The rationale behind the design of the purple set stations

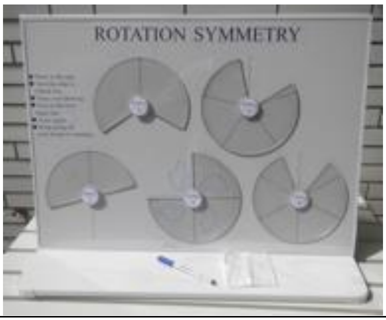
The general principle behind all Magic Mathworks stations is that the children are required to perform an action, predict the consequences and observe the result. To that extent each station is a scientific experiment. They may need to predict how a numerical sequence continues or the locus of a point imposed by some simple geometrical constraint. The tasks are ‘low threshold, high ceiling’: ‘low threshold’ because following the instructions and achieving the physical operation specified is already a measure of success, ‘high ceiling’ because older children can, for example, generalise the arithmetic algebraically or represent a locus by an equation.

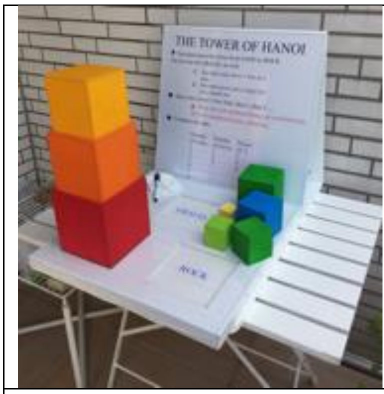
The station title	**The physical experience**	**The mental activities encouraged**	**The underlying mathematics and explicit content**
Handshakes 	Looping rubber bands over pegs.	Visualising the correspondences between the real situation and the model: person = peg, handshake = rubber band.	A triangular number first as a dot pattern, then as a number with a particular algebraic form. The triangular numbers as a sequence whose differences are the natural numbers.
Fibonacci numbers 	Moving a slide rule cursor.	Adding pairs of numbers.	The Fibonacci numbers as a recursive sequence.
Fibonacci spirals 	Tracing continuous curves (spirals) through discrete points (pine cone seed scales, daisy florets).	Observation, the concentration needed in order not to lose your place in the design.	The physical result of the irrationality of the golden mean is that no unit (a clump of cells called a *primordium*) is favoured in the competition for space.

The feely box  	Feeling a polycube.	Visualising a spatial arrangement from a shape explored only by touch. Realising which prepositions are spatially ambiguous. Improvising a coordinate system by which the location of the parts can be communicated.	There is no explicit content. The object of the exercise is simply to make the child aware that the task requires precise geometrical language.
Left & right  	Matching an object and its reflection.	The children must remember which way they folded their arms the first time. This knowledge depends on *proprioception*, the awareness of the position of one's body in space.	Solids lacking a symmetry plane are *chiral*/non-superimposable. To turn a closed surface inside out (as happens with the cube here) is to reverse its chirality.
The sliding ladder  	The child must move a stick in a right-angled frame, keeping the ends in contact with the frame.	Visualisation of the motion of the stick, concentrating on the position of the mid-point. Where an off-centre point is chosen, the child can assist this visualisation with a consideration of how the reduced symmetry of the stick will dictate the reduced symmetry of the resulting figure.	Our apparatus represents one quadrant of the Trammel of Archimedes.

Tables race  	The child must turn a tetrahedral block until its orientation is such that, when placed in a matching hole, it presents to the viewer the correct number in the chosen ‘times table’.	The successful children realise that every block bears a number they need. Upon finding this number, they can place it straight in the hole to which it belongs. The slower strategy is to seek the numbers in order.	Alternative factorisations mean that the same number may occur in several positions on the multiplication square. That the task is possible means that there are just 9 x 4 = 36 distinct products on the 9 x 9 multiplication square.
The seesaw  	The children place hangers on the pegs of a mathematical balance so that they achieve equilibrium.	The mathematical balance is a key piece of apparatus in experimental cognitive psychology because two attributes, the choice of a peg and the choice of the number of hangers to place on it, can be varied independently. The finding is that a certain mental maturity must be attained before the child can bear the two in mind simultaneously.	‘The law of the lever’.
Times chimes  	A child rings a bell on each multiple of a chosen number as a count is made – and may attempt to do so with each hand independently.	Realising that numbers contribute to their common multiples: that, for example, the factors 3 and 4 occur in all multiples of 12. Observing that the numbers which sound alone are prime.	Common multiples. A common multiple of two numbers corresponds to a musical chord (a ‘chime’). The musical score used also displays the following property geometrically (in the slanting lines joining squares of the same colour):

