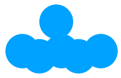
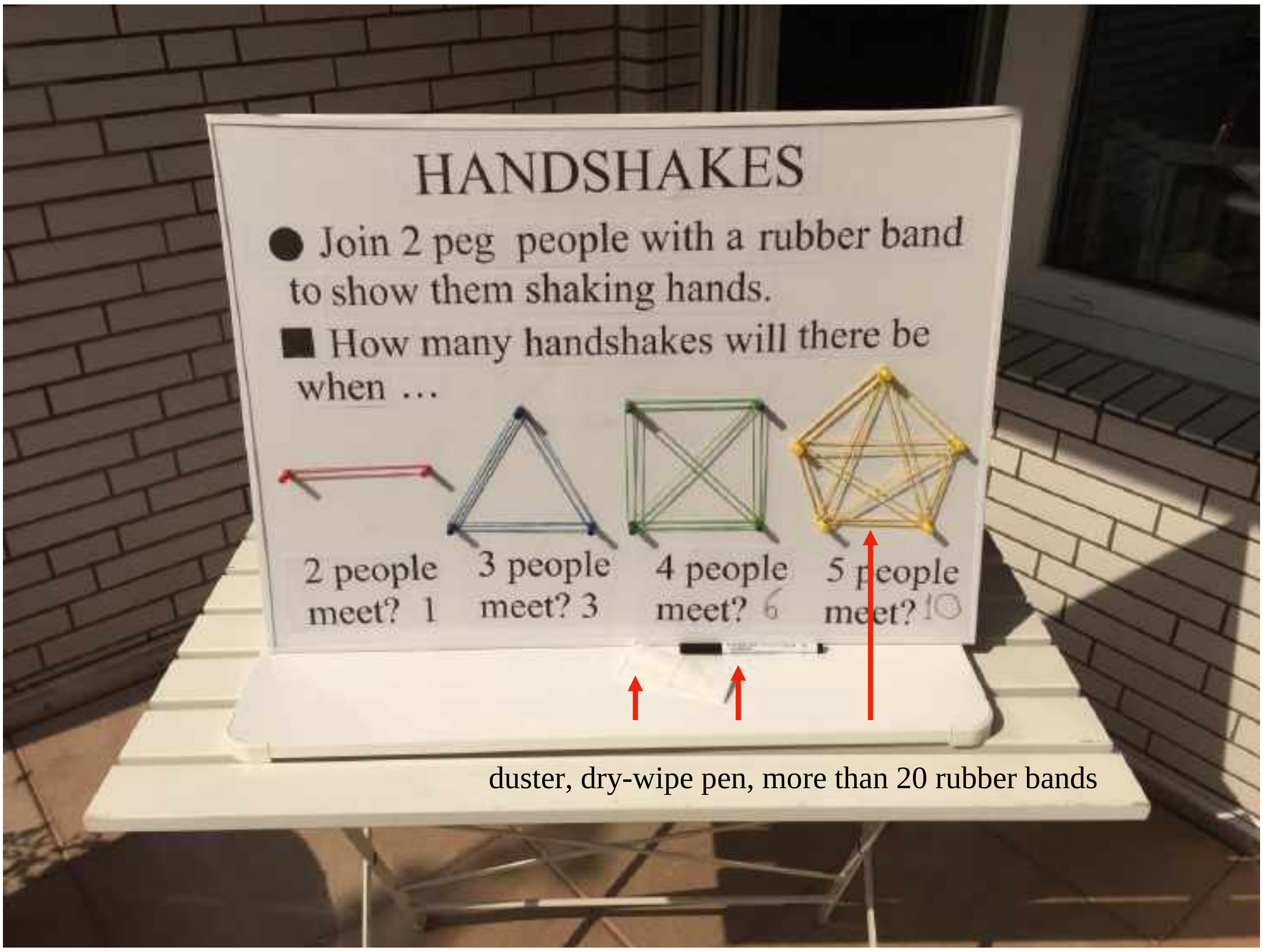
The Purple Set
Manual

Station *Handshakes*

Physical experience Looping rubber bands over pegs

Mental activities Visualising correspondences: person = peg, handshake = rubber band

Explicit content A triangular number as a dot pattern: 



Care Pack facing underside of metal base to protect pegs.

Station *Handshakes*

Physical experience Drawing

Mental activities Visualising correspondences: person = dot, handshake = line

Explicit content The triangular numbers as a sequence whose differences are the natural numbers

Interactivities Go to http://nrich.maths.org/2883 .

dry-wipe pen, duster

Station	*Fibonacci numbers*
Physical experience	Moving a slide rule cursor
Mental activities	Adding pairs of numbers of increasing size
Explicit content	The Fibonacci numbers as a recursive sequence

dry-wipe pen slider duster

Station *Fibonacci numbers*

Possibilities for collaboration

We often observe that, when the child doing the sums gets stuck, the child behind the wedge will try to help, and to do so in a way that adult teachers would envy.

Interactivities

Go to www.magicmathworks.org, then ‘Virtual Circus’, ‘Number Patterns’, ‘Fibonacci Patterns’.

Station *The feely box*

Physical experience Feeling a polycube

Mental activities (for 'B') Visualising a three-dimensional spatial arrangement from a shape explored only by touch. Realising which prepositions - 'opposite', 'over', 'next to', …, are spatially ambiguous. Improvising a coordinate system by which the location of the parts can be communicated.

Explicit content There is no explicit content. The object of the exercise is simply to make the child aware that the task requires precise geometrical language.

Up to a dozen Multilink cubes, Polydron 'Framework' triangles as an alternative

Corresponding pieces the other side

Possibilities for collaboration

If 'C' is not receiving unambiguous instructions, s/he should complain and seek clarification: "When you say 'underneath', do you mean 'on a lower level' or 'towards me'?"

