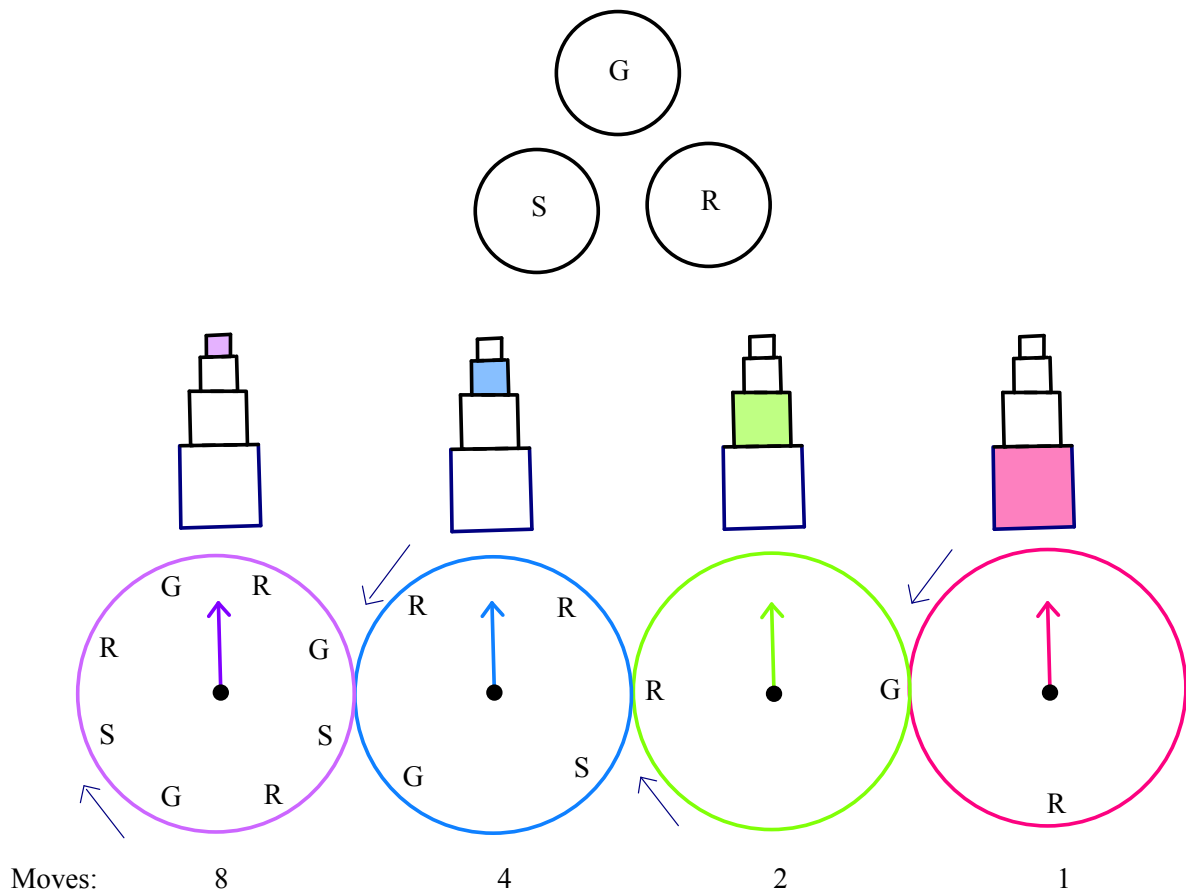


Topic: Sequences

Station: The tower of Hanoi

Moving a tower 4 beakers high with
THE HANOI COMPUTER

The Hanoi computer has 4 meshing gear wheels with arrows mounted on them, one for each beaker in the tower. The wheels are transparent. Through them you see letters printed behind: S (for SAND), G (for GRAVEL), R (for ROCK). When an arrow points to a letter, it tells you the move to make. For example, when the lilac arrow points to R, you must move the lilac beaker to ROCK.

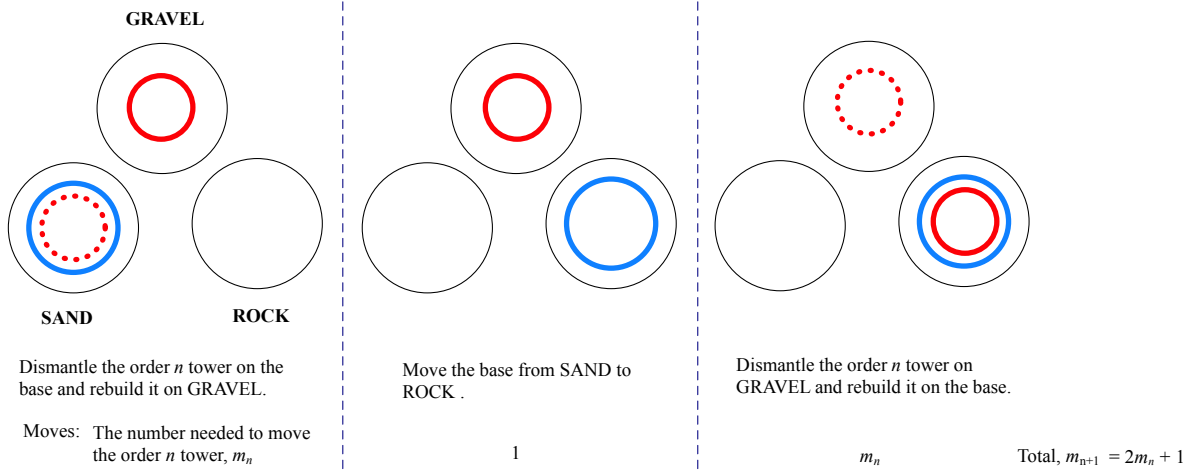


The design displays two properties of the tower: **(1)** alternate beakers move in opposite senses: even-numbered ones clockwise, odd-numbered ones anticlockwise, **(2)** a beaker moves twice as many times as the beaker beneath.

What it doesn't show is the nested (*recursive*) structure of the operation. But you may have realised this when doing the experiment. Every time you build a new tower, you have to dismantle and rebuild an older one. This is shown next.

The computer obtains the total 15 for the number of moves by adding $8 + 4 + 2 + 1$. Below we get it by doubling the last total and adding 1 each time.

How to move an order (n+1) tower



Since $m_0 = 0$, we have the m_n sequence

$$m_1 = 2(0) + 1 = 1,$$

$$m_2 = 2(1) + 1 = 3,$$

$$m_3 = 2(3) + 1 = 7,$$

$$m_4 = 2(7) + 1 = 15,$$

and in general,

$$m_n = 2^n - 1.$$

The representation below shows what happens on the board for each position of the four wheels. It also brings out the nesting.

