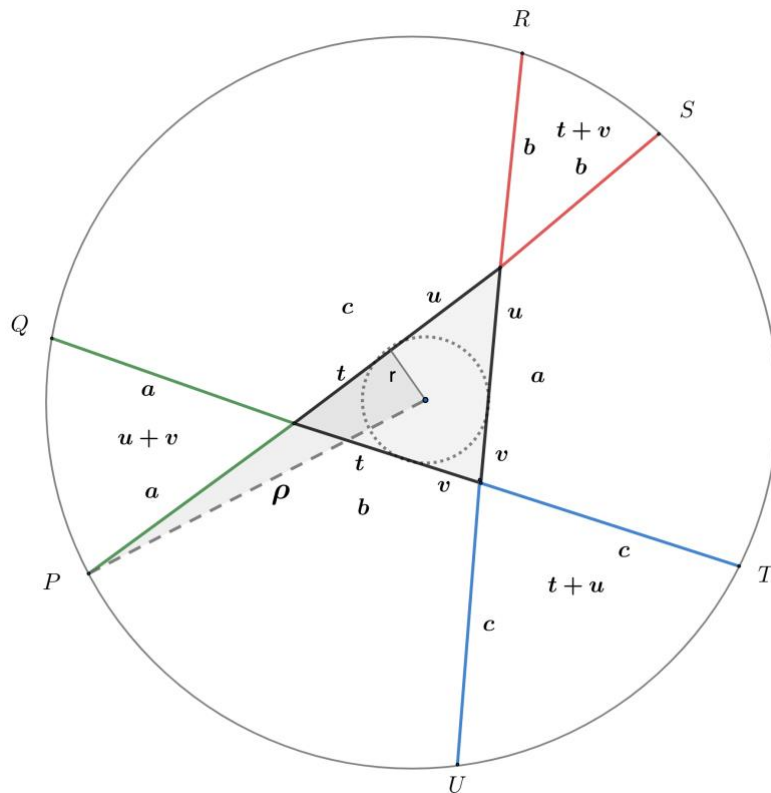# 3.6 The Conway circle

The following circle theorem due to John Horton Conway is a surprising example of a piece of elementary geometry only discovered in the last century.

The sides of a triangle are extended as shown.

We have 6 congruent right triangles, each with short side r, middle side $t + u + v = s$, where $s = \frac{a+b+c}{2}$, the semiperimeter of the triangle. P, Q, R, S, T, U therefore lie on a circle with radius $\sigma = \sqrt{s^2 + r^2}$. In **4.2: From Heron to von Staudt** we meet two formulas for the area A of a triangle:

$$A = \sqrt{s(s - a)(s - b)(s - c)} \text{ and } A = rs.$$

Combining these, $r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$.

Substituting in the top equation and simplifying, $\boldsymbol{\sigma = \sqrt{\frac{s^3 + (s-a)(s-b)(s-c)}{s}}}$.