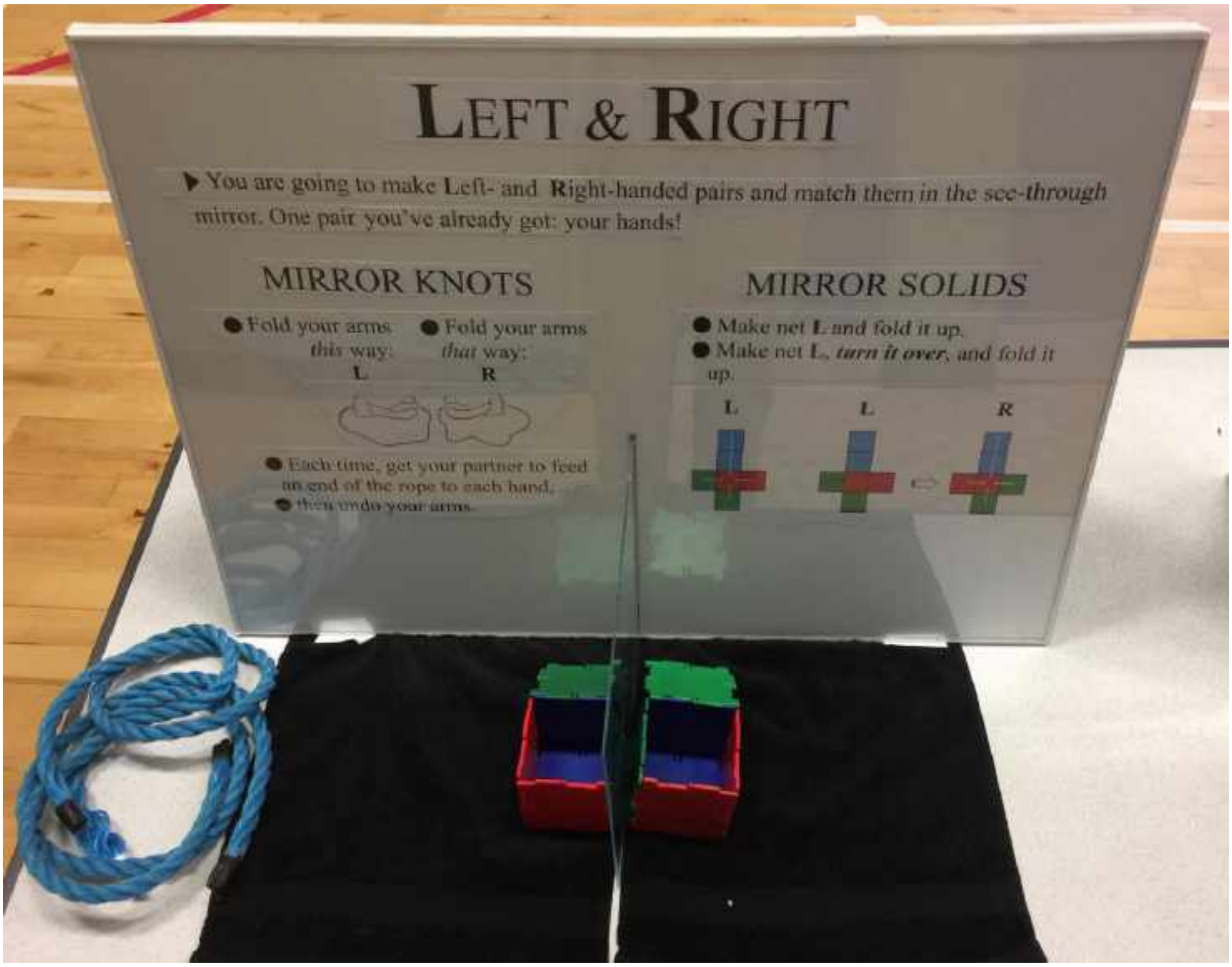
Station *Left & Right*

Physical experience Matching an object and its reflection.

Mental activities In the rope activity the children must remember which way they folded their arms the first time. This knowledge depends on *proprioception*, the awareness of the position of one's body in space.

The mental and physical activities are therefore inseparable.

Explicit content Solids lacking a symmetry plane are *chiral*/non-superimposable. To turn a closed surface inside out (as happens with the cube here)



is to reverse its chirality.

Possibilities for collaboration

With three children, the two who are to make the knots can face each other as if in a mirror and be fed by the third.

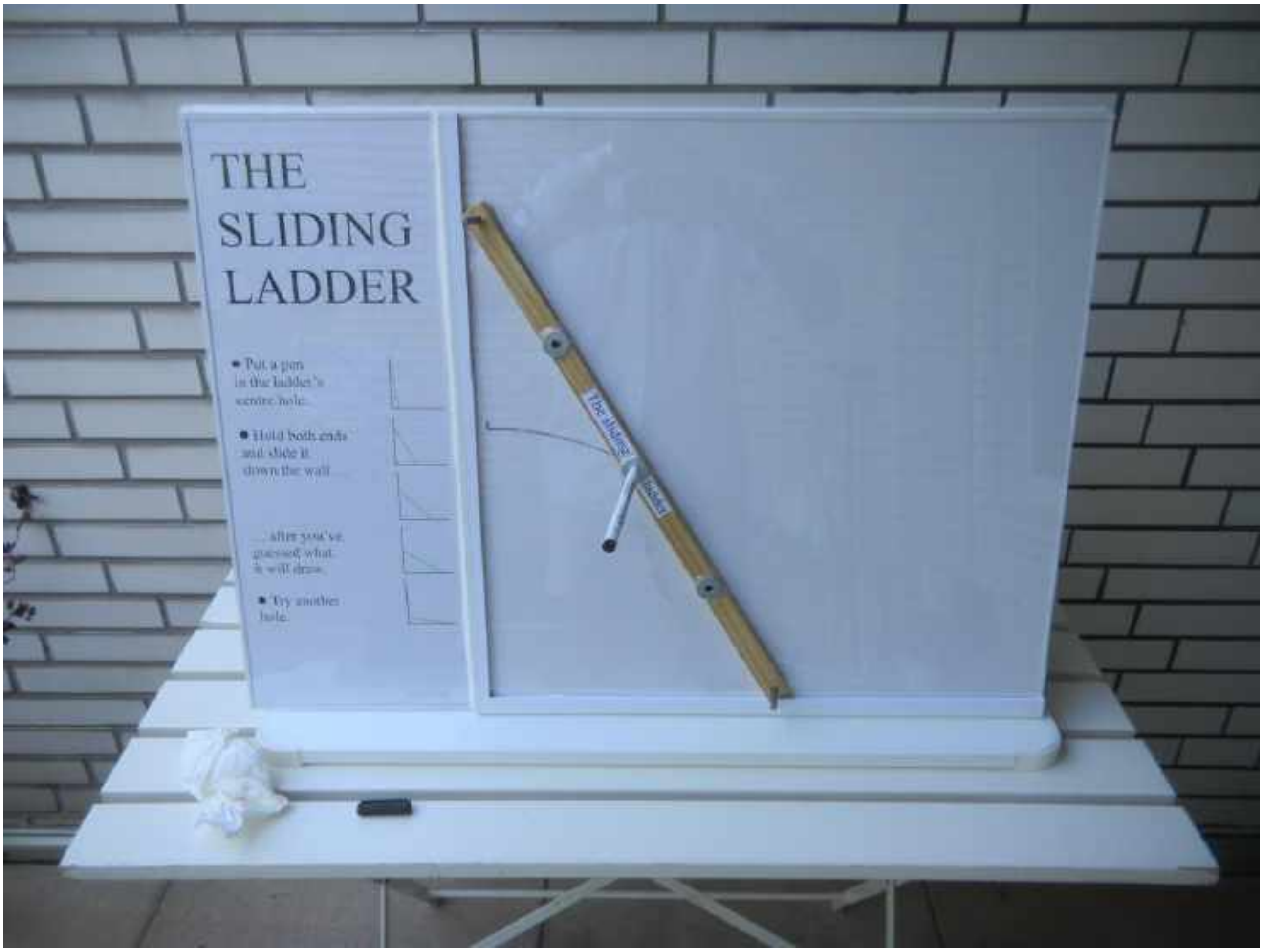
Station *The sliding ladder*

Physical experience The child must move a stick in a right-angled frame, keeping the ends in contact with it.

Mental activities Visualisation of the motion of the stick, concentrating on the position of the mid-point. Where an off-centre point is chosen, the child can assist this visualisation with a consideration of how the reduced symmetry of the stick will dictate the reduced symmetry of the resulting figure.

The locus is counterintuive, perhaps because one imagines lines drawn along the stick, forming a parabola as an envelope which bends the other way. This is itself an instructive curve, activity.

Explicit content Our apparatus represents one quadrant of the Trammel of



Archimedes.

Possibilities for collaboration

The second child can help by applying gentle pressure to the pen.

duster

dry-wipe pen of correct size to fit hole

stick

Station *Tables race*

Physical experience The child must turn a tetrahedral block until its orientation is such that, when placed in a matching hole, it presents to the viewer the correct number in the chosen 'times table'.

Mental activities The successful children realise that every block bears a number they need. Upon finding this number, they can place it straight in the hole to which it belongs. The slower strategy is to seek the numbers in order.

Explicit content Alternative factorisations mean that the same number may occur in several positions on the multiplication square. That the task is possible means that there are just 9 x 4 = 36 distinct products on the 9 x 9 multiplication square.

Interactivities

Go to www.magicmathworks.org , then 'Virtual Circus', 'Multiplication', 'Tables Race'.

Station *The seesaw*

Physical experience The children place hangers on the pegs of a mathematical balance so that they achieve equilibrium.

Mental activities The mathematical balance is a key piece of apparatus in experimental cognitive psychology because two attributes, the choice of a peg and the choice of the number of hangers to place on it, can be varied independently. The finding is that a certain mental maturity must be attained before the child can bear the two in mind simultaneously.

Explicit content ‘The law of the lever’.

Interactivities

Go to www.magicmathworks.org , then ‘Virtual Circus’, ‘Multiplication’, ‘The Seesaw’.

Station *Times chimes*

Physical experience A child rings a bell on each multiple of a chosen number as a count is made – and may attempt to do so with each hand independently.

