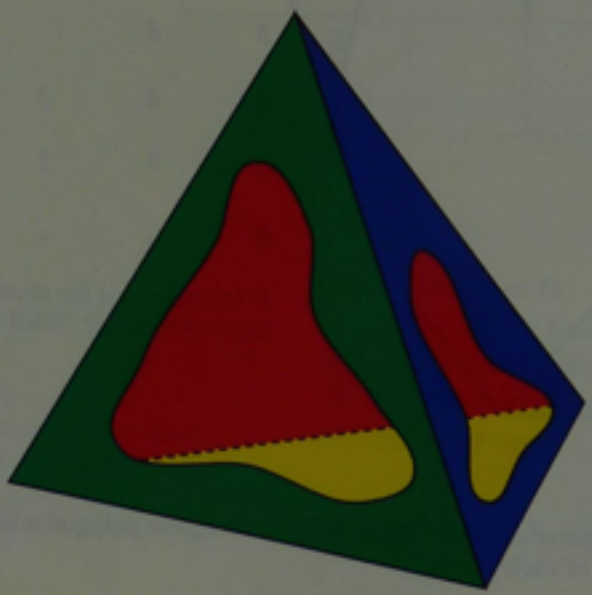
I have to admit that topple-printing is a Very Messy Business. To make a potato print you hold the uninked bit; in topple-printing you have inked all the polyhedron and to roll it is, necessarily, to get inked. I shall spare you this.

Build the polyhedron from Framework Polydron. 'Develop the print' from ordinary Polydron. If you topple the polyhedron and it lands on a red square face, add a red square in that position to your Polydron mat.

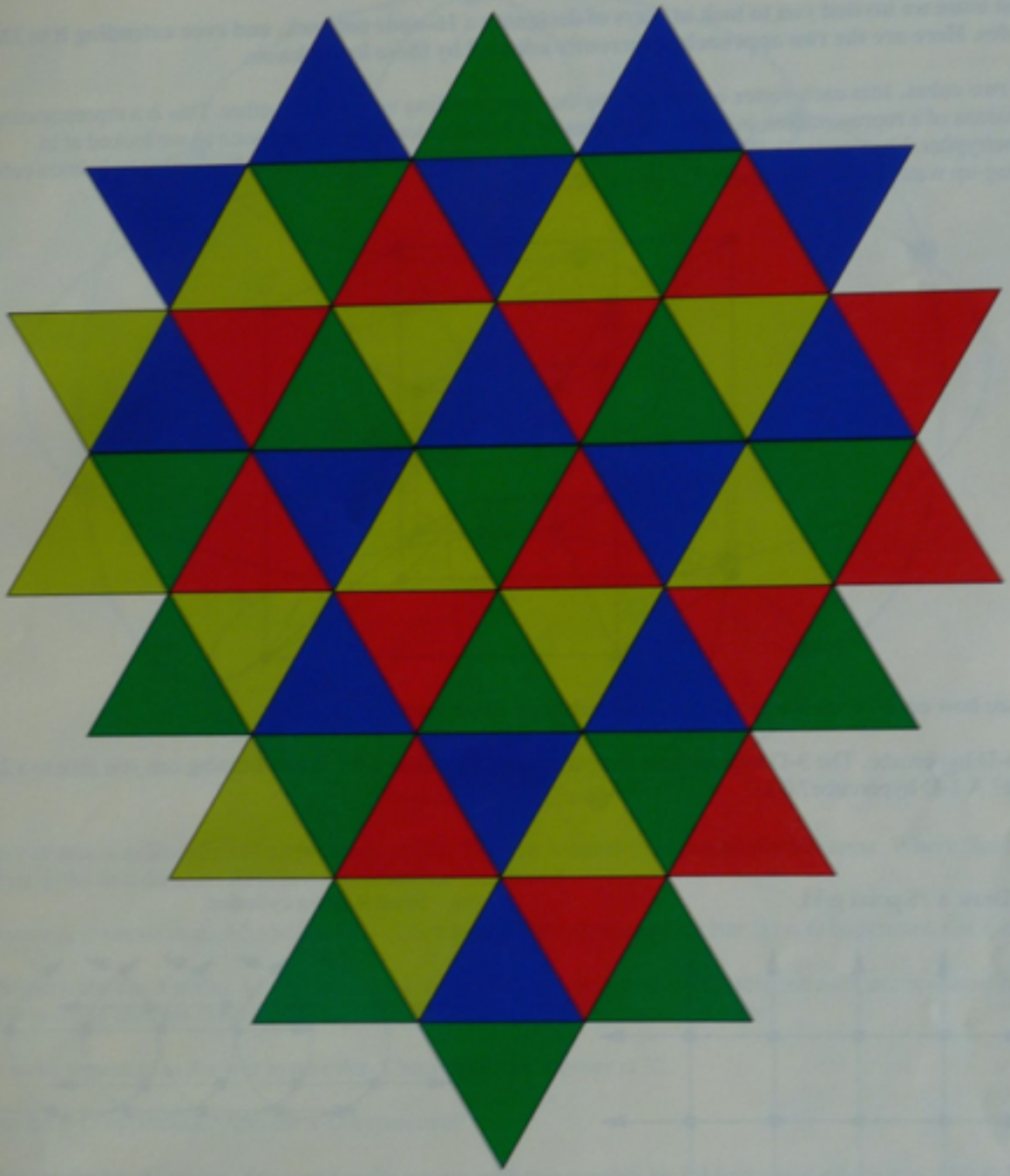
Choose for your polyhedron the regular tetrahedron:



