Scaling equilateral triangles

Work in terms of the area of a single set square triangle, call it u, so that the area of the equilateral triangle is $2u$.

The area of each new triangle goes as the square of the side length, s, so the new area is

$$2u\left(\frac{s}{2}\right)^2 = \frac{s^2}{2}u.$$

What we want is a whole number of set square triangles, so we require s^2 to be even.

Let's see which of the cases I listed qualify.

1. s is made of 2s. **No** problem.

2. s is made of 2s and 1s, making up an odd number. **Problem**. If s is odd, so is s^2.

3. s is made of an even number of $\sqrt{3}$ s, say $2a$, so
$s = 2a\sqrt{3}$.
$s^2 = 12a^2$.
No problem.

4. s is made of an odd number of $\sqrt{3}$ s, say $2a + 1$, so
$s = (2a + 1)\sqrt{3}$.
$s^2 = 3(4a^2 + 4a + 1) = 12a(a + 1) + 3$.
Problem.

5. s is made of some whole number, n, plus some multiple, k, of $\sqrt{3}$.
$s = n + \sqrt{3}$ k.
$s^2 = (n + \sqrt{3}k)^2 = (n^2 + 3k^2) + 2nk\sqrt{3}$.
Problem: no multiple of a surd is a whole number.