Mental activities Realising that numbers contribute to their common multiples: that, for example, the factors 3 and 4 occur in all multiples of 12.

Observing that the numbers which sound alone are prime.

Explicit content A positive integer realised as a product of factors.

The musical score used also displays the following property geometrically (in the slanting lines joining squares of the same colour):

If $q - p = 1$,
then $nq - np = n$.

Interactivities

Go to www.magicmathworks.org , then 'Virtual Circus', 'Multiplication', 'Times Chimes'.

Possibilities for collaboration

This is by its nature a collaborative activity. For the younger, less experienced musicians, each child should ring a separate bell. Older, more practiced musicians can try two bells each, one in each hand.

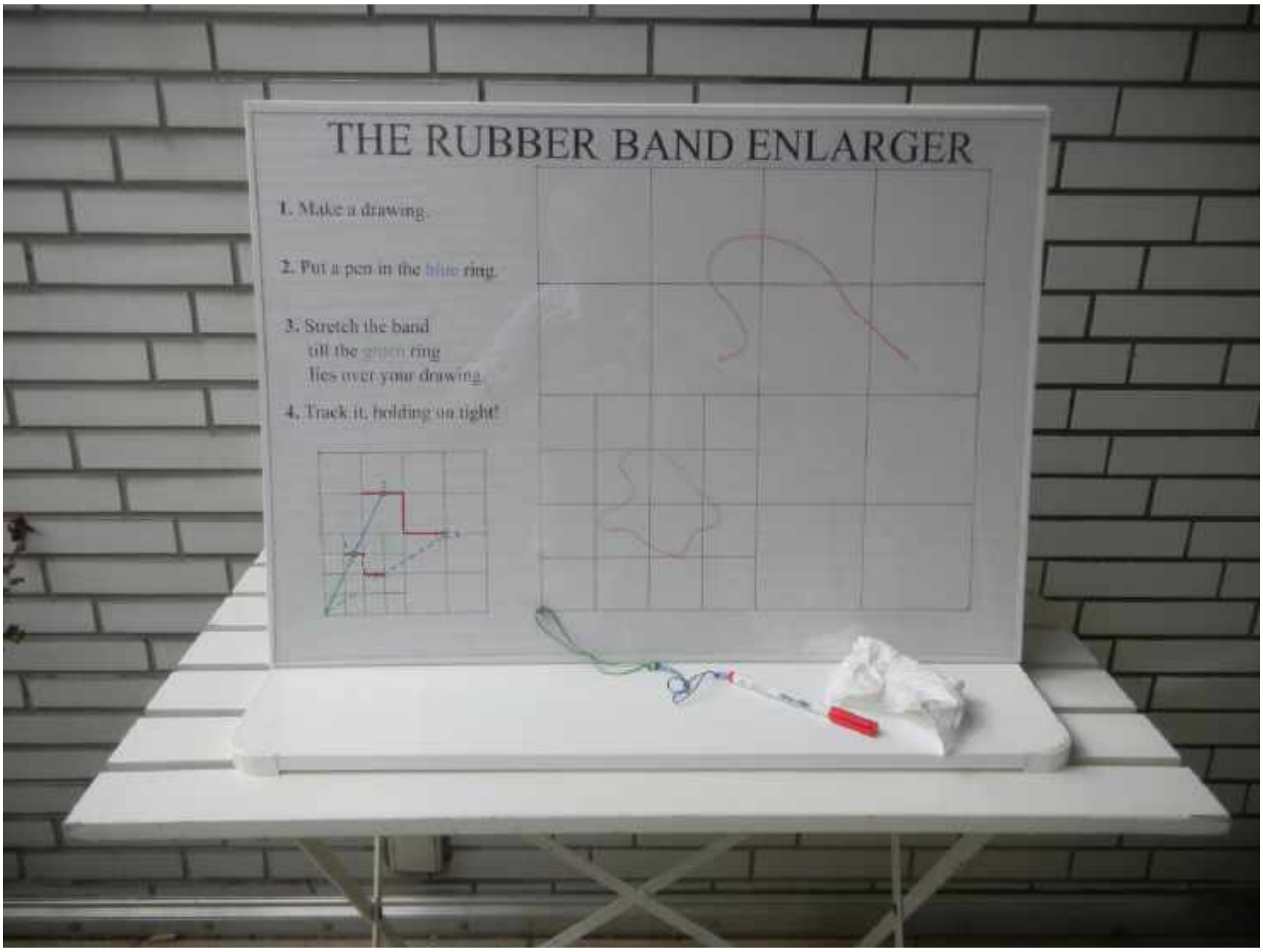
The bells are colour-coded on the musical score. It is only necessary to tap the bells to ring them.

Station *The rubber band enlarger*

Physical experience The child stretches a rubber band of double length from an anchor peg. With the mid-point s/he tracks by eye the outline of a drawing. The pen at the end makes an enlarged copy.

Mental activities Realising that, whatever the properties of the rubber, two bands will stretch twice as far as one, and that the result is therefore an enlargement by scale factor 2. Extrapolating from this to scale factors 3, 4, 5, … or, imagining that the sighting point and pen point are swapped, to fractional enlargements.

Explicit content If, from a given point, radiating lines of length a, b, c, … are drawn, multiplying their lengths by k produces a figure enlarged by that factor.



Interactivities Go to **www.magicmathworks.org** , then ‘Virtual Circus’, ‘Transformations’, ‘Rubber Band Enlarger’.

Station *Spirals*

Physical experience *Archimedean:*

The child wraps a stick round a peg.

Equiangular:
The child moves a stick round a peg according to a rule which ensures the required property.

Mental activities In each case the child should consider why the curve turns out the way it does.

double rubber band, with rings at centre and end

Explicit content

Archimedean:
The radius is directly proportional to the angle through which the stick turns.

Equiangular:
Each segment makes the same angle with the radius.

The Archimedean spiral:

The equiangular spiral:

duster, dry-wipe pen


SPIRALS
The Archimedean spiral
Put a pen in the sticky stick.
Wind it round the hub.
The equiangular spiral
Take the sliding stick.
Lay it along the hub.
Mark the ends of the arrows.
Move the red arrow to where the green arrow was.
Make a mark.
Keep doing that.
Join your marks.

Station *Anamorphs*

Physical experience

The child attempts to draw a letter in such a way that, upon reflection in a mirror, it appears correct. The mirrors are respectively plane and cylindrical. Because s/he watches the reflection form, he has immediate feedback and can correct errors as s/he proceeds.

Mental activities

The child may extrapolate from the two instances here to the thought that different mirrors might produce other transformations. If s/he has used the *Rubber band enlarger*, s/he can entertain the wider thought that many different pieces of apparatus might produce their own transformations.

Explicit content

The geometrical optics predicts that, in a plane mirror, front and back are reversed. In the experimental set-up here, this means a swap of top and bottom. In the case of the cylindrical mirror, there is an additional transformation: parts of the drawing furthest from the mirror and tangential to the curve must be drawn proportionally longer. The upshot is that a square grid can be used in the case of the plane mirror but must be swapped for a polar grid in the case of the cylindrical one.