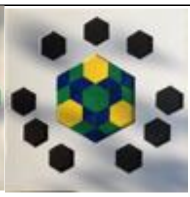
			If $q - p = 1$, then $nq - np = n$.
The rubber band enlarger 	The child stretches a rubber band of double length from an anchor peg. With the mid-point he tracks by eye the outline of a drawing. The pen at the end makes an enlarged copy.	Realising that, whatever the properties of the rubber, two bands will stretch twice as far as one, and that the result is therefore an enlargement by scale factor 2. Extrapolating from this to scale factors 3, 4, 5, … or, imagining that the sighting point and pen point are swapped, fractional enlargements.	If, from a given point, radiating lines of length *a*, *b*, *c*, … are drawn, multiplying their lengths by *k* produces a figure enlarged by that factor.
Spirals 	The child winds a stick around hubs of two diameters. Velcro ensures the stick does not slide but rolls.	Having seen what happens with the thick hub, he should predict what will happen with the thin one.	The spiral is Archimedean: The radius, *r*, is directly proportional to the angle, θ, through which the stick turns, giving the polar equation $r = k\theta$ for some *k*.
Anamorphs 	The child attempts to draw a letter in such a way that, upon reflection in a mirror, it appears correct. The mirrors are respectively plane and cylindrical. Because he watches the reflection form, he has immediate feedback and can correct errors as he proceeds.	The child may extrapolate from the two instances here to the thought that different mirrors might produce other transformations. If he has used the **Rubber band enlarger**, he can entertain the wider thought that many different pieces of apparatus might produce their own transformations.	The geometrical optics predicts that, in a plane mirror, front and back are reversed. In the experimental set-up here, this means a swap of top and bottom. In the case of the cylindrical mirror, there is an additional transformation: parts of the drawing furthest from the mirror and tangential to the

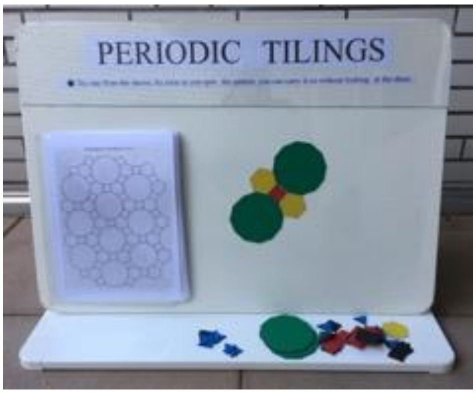
			curve must be drawn proportionally longer. The upshot is that a square grid can be used in the case of the plane mirror but must be swapped for a polar grid in the case of the cylindrical one.
Mirror symmetry 	One child adjusts the angle between hinged mirrors till the other announces that he can see the whole design.	The realisation that a 2-dimensional figure may possess more than one symmetry axis.	The geometrical optics of single reflections in a mirror and multiple reflections between mirror pairs.
Rotation symmetry 	By means of the actions described, the child completes drawings with rotation symmetry of orders 2 through 6.	The realisation that a 2-dimensional figure can have rotation symmetry of any order, the limiting case being the circle.	If the order of rotation symmetry is *k*, the figure may be brought into coincidence with itself in *k* positions.
Perspective drawing 	With an eye to the sight, the children trace the outline of some chosen object behind the Dürer screen.	Having got over their surprise that a mindless procedure has such an accurate result, the children can think about how the distance of an object from the screen affects its apparent size.	Projective geometry. How the picture plane cuts the pyramid of vision, with the consequence referred to in the next box.
The tower of Hanoi	A legal move is to take a box from the top of a pile and place it	The fractal structure referred to in the next box may not be apprehended	There is a fractal structure to the activity whereby the same sequence

	on a larger box or an empty one of the three sites. In fact, given the progress of the top box (clockwise or anticlockwise), the moves are forced.	explicitly. Nevertheless, quite young children may be experts, suggesting implicit recognition of an underlying pattern.	of moves is repeated on different scales. The number of moves, m, is an exponential function of the number of boxes, b, viz. $m = 2^b - 1$.
3-D Os & Xs 	A player places a ball in a hole with the object of completing a line of three.	In the 2-player game, by commanding the centre hole, the first player can force the situation that, on the fourth move, his opponent can only block one of two lines, ensuring victory on the next move. The important realisation for the game in general is that the aim should be to create as many potential lines as possible.	The essential geometry of the playing grid, which players are encouraged to master on the back of the main board, is that of the cube itself. Though the strategy for the 2-player game is clear, the varying allegiances possible complicate the 3- and 4-player versions.
Nim 	A move consists in choosing a row and the number of matches to be removed from it.	The children must anticipate the result of their move, but, as indicated in the next box, it is equally important to think how they arrived at the current position.	The game can be analysed in terms of the binary numbers represented by the rows, but even sixth formers find the analysis difficult. The strategies suggested on the back of the main board encourage the players to think backwards from a potentially winning situation.
Number-building: the triangle family	The base grid forces a close packing of the balls, which the child follows in	If the children have made the **Handshakes** investigation, they may recognise the	The following number types may be realised: *Triangular*: $T_n = \frac{n(n+1)}{2}$,