Choose for your colour scheme a different colour for each face, i.e. the tetrahedron is 'map-coloured': faces sharing an edge are of different colours.



I hope you'll be pleased with your print:



How many different patterns can you spot?

The Steinhaus topple-print has 4 colours. I offer you a conjecture that of all solids the regular tetrahedron yields the only topple-print of more than two colours.

Look into the whole matter. Consider angles round a vertex on the paper and on the solid. It will be useful if you have to hand a good atlas of polyhedra, e.g. chapter 3 of *Mathematical Models – H.M. Cundy & A.P. Rollett, publ. Oxford/Tarquin (2nd edition)*. Likewise tessellations. The same book gives you the regular and semiregular ones, but the next work is more of an atlas and also gives you the demisemiregular tessellations:

Tessellations, the Geometry of Pattern – S. Bezuszka, M. Kenny & L. Silvey, publ. Creative Publications.

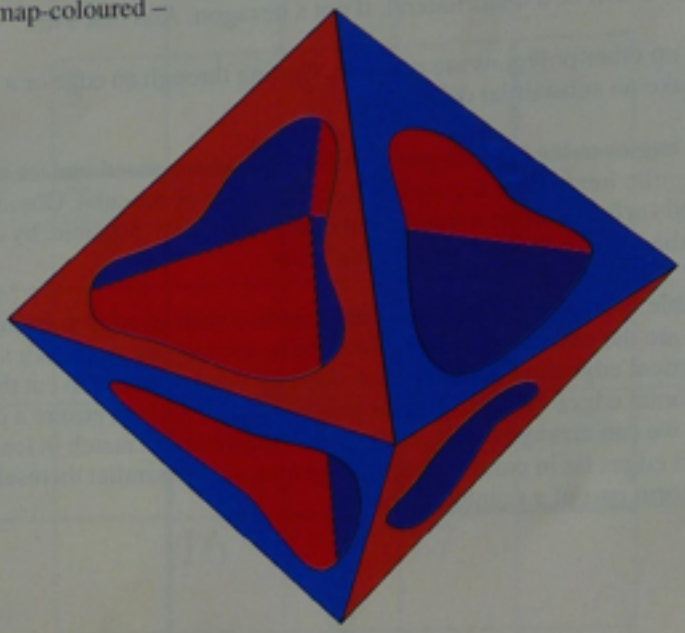
I'm going to challenge you to find two solids which yield 2-colour prints. One, like the regular tetrahedron, can be rolled freely; the other must be directed.