duster, dry-wipe pen

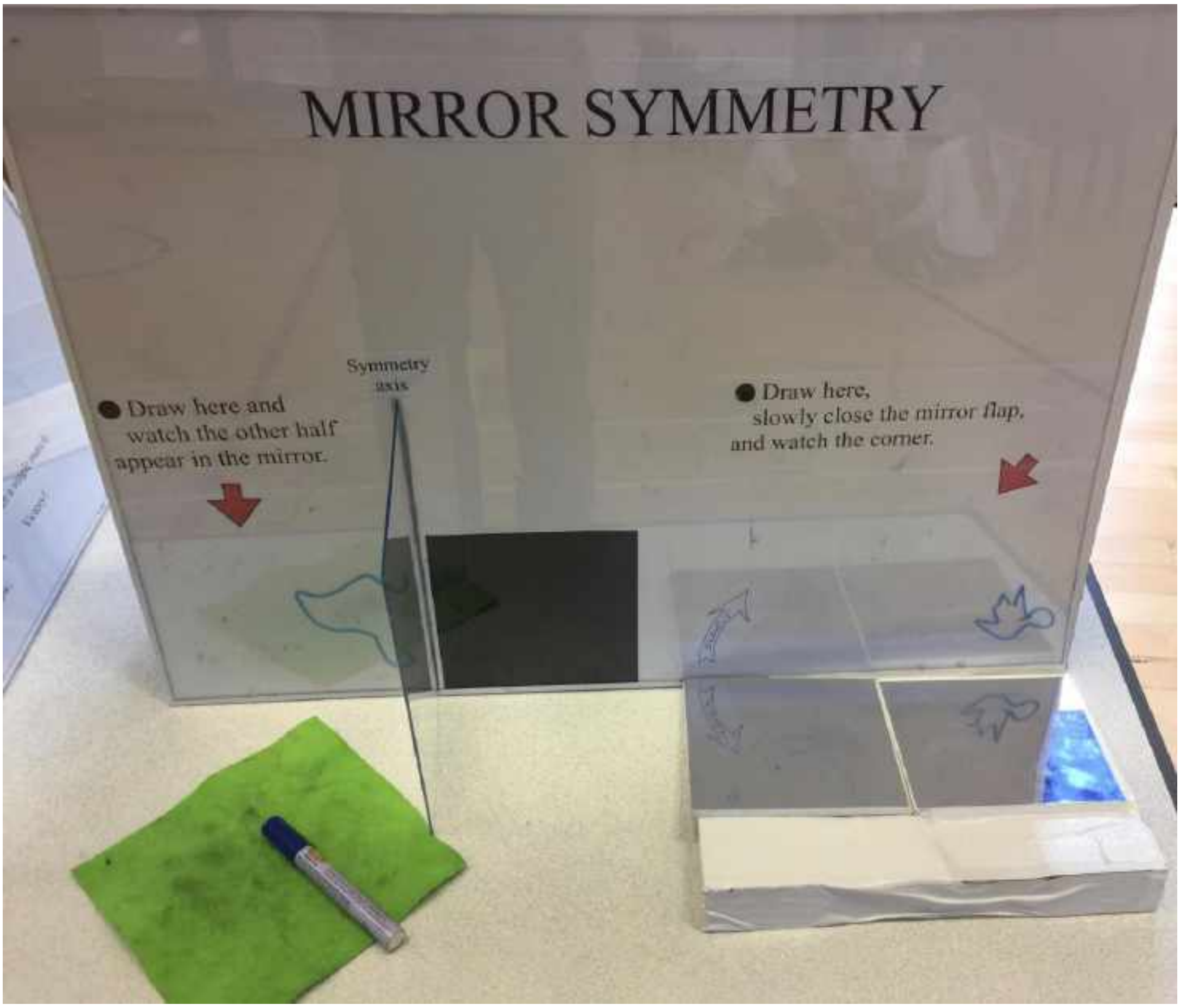
Interactivities Go to www.magicmathworks.org , then ‘Virtual Circus’, ‘Transformations’, ‘Curved Mirror’.

Station *Mirror symmetry*

Physical experience The child draws half a picture and watches the symmetrical half appear. In the kaleidoscope s/he observes how symmetry axes multiply as s/he reduces the angle between the mirrors.

Mental activities The realisation that a 2-dimensional figure may possess more than one symmetry axis.

Explicit content The geometrical optics of single reflections in a mirror and multiple reflections between mirror pairs.



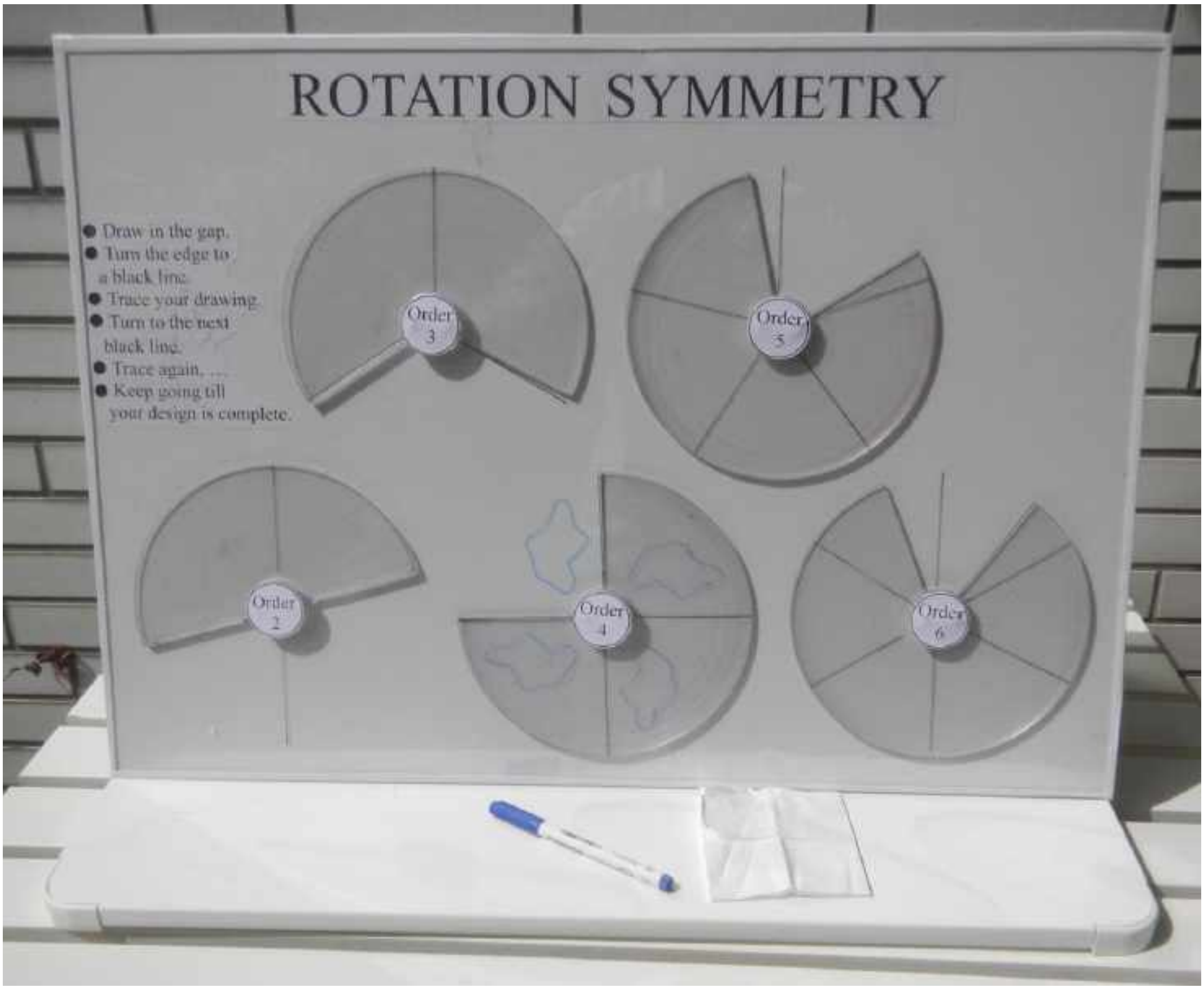
angled support to ensure kaleidoscope mirror flaps are perpendicular to the board

Station *Rotation symmetry*

Physical experience By means of the actions described, the child completes drawings with rotation symmetry of orders 2 through 6.

Mental activities The realisation that a 2-dimensional figure can have rotation symmetry of any order, the limiting case being the circle.

Explicit content If the order of rotation symmetry is k, the figure may be brought into coincidence with itself in k positions.



dry-wipe pen, duster

Station	*Perspective drawing*
Physical experience	With an eye to the sight, the children trace the outline of some chosen object behind the Dürer screen.
Mental activities	Having got over their surprise that a mindless procedure has such an accurate result, the children can think about how the distance
of an	object from the screen affects its apparent size.
Explicit content	Projective geometry. How the picture plane cuts the pyramid of vision, with the consequence observed.

