# 3.4 The Euler line

We draw a figure and an alignment occurs. Is it spurious or significant? In these circumstances it is useful to draw a limiting case. Here are the main centres of a triangle and the lines on which they lie. Four centres could plausibly be in line here. The right triangle bottom right shows that *I* is certainly not one: it would remain on a 45° line however far we stretched the triangle to the left.
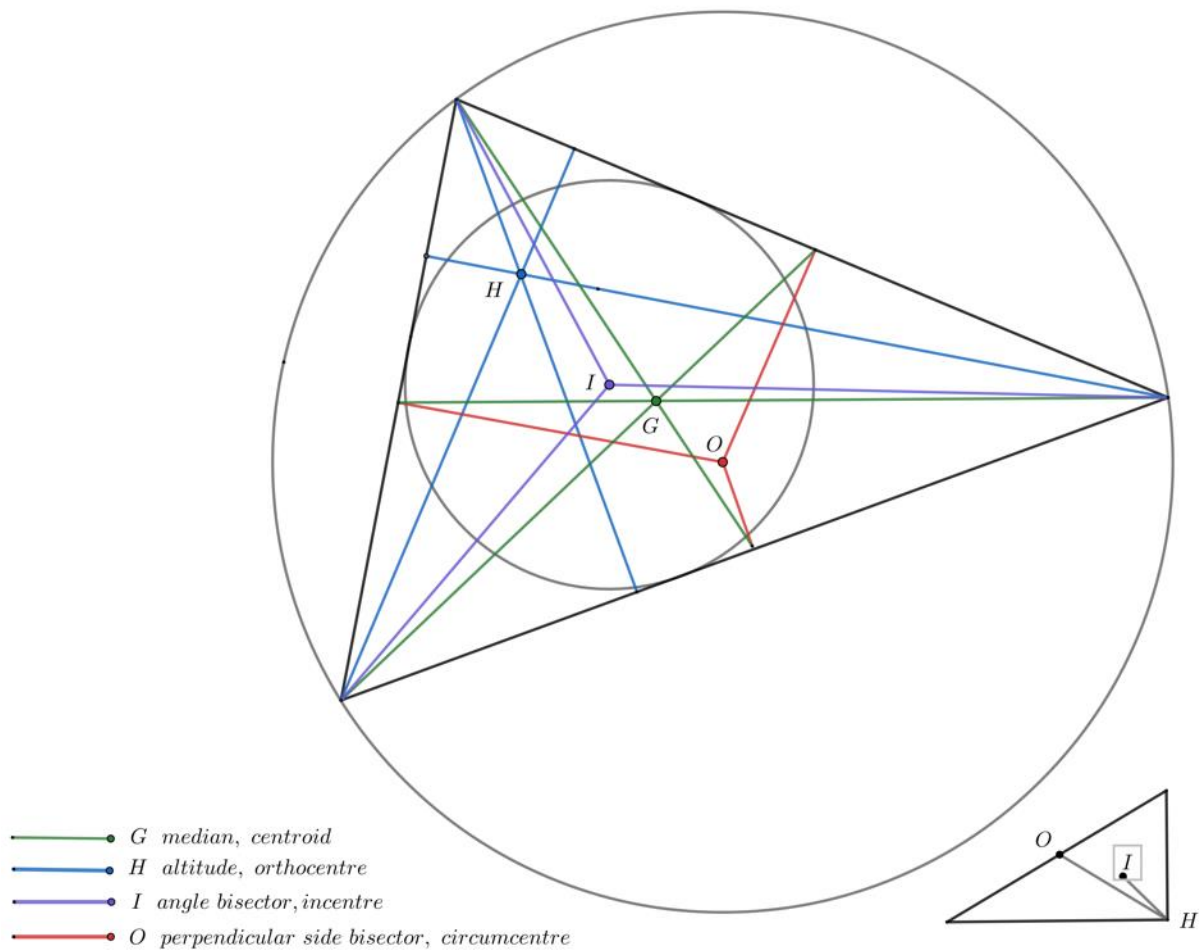
We now show that *H, G, O* are indeed collinear.