Paul Stephenson
The Magic Mathworks
Travelling Circus

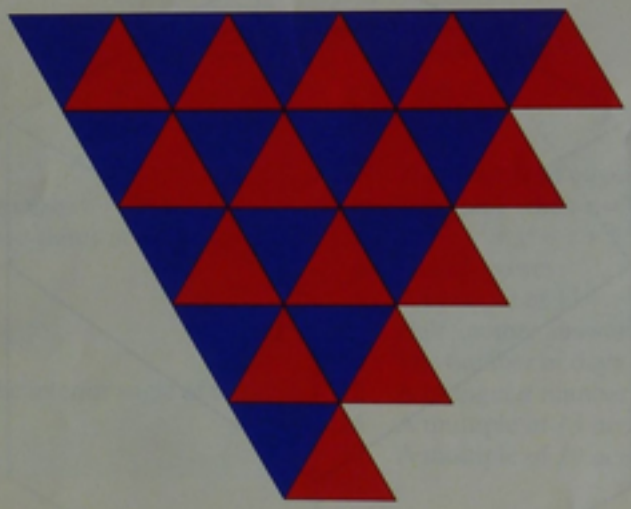
TOPPLE-PRINTING

In our last issue we asked you to find two solids which yielded 2-colour prints. Here they are.

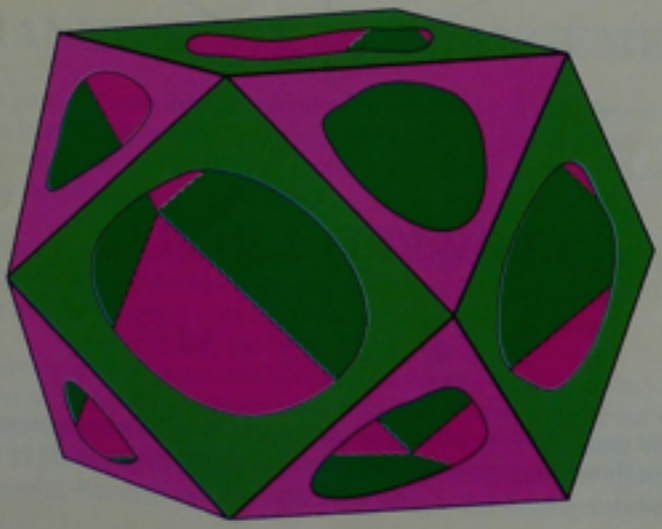
1. The regular octahedron, map-coloured –



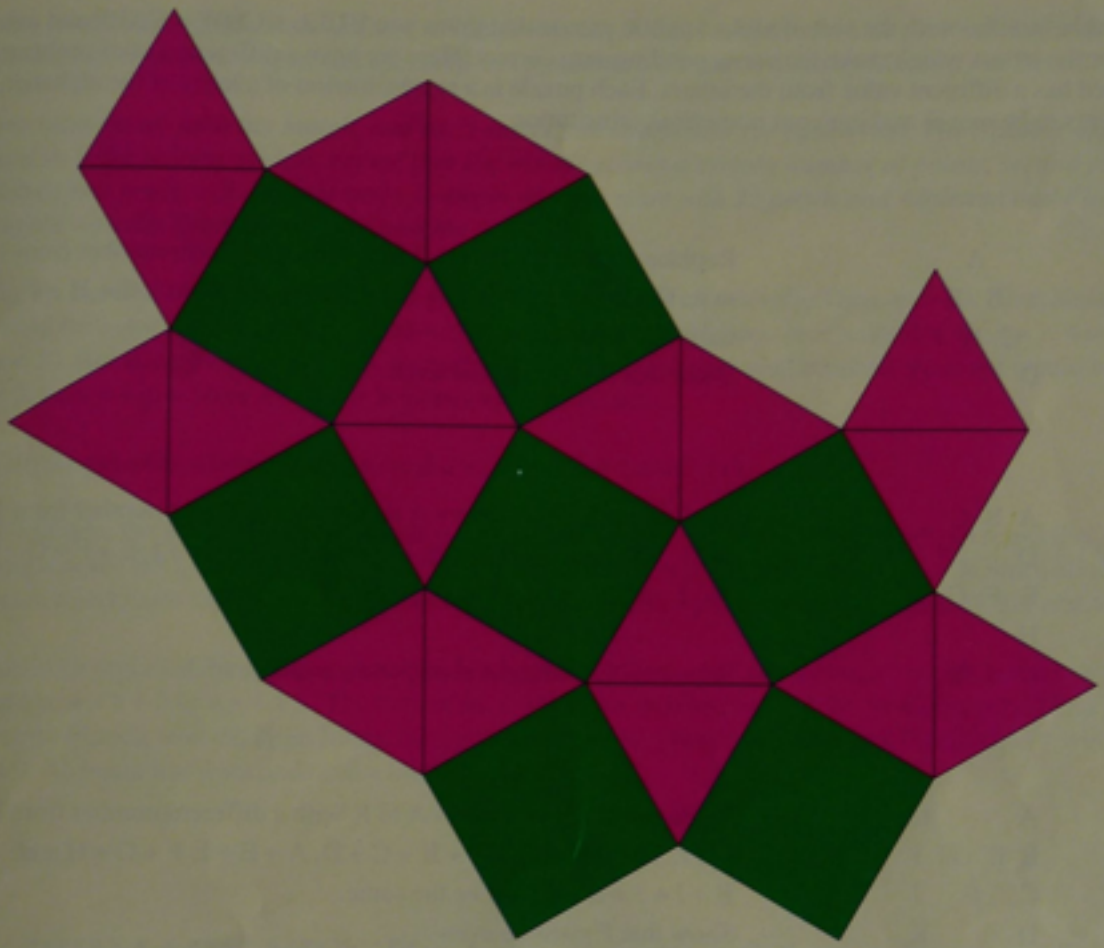
yields a corresponding tessellation:



