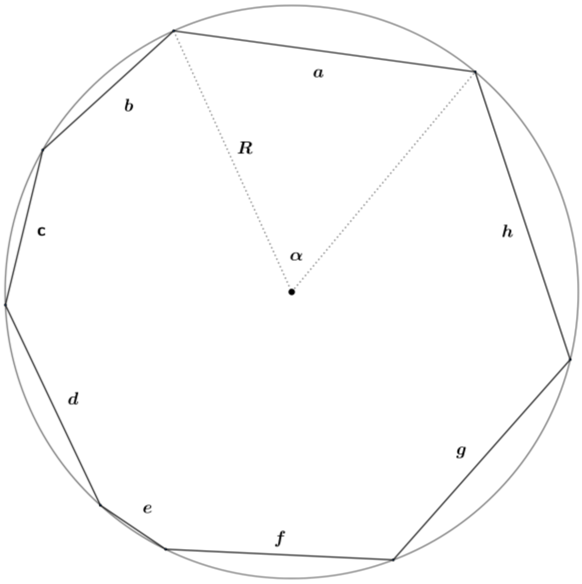
5.3 Irregular cyclic polygons

5.3.1 The circumradius of a cyclic polygon given the sides

This section is here to make the distinction between local and global properties through a very disappointing example.



We have:

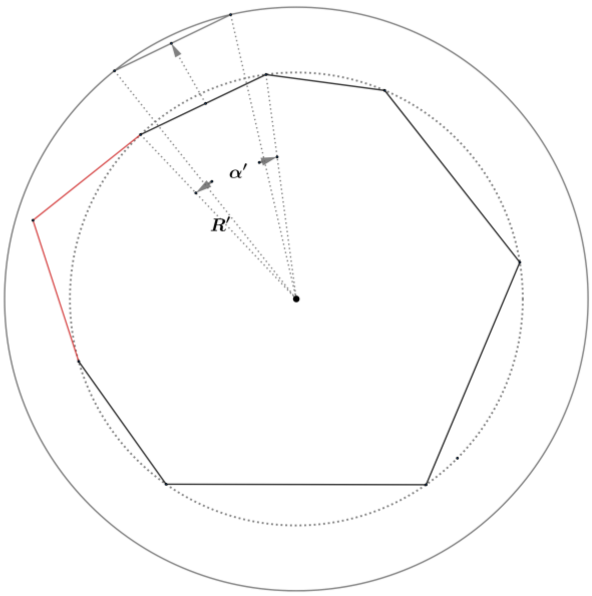
$$R = \frac{a}{2} cosec \frac{\alpha}{2} = \frac{b}{2} cosec \frac{\beta}{2} = \frac{c}{2} cosec \frac{\gamma}{2} = \cdots$$

But what are those angles?

Imagine that the polygon is a linkage and we try to fit it to a given circle, which we can expand from the centre. In the position shown we have still to accommodate the red sides. Until we do so, all the angles, $\alpha', \beta', \gamma', \ldots$, and therefore our circumradius value, *R'*, are undecided.

In the general case, there is no expression of closed form for the circumradius of an irregular cyclic polygon in terms of its sides.

We see that the reason for this is the global nature of the variable: it depends on a property of the whole figure; it cannot be calculated from any one part.



The triangle is the exception. It is a rigid shape. We know that, as the circle expands, the point where the red-red vertex lies on the circle is the point where *R'* = *R''* = *R*.

(All regular polygons are cyclic. The angles at the centre are equal so there is no doubt about the value of *R*.)