Interactivities	Go to <u>www.magicmathworks.org</u> , then ‘Virtual Circus’, ‘Transformations’, ‘Perspective Drawing’.
Station	*The tower of Hanoi*

Physical experience	A legal move is to take a box from the top of a pile and place it on a larger box or an empty one of the three sites. In fact, given the progress of the top box (clockwise or anticlockwise), the moves are forced.
Mental activities	The fractal structure referred to below may not be apprehended explicitly. Nevertheless, quite young children may be experts, suggesting implicit recognition of an underlying pattern.
Explicit content	There is a fractal structure to the activity whereby the same sequence of moves is repeated on different scales. The number of moves, *m*, is an exponential function of the number of boxes, *b*, viz. $m = 2^b - 1$.

Station *3-D Os & Xs*

Physical experience A player places a ball in a hole with the object of completing a line of three.

Mental activities In the 2-player game, by commanding the centre hole, the first player can force the situation that, on the fourth move, his opponent

can only block one of two lines, ensuring victory on the next move. The important realisation for the game in general is that the

aim should be to create as many potential lines as possible. For the

older children

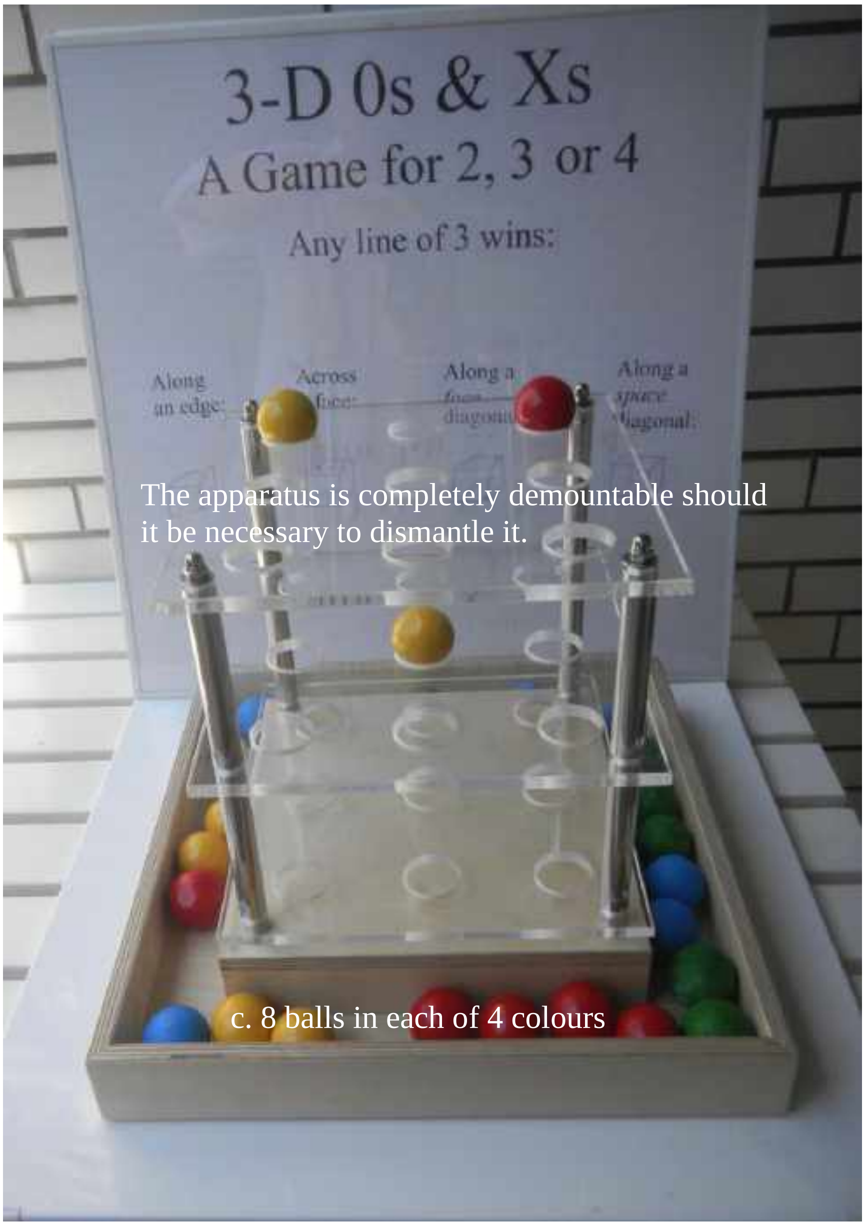
the task on the

back of the board focuses attention on the four

special base

types of site and the number of potential lines through each.

Explicit content The essential geometry of the playing grid, which players are encouraged to master on the back of the main board, is that of the cube itself. Though the strategy for the 2-player game is clear, the varying allegiances possible complicate the 3- and 4-player versions



The apparatus is completely demountable should it be necessary to dismantle it.

c. 8 balls in each of 4 colours

Station *Nim*

Physical experience A move consists in choosing a row and the number of matches to be removed from it.

Mental activities The children must anticipate the result of their move, but, as indicated in the next box, it is equally important to think how they arrived at the current position.

Explicit content The game can be analysed in terms of the binary numbers represented by the rows, but even sixth formers find the analysis difficult. The strategies suggested on the back of the main board encourage
the players to think backwards from a potentially winning
situation.

duster, dry-wipe pen

Station *Number-building: the triangle family*

Physical experience The base grid forces a close packing of the balls, which the child follows in building and extending figurate expressions of the
numbers under investigation.

Mental activities If the children have made the *Handshakes* investigation, they may recognise the triangular numbers. More hidden is the fact that
6 of these plus 1 comprise a centred hexagonal number.
The main realisation is that figurate numbers match an
arithmetical (or rather algebraic) pattern with a geometric one.

Explicit content The following number types may be realised:

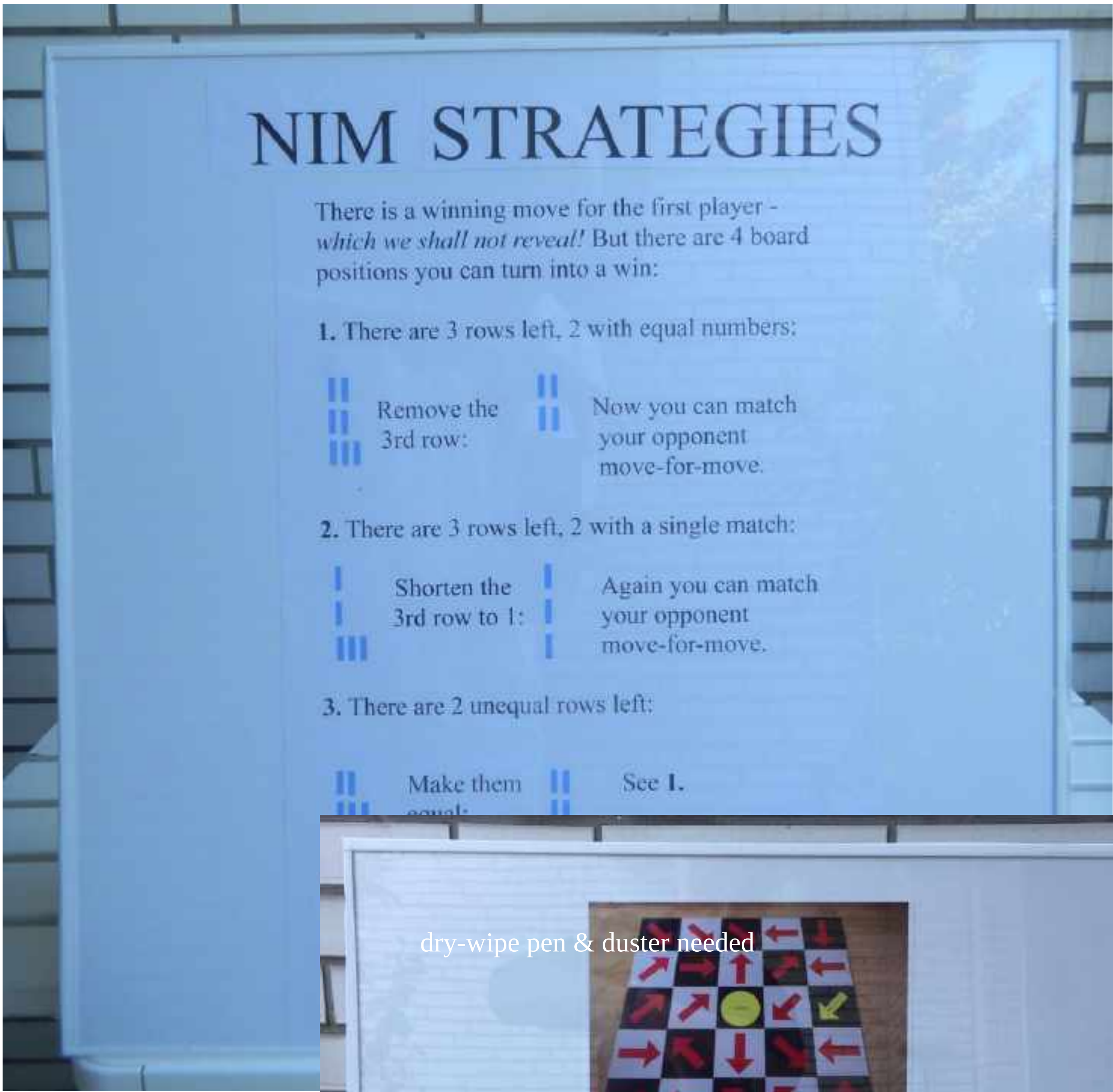
Triangular
Square (= rhombic)