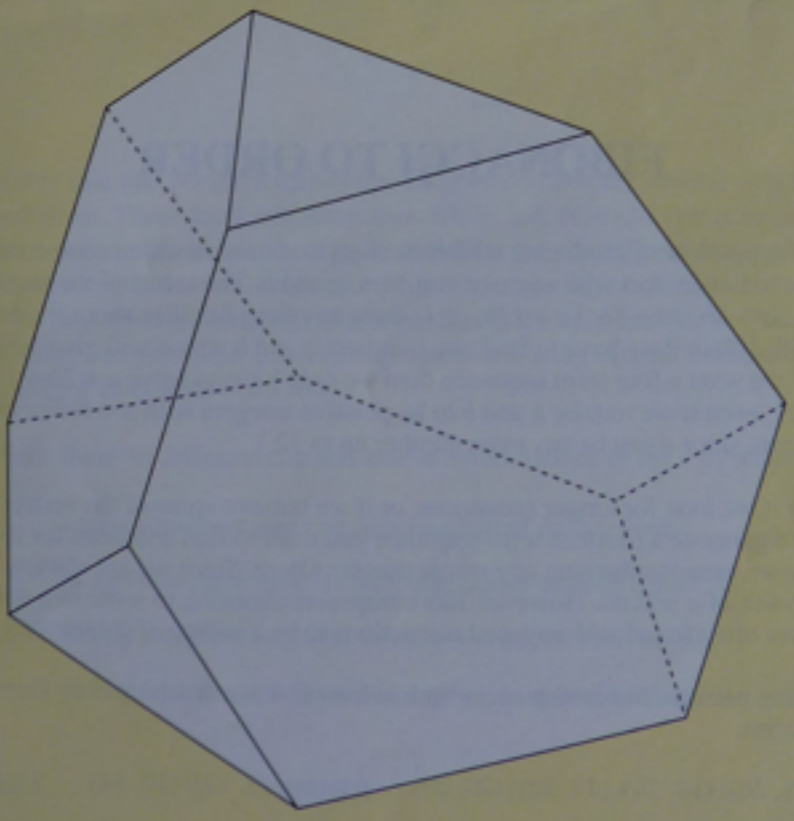
2. The cuboctahedron, map-coloured –



colour-codes triangles and squares in the semi-regular tessellation $3.4.3^2.4$. But it cannot be rolled freely: every shared triangle side is a "stop" line.



Investigate also the curious case of the truncated tetrahedron.

