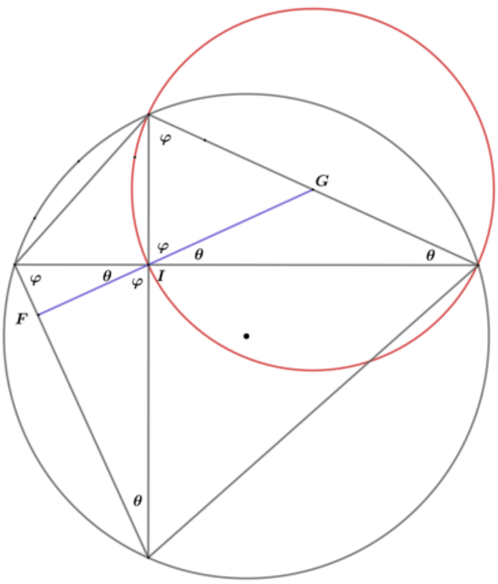
4.3.7 Further properties of a *CO*

A We trace a line from the foot of an altitude (*F*) through the intersection of the diagonals (*I*) to a point on the opposite side (*G*). Tracking the complementary angles θ and φ, we see that we have two isosceles triangles whose equal sides are radii of the red circle, centre *G*. This is Brahmagupta's theorem.

B displays the relationships arising. We have labelled circles by their centres. What we have observed in ***A*** is the dual relation indicted by the lines $F_{PQ}IC_{RS}$ and $F_{RS}\mathrm{I}C_{PQ}$, and the corresponding relation concerning the other pair of opposite sides *PS* and *QR*. Because they have vertically opposite angles, the right triangles $IF_{RS}C_{RS}$ and $IF_{PQ}C_{PQ}$ are similar. And the same applies therefore to the corresponding pair in the other quadrants.

A



The 4 blue circles have *IP*, etc. as diameters. They necessarily meet in pairs in the feet of the altitudes, F_{PQ}, etc. and all meet in *I*, where one pair share *PR* as tangent, the other pair, *QS*. The 4 red circles have the sides, *PQ*, etc. as diameters and therefore meet in pairs in the vertices, *P*, etc., and all pass through *I*.

B