### 4.3.4 The diagonals of a $C T$

Show that the diagonals of a $C T$ cut on the line of centres of the in- and circum-circles.

$\boldsymbol{A}, \boldsymbol{B}$ show the two diagonals of a $C T$. One cuts the line of centres of the inner and outer circles at $P$, the other at $Q$. The red lines are perpendicular to the line of centres.
A
B


By the intersecting chord theorem,
$e g=s^{2}=r^{2}-p^{2}($ Equation 1),
$f h=t^{2}=r^{2}-q^{2}($ Equation 2).
These two equations establish the following set of consistent conditions. The intersection point cannot lie off the line of centres without upsetting this equivalence.
$e g=f h \Leftrightarrow s=t \Leftrightarrow p=q \Leftrightarrow P, Q$ are the same point $\Leftrightarrow$ The two centres and the intersection of the diagonals are collinear.
(When the $C T$ has a symmetry axis, it is also true that the diagonals of the inscribed $C O$ cut on this line.)