Tetrahedral
Pyramidal (= skew pyramidal)
Centred hexagonal

and 

their

interrelations.



dry-wipe pen & duster needed

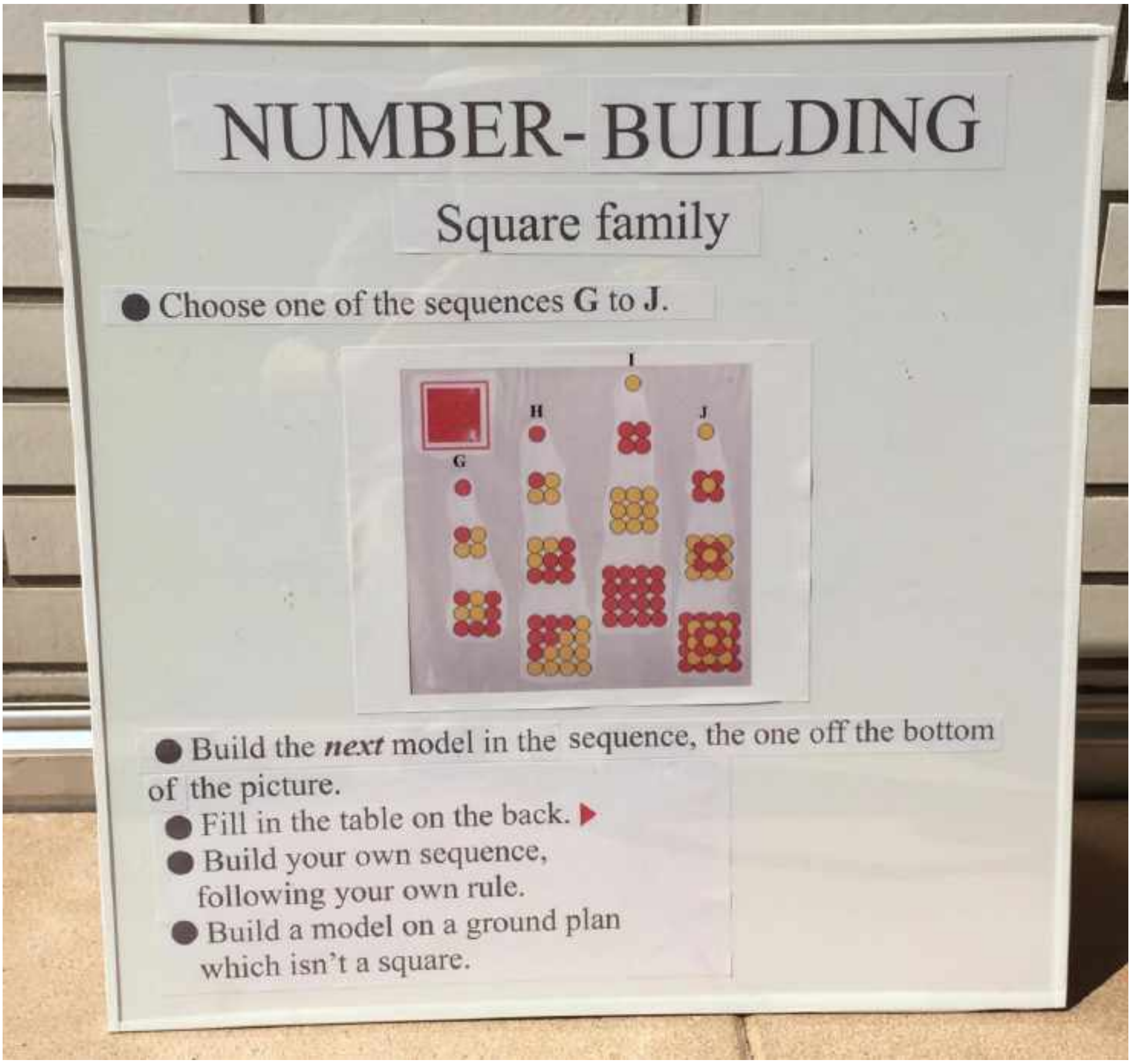
Station

Number-building: the square family

Physical experience Again, there is a close packing but the orientation differs from that on the blue board so that, for example, the sloping face of a pyramid on the red board corresponds to the base of a tetrahedron on the blue board.

Mental activities Sequences **G** and **H** are the important patterns to think about. In the second case the children find that two consecutive triangular numbers form a square. By suitable colouring they can also show that two consecutive tetrahedral numbers make a pyramid.

Explicit content From the above remark we realise that the relations here are not new. However, some are clearer in the ‘blue’ orientation of the layers; some, in the ‘red’ orientation.



At both the above stations this table occurs on the back:

This is the suggested layout for the combined station:

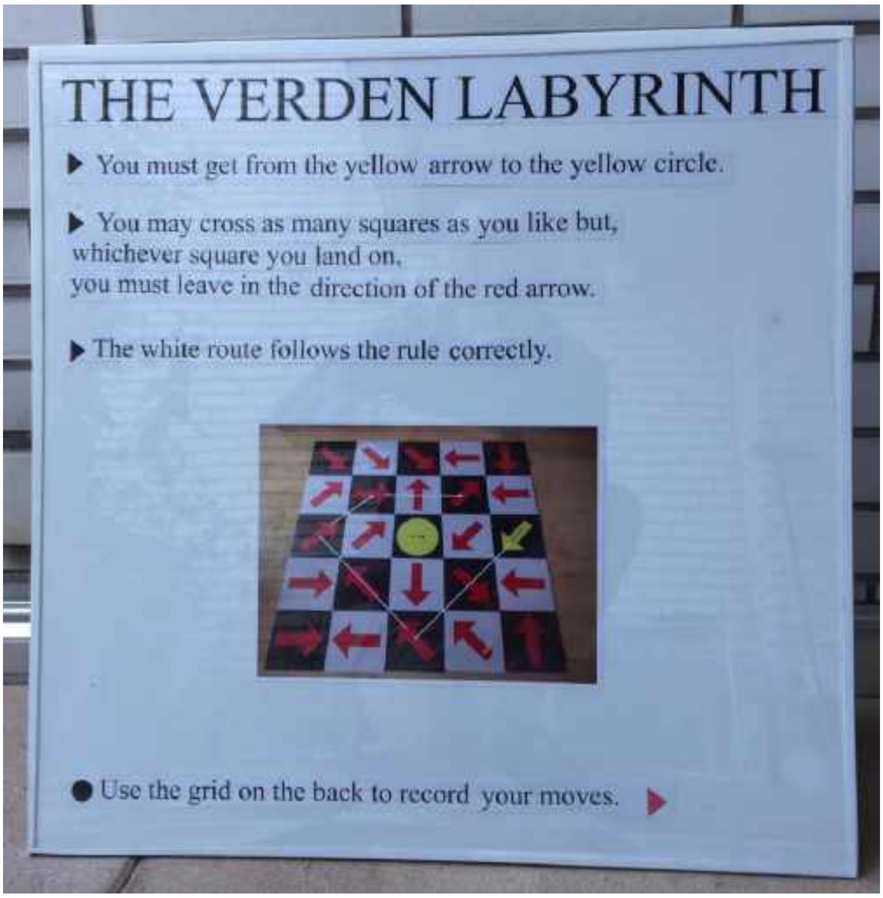
Station *The Verden labyrinth*

Physical experience A legal move is to follow the arrow on the tile on which you land from a previous one.