 	building and extending figurate expressions of the numbers under investigation.	triangular numbers. More hidden is the fact that 6 of these plus 1 comprise a centred hexagonal number. The main realisation is that figurate numbers match an arithmetical (or rather algebraic) pattern with a geometric one.	Rhombic = *Square*: $S_n = n^2$, *Tetrahedral*: $Tet_n = \frac{n(n+1)(n+2)}{6}$, Skew pyramidal = *Pyramidal*: $P_n = \frac{n(n+1)(2n+1)}{6}$, *Centred hexagonal*: $H_n = 3n^2 - 3n + 1$, and their interrelations.
Number-building: the square family  	Again, there is a close packing but the orientation differs from that on the blue board so that, for example, the sloping face of a pyramid on the red board corresponds to the base of a tetrahedron on the blue board.	Sequences **G** and **H** are the important patterns to think about. In the second case the children find that two consecutive triangular numbers form a square. By suitable colouring they can also show that two consecutive tetrahedral numbers make a pyramid.	From the remark in the first box we realise that the relations here are not new. However, some are clearer in the 'blue' orientation of the layers; some, in the 'red' orientation.
The Verden labyrinth 	A legal move is to follow the arrow on the tile on which you land from a previous one.	Children struggling to find a solution should be encouraged to work backwards, a heuristic advocated for **Nim** above.	How many steps you take on a given move dictates the direction you have to take next. This is not quite like a *maze*, where, at a decision point, one has the choice of two directions. In fact the solution follows a unique route, making this a *labyrinth*, of which that at Cnossos is the original.

Safe queens 	In any line - vertical, horizontal or diagonal - there must be no more than 1 queen.	Along with the explicit rule in the box on the left, there is a condition the children may not realise but helps in the solution of the puzzle: every row and every column must contain a queen, otherwise the first condition could not be met.	The back of the main board encourages the children to think in terms of periodic arrays – even though, in the case of the 8 queens, they may need to violate a pattern.
Leapfrog 	There are just two valid moves: a slide onto an adjacent empty stripe, and a jump over a single frog of the other colour onto an empty stripe.	When the children are failing to find the optimal strategy, they should be encouraged at each decision point to make the move which maintains an alternating colour pattern. This permits the ‘leapfrogging’ of the title.	The optimal strategy produces the following number, n, of moves when there are f frogs each side: $n = f(f+2)$, though it is rare even for older students to arrive at this or an equivalent expression without being steered.
Over the phone 	Arranging polygonal tiles.	This is a sister station to **The feely box**. The challenge is to put yourself in the position of the other child and thereby realise how precise you have to be in your description.	As with **The feely box**, there is no explicit mathematical content.
9 hexagons to 1	Arranging polygonal tiles in an outline.	Though there is no instruction board, no compulsion, the older children will strive to make a	A dissection puzzle like this is predicated on the conservation of area.

 		pattern with symmetry. See **Mirror symmetry**, **Rotation symmetry**. And the younger children will be able to construct a narrative, describing – to themselves and others - how they made their pattern.	There are orders of magnitude in the dissection, which govern the possible patterns: 2 triangles = 1 rhombus, 3 rhombuses = 1 hexagon.
Building and map-colouring polyhedra 	Clicking together interlocking tiles.	Recognising like configurations, rotational symmetry.	The Four Colour Map Theorem.
Uniform polyhedra 	Clicking together interlocking tiles.	Recognising like configurations, reflective and rotational symmetry.	Number of vertices x angle defect at vertex = $720°$. Total number of *n*-gons = Number of vertices x number of *n*-gons at vertex $\div n$.
Teashop Sudoku	Arranging coloured objects in a square array and scanning the rows and columns which result.	Predicting the effect of transpositions on the pattern.	For prime *n*, or a power of it, one can form (*n* - 1) Latin squares, every pair of which is orthogonal.

			
Periodic tilings 	Fitting polygonal tiles together edge-to-edge.	Having estimated the sizes of their interior angles, judging which polygons can be arranged around a vertex. Identifying the pattern which results so that it may be repeated.	No periodic tiling can be formed from regular *n*-gons where *n* is a multiple of a prime other than 2 or 3. Possible angle sums round a vertex restrict the possibilities further. The net result is that only regular 3-, 4-, 6-, 8- and 12-gons can contribute to a vertically transitive tiling.
Aperiodic tilings 	Fitting polygonal tiles together edge-to-edge.	Following the required edge-matching rules. Back-tracking in anticipation of an impossible vertex arrangement.	Following Roger Penrose, we use rhombuses whose interior angles are multiples of $\frac{\pi}{5}$. To guarantee an aperiodic tiling with rhombuses, it is necessary to mark the edges which may meet. In addition, only 8 vertex arrangements allow the infinite extension of the tiling.

Triangle number race 	Dropping marbles into V-shaped slots so that the top rows are complete.	Being systematic in choosing the biggest triangle number below the randomly-chosen total and reassigning the remainder in the same way.	Gauss' 'Eureka' theorem: no positive integer needs more than three triangle numbers for its representation as a sum.
Magic masks 	Folding 4 square acetates on to a central square (the multiplication table up to 15 x 15) in various combinations, and observing the result.	Being prepared to revise a hypothesis if falsified by experiment, framing the new hypothesis in the light of the contradictory observation.	Common multiples. When two acetates overlap, the smallest number which appears is the LCM of the smallest numbers which appear when the acetates are chosen individually.