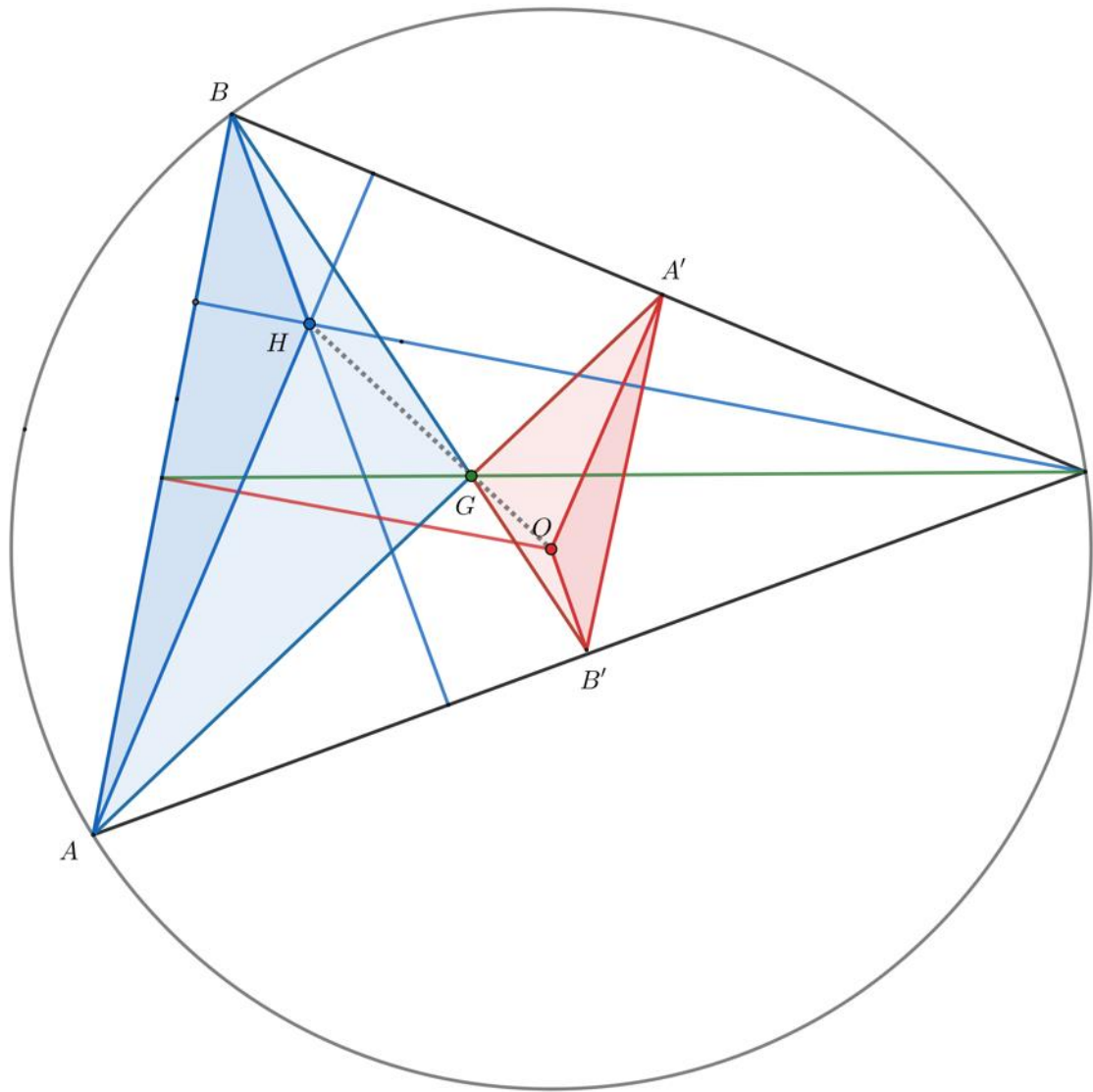
Each side in the blue triangle is parallel to a side in the red triangle. The two are therefore similar. The blue triangle is rotated a half turn with respect to the red triangle and scaled by 2. If we join corresponding points, the lines will intersect in a point which is a centre of enlargement for the red triangle with factor -2. This point will divide the lines in the ratio 2 : 1 so that, in particular, $|HG| = 2|GO|$. But two of these lines are medians. The point of intersection is thus G and our claim is proved.