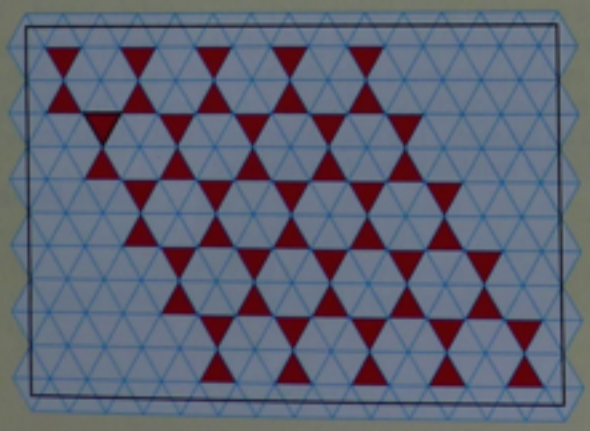


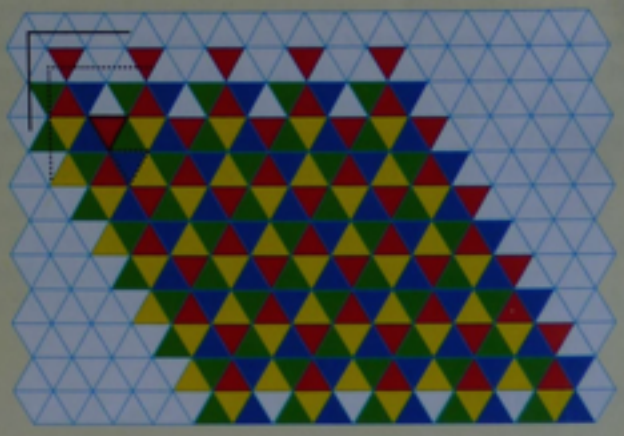
You can manage three colours, but only by cheating: one is the colour of your paper, revealed in triangles the solid cannot reach.

Paul Stephenson, *The Magic Mathworks*

More on Topple-Printing

In a previous issue* I mentioned a beautiful property of the regular tetrahedron: you put a different colour on each face and tumble it about edges on a sheet of paper, and it prints a 4-colour tiling without smudging.
If only one face were coloured, you would get the tessellation on the left, 3.6.3.6 – can you see how the naming works?
The 4-colour tiling is what you'd see if you made 4 acetates, one in each colour, and overlapped them so that none of the white background showed through:



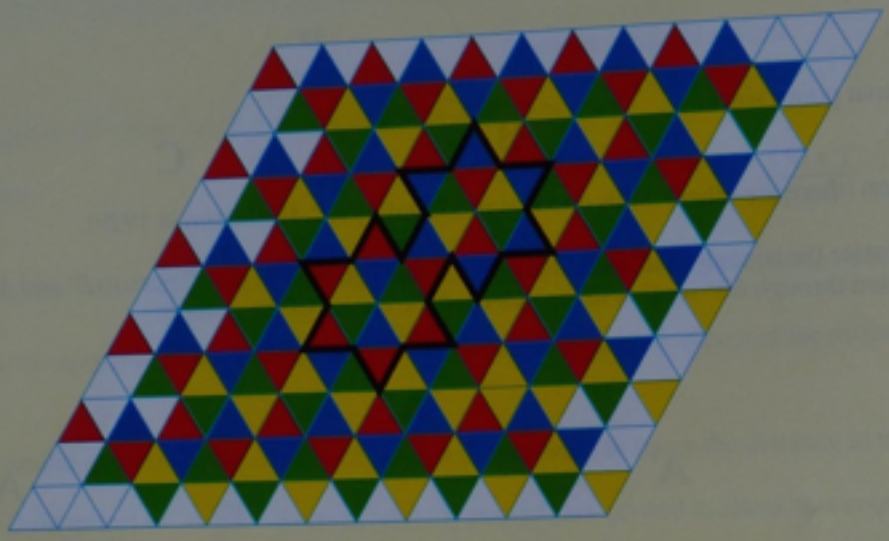


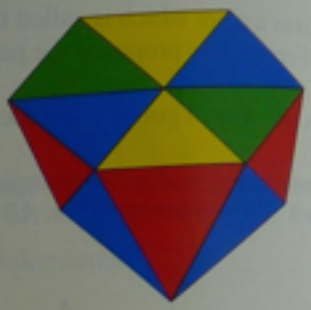
(It works too for a tetrahedron whose faces are scalene, but congruent, triangles)
* In issue No 5 Spring 1998 ….