Mental activities Children struggling to find a solution should be encouraged to work backwards, a heuristic advocated for *Nim* above.

Explicit content How many steps you take on a given move dictates the direction you have to take next. This is not quite like a *maze*, where, at a decision point, one has the choice of two directions. In fact the solution follows a unique route, making this a *labyrinth*, of which that at Cnossos is the original.


THE VERDEN LABYRINTH
ENDE
ANFANG



Station *Safe queens*

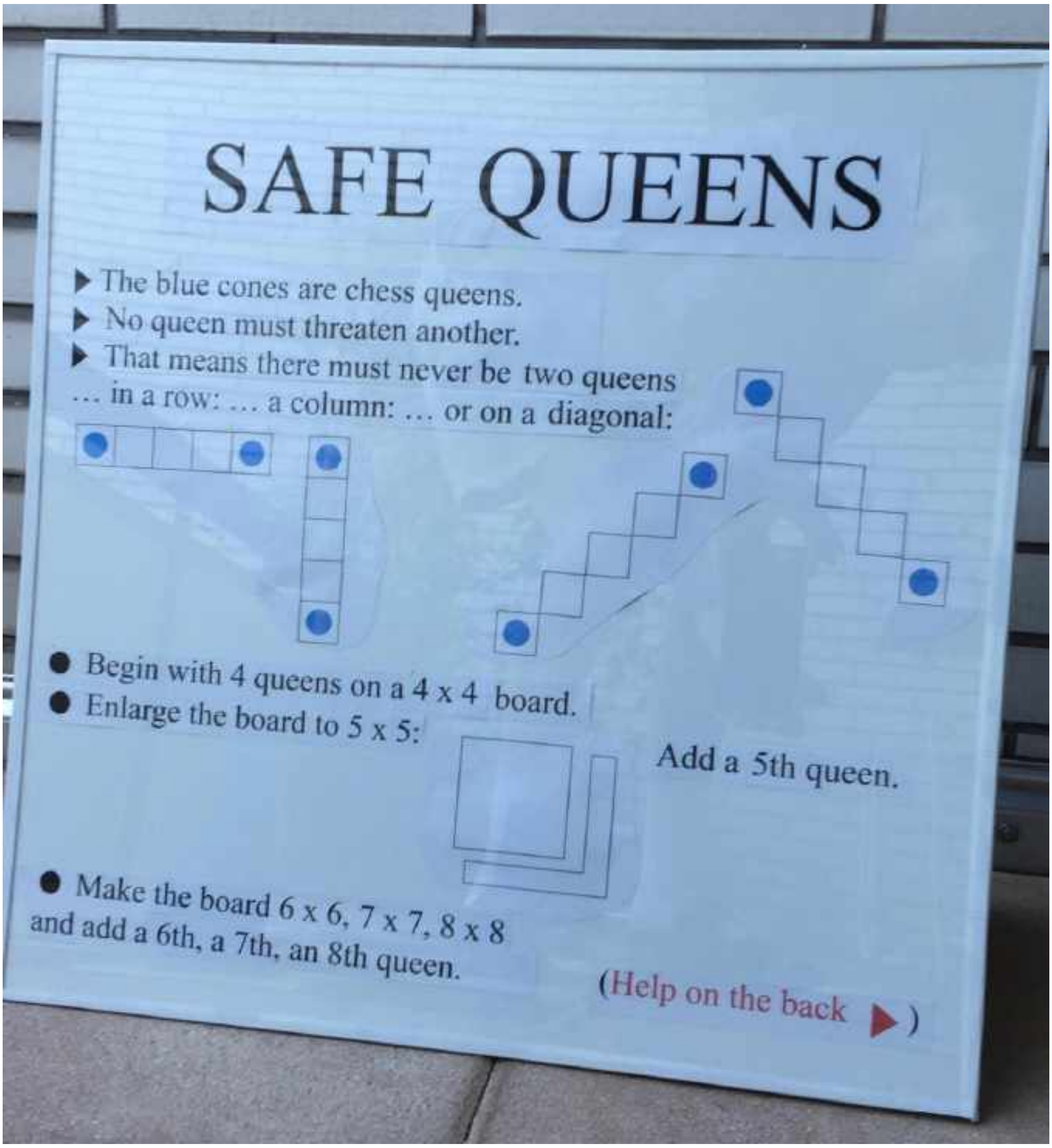
Physical experience Placing the cones. In any line - vertical, horizontal or diagonal - there must be no more than one queen.

