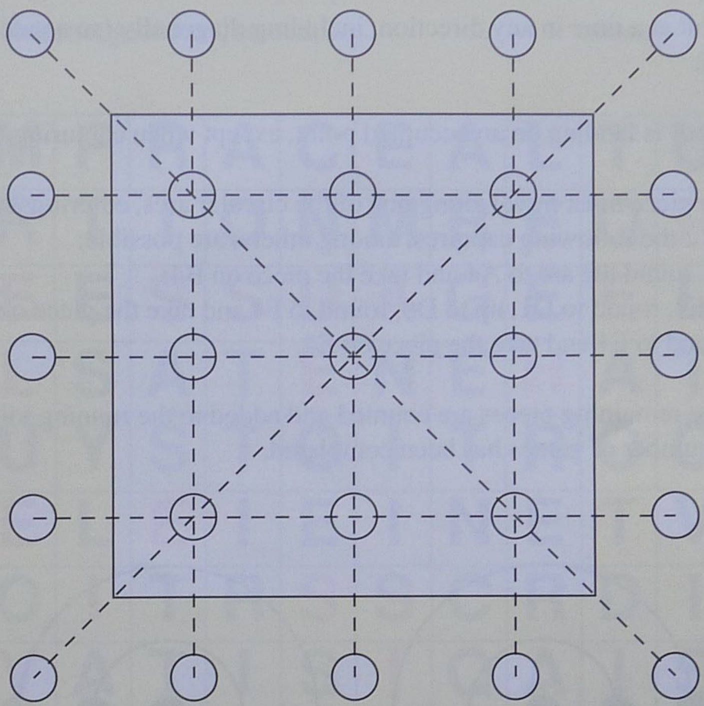
THINKING OUTSIDE THE BOX
Counting Lines on a 3-D ‘0s & Xs’ Board
Part Two

Outside The Box

The mathematician Leo Moser had a brainwave. I’ll illustrate his method on the ordinary 3×3 grid:



He embeds the 3×3 square in a 5×5 one. There are $5^2 - 3^2$ border cells. Each line runs between a unique pair of those border cells (and uses up all of them). The number of possible lines is therefore $\frac{1}{2}(5^2 - 3^2) = 8$ (as you can easily check)

This method generalises, so we embed the $3 \times 3 \times 3$ cube in a $5 \times 5 \times 5$ one. The number of surrounding cells is now, of course, $5^3 - 3^3$, and the answer we seek is therefore $\frac{1}{2}(5^3 - 3^3) = 49$.

Don’t you wish you’d thought of that?

PS

ANSWERS *for this issue*

Twelfths answer: 8 cu.ft. $4' \; 6'' \; 8''' \; 4^{iv} \; 6^{v}$ or 8 cu.ft. $+ 656\frac{3}{8}$ cu.in

How many houses:

(1) There are clearly 32 even numbers up to 64. Adding one to each of the odd numbers gives us the sequence 2, 4, 6, ..., 54 so there are 27 odd-numbered houses. This gives us $32 + 27 = \underline{59}$.

(2) In the general case, with the highest even number a and the highest odd number b, we repeat the same trick to give us $\frac{1}{2}a + \frac{1}{2}(b + 1)$, which is more simply written as $\frac{1}{2}(a + b + 1)$.

(3) The number of houses is just $60 \div 3 = 20$. This time, adding **2** to each of the ground floor accommodation on the other side of the road will give the sequence 3, 6, 9, ..., 75, which of course gives $75 \div 3 = 25$; and adding 1 to anything on the first floor gives the sequence 3, 6, 9, ..., 45, which gives a further 15. In total, then, there will be $20 + 25 + 15 = 60$. The general problem of this type gives $\frac{1}{3}(a + b + c + 3)$.