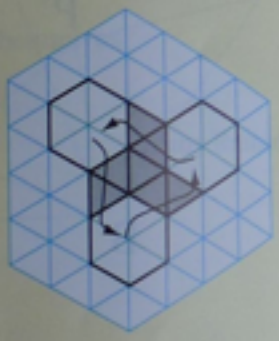
continued on page 4

More on Topple-Printing – *continued*

In that article I suggested a less messy alternative: you use Framework Polydron for your toppling tetrahedron and lay down ordinary Polydron pieces for the print itself. Unfortunately, because of its very nature, Polydron – in either form – invites you to fold it into 3-D shapes; this is what people do when I put out the activity. Three year 8 (Klasse 7) girls at Gymnasium am Stadpark, Krefeld, in the Ruhr- Alexandra, Christine and Fiona – also did so, but found in the print nets for hexagonal antiprisms:







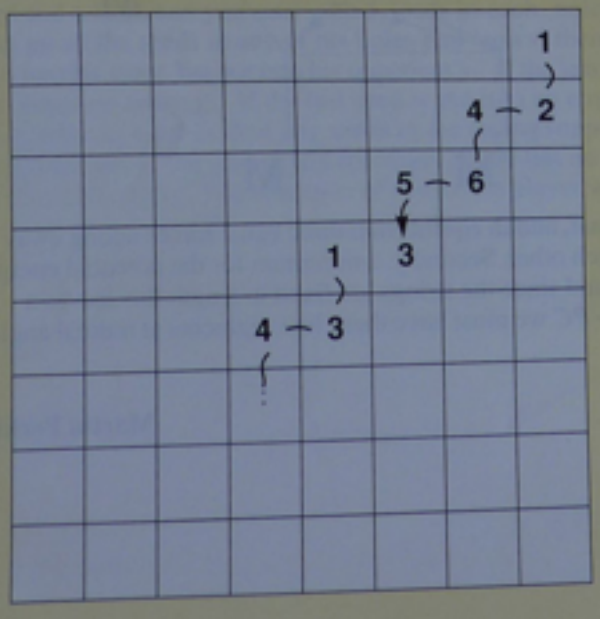
Does this solid itself 'topple-print'? No, but if you use one colour for the triangles and another for the hexagons, it prints the tessellation you see here, in which hexagons are completely separated by triangles – can you name it? (This tiling exists in mirror image forms; I show one only.) However, the routes the toppling solid can follow are restricted: there are STOP signs. The central triangle is a cul-de-sac: you can enter it, but must leave the same way. To print it you therefore need to make a small detour from the route shown:

We've run together two questions here, which should really be considered separately. This gives me the opportunity to prepare you more carefully for the challenge I casually threw at you at the end of my last piece on the topic. Take the cube. What prints can you make if:

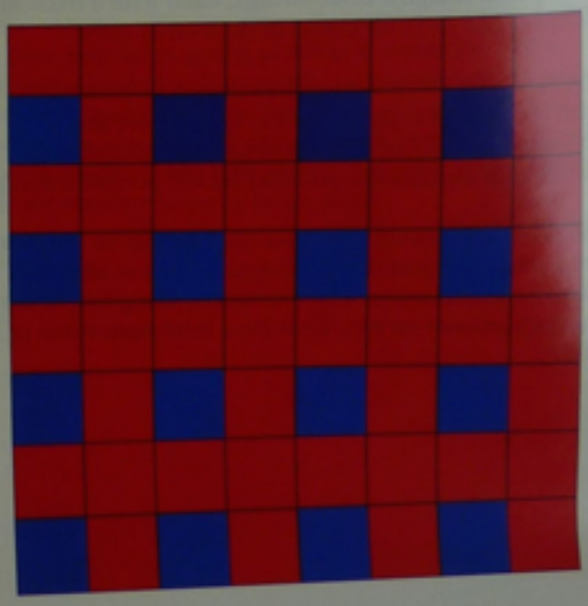
1 you give each face a different colour, but restrict the toppling routes?
2 you give some faces the same colour, but allow the solid to topple freely?

Only then is the time to try combining the restrictions in some way. Look at the diagrams below.

The left-hand diagram I shows a dice rolled under restriction (1). I've given the faces their spot numbers.



The right-hand diagram shows a cube rolled under restriction (2). One pair of opposite sides are blue, the rest red.



See what you can do with the cube by combining (1) and (2). Then try other shapes.

Before I finish, I want to return to the tetrahedron tiling and show you something remarkable about it – and 'show' is all I'm going to do. In the two photographs, the thing on the left is the tetrahedron used, the thing on the right is part of the print, namely that produced by one "topple" about each edge of the base triangle … and this is the *same* print, the print shown at the beginning of this article and in the last.

Imagine, then draw on isometric paper, what a third photograph would show of the print.

Paul Stephenson
The Magic Mathsworks Travelling Circus