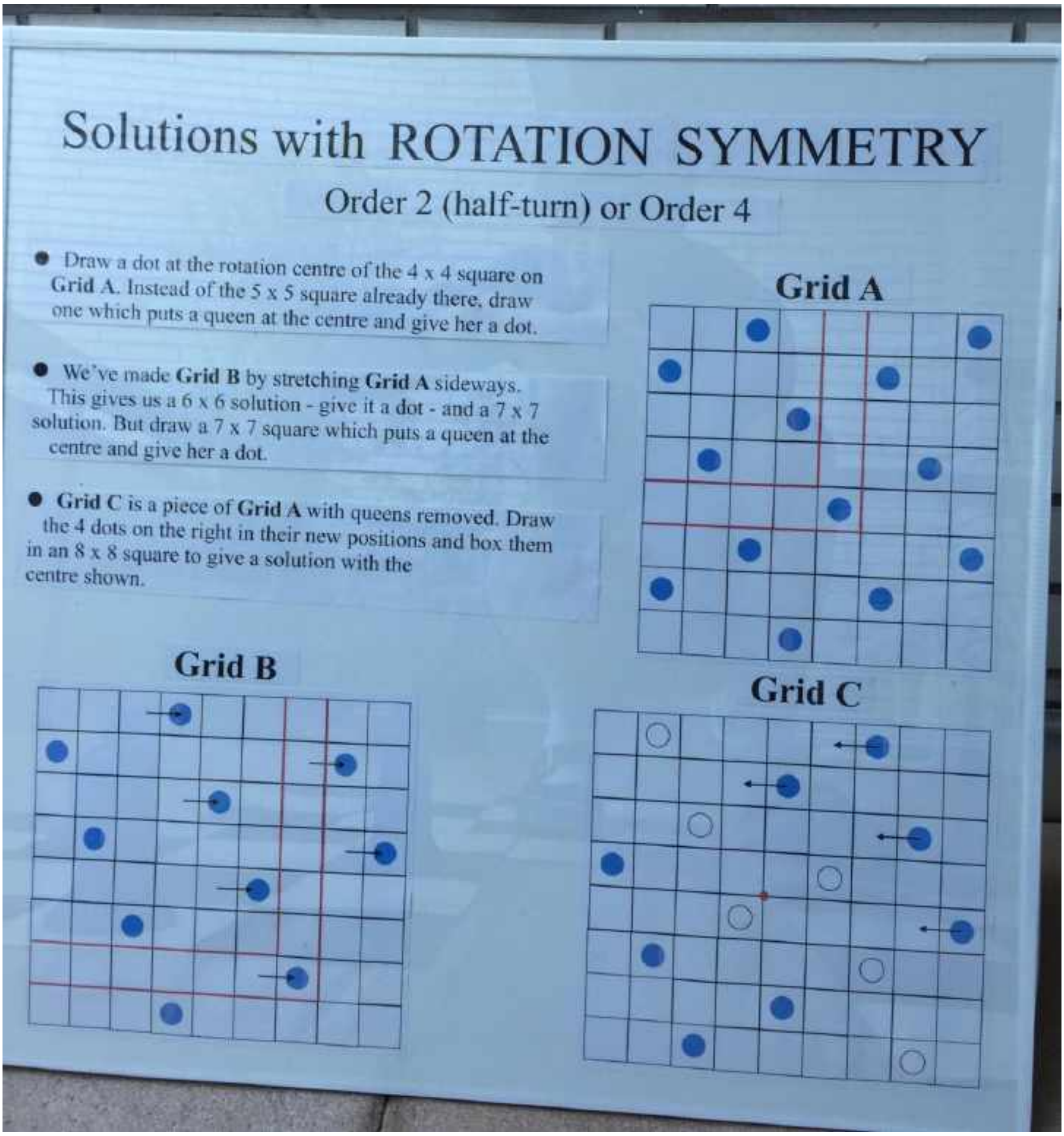
Mental activities

Along with that explicit rule, there is an implicit one, a condition the children may not realise but helps in the solution of the puzzle: every row and every column must contain a queen, otherwise there would be one with more than one queen, breaking the rule above.

Explicit content

The back of the main board encourages the children to think in terms of periodic arrays – even though, in the case of the 8 queens, they may need to violate a pattern.


SAFE QUEENS
▶ The blue cones are chess queens.
▶ No queen must threaten another.
▶ That means there must never be two queens
… in a row: … a column: … or on a diagonal:
● Begin with 4 queens on a 4 x 4 board.
● Enlarge the board to 5 x 5:
Add a 5th queen.
● Make the board 6 x 6, 7 x 7, 8 x 8
and add a 6th, a 7th, an 8th queen.
(Help on the back ▶)



Station *Leapfrog*

Physical experience There are just two valid moves: a slide onto an adjacent empty stripe, and a jump over a single frog of the other colour on to an empty stripe.

Mental activities When the children are failing to find the optimal strategy, they should be encouraged at each decision point to make the move which maintains an alternating colour pattern. This permits the ‘leapfrogging ’of the title.

Explicit content The optimal strategy produces the following number, n, of moves when there are f frogs each side: $n = f(f + 2)$, though it is rare even for older students to arrive at this or an equivalent expression without being steered. However, by tabulating differences, as in the table, they can predict the next n values.

These two boards mark the edges of the road.

strip of vinyl, which rolls up for storage


LEAPFROG
The red and yellow frogs must swap places on the zebra crossing.
A frog can slide on to an empty stripe or jump over a frog of the opposite colour.
How many moves does it take if you start with 1 frog each side? 2 frogs? 3 frogs? … ?

Station *Over the phone*

Physical experience Arranging polygonal tiles.

Mental activities This is a sister station to *The feely box*. The challenge is to put yourself in the position of the other child and thereby realise how precise you have to be in your description.

Explicit content As with *The feely box*, there is no explicit mathematical content.

assorted polygons in two identical sets, both coloured and black

Station *9 hexagons to 1*

Physical experience Arranging polygonal tiles in an outline.

Mental activities

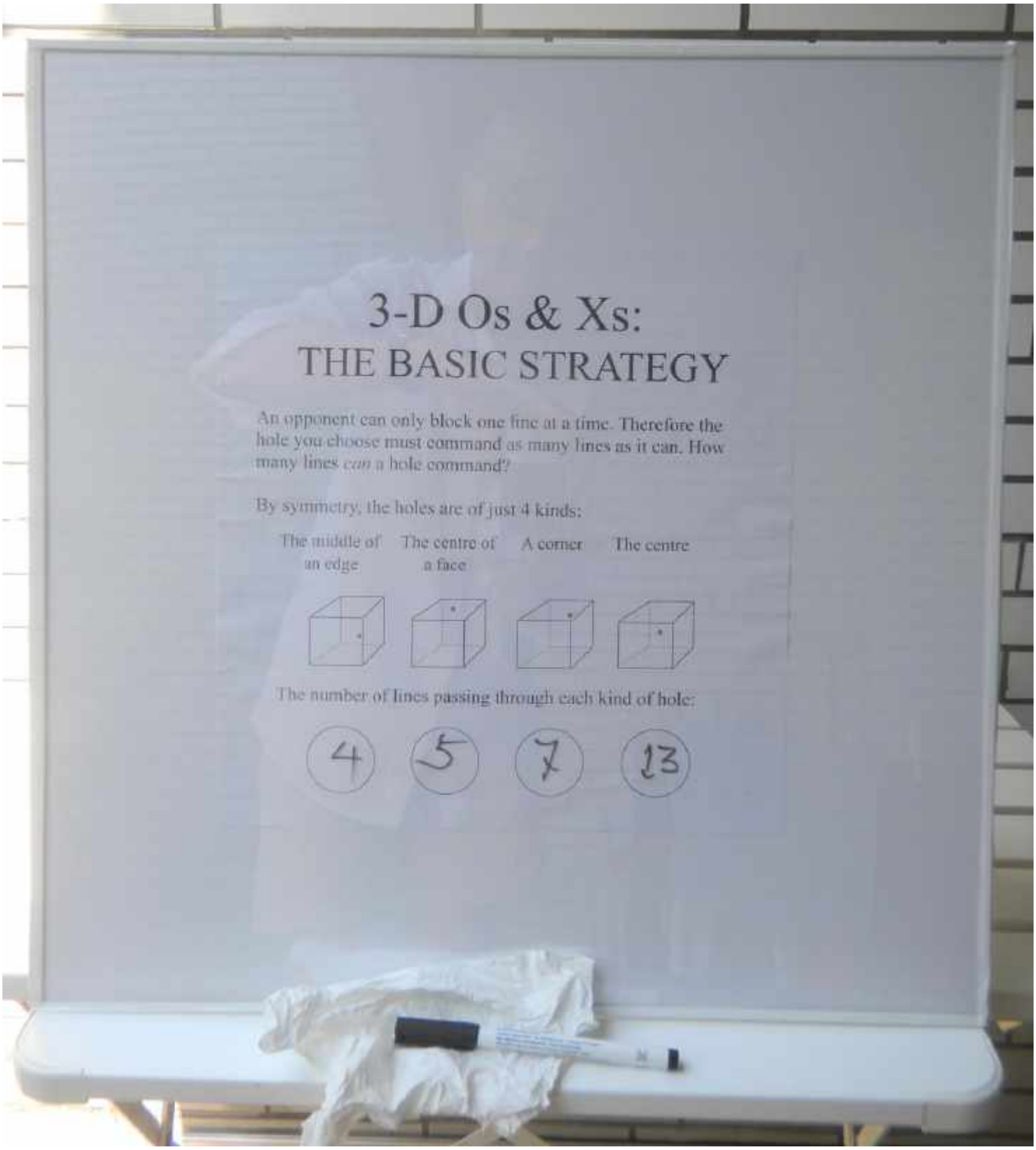
Though there is no instruction board, no compulsion, the older children will strive to make a pattern with symmetry. (See *Mirror symmetry*, *Rotation symmetry*). And the younger children will be able to construct a narrative, describing – to themselves and others - how they made their pattern.

Explicit content

A dissection puzzle like this is predicated on the conservation of area.

There are orders of magnitude in the dissection, which govern the possibe patterns:

2 triangles = 1 rhombus,
3 rhombuses = 1 hexagon.


3-D Os & Xs:
THE BASIC STRATEGY
An opponent can only block one line at a time. Therefore the hole you choose must command as many lines as it can. How many lines *can* a hole command?
By symmetry, the holes are of just 4 kinds:
The middle of an edge
The centre of a face
A corner
The centre
The number of lines passing through each kind of hole:
4
5
7
13