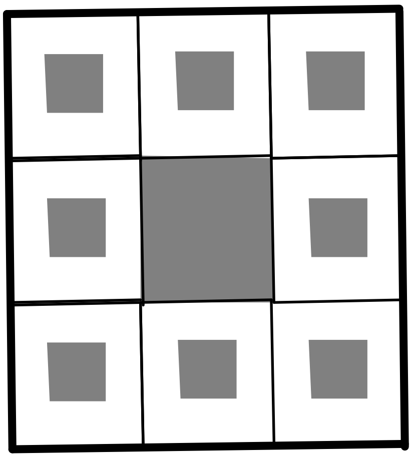
MegaMenger

Google 'MegaMenger' and find pictures of the 'distributed construction' project of Laura Taalman and Matt Parker, beginning on Martin Gardner Day, October 21, 2014, his centenary, and ending on the day I write, October 26.

Here are two levels of the fractal called the Sierpinski Carpet. The level 1 panel is a square hole bordered by 8 like, solid squares. The level 2 panel is a square hole bordered by 8 level 1 panels. And so on up.



The Menger Sponge is the 3-dimensional analog.

Matt and Laura arranged for business cards 55 mm wide to be sponsored, made and delivered to 20 centres round the world. Each level 1 Sponge needed 1 blank card for each face of its 20 solid cubes; and a further 2 with the level 2 Sierspinki Carpet motif overprinted for each of its 12 edges. At the 20 centres volunteers built level 1 Sponges, assembled each batch of 20 into a level 2, and every 20 level 2s into a level 3. The total number of blank cards needed was therefore 6 x 20^4 = 960,000; the total number of overprinted cards, 2 x 12 x 20^3 = 192,000. The printed panels effectively increased the level by 2, so that the (imaginary) World Sponge was level 6, a cube $\approx$ 4.5 metres high, punctured by holes only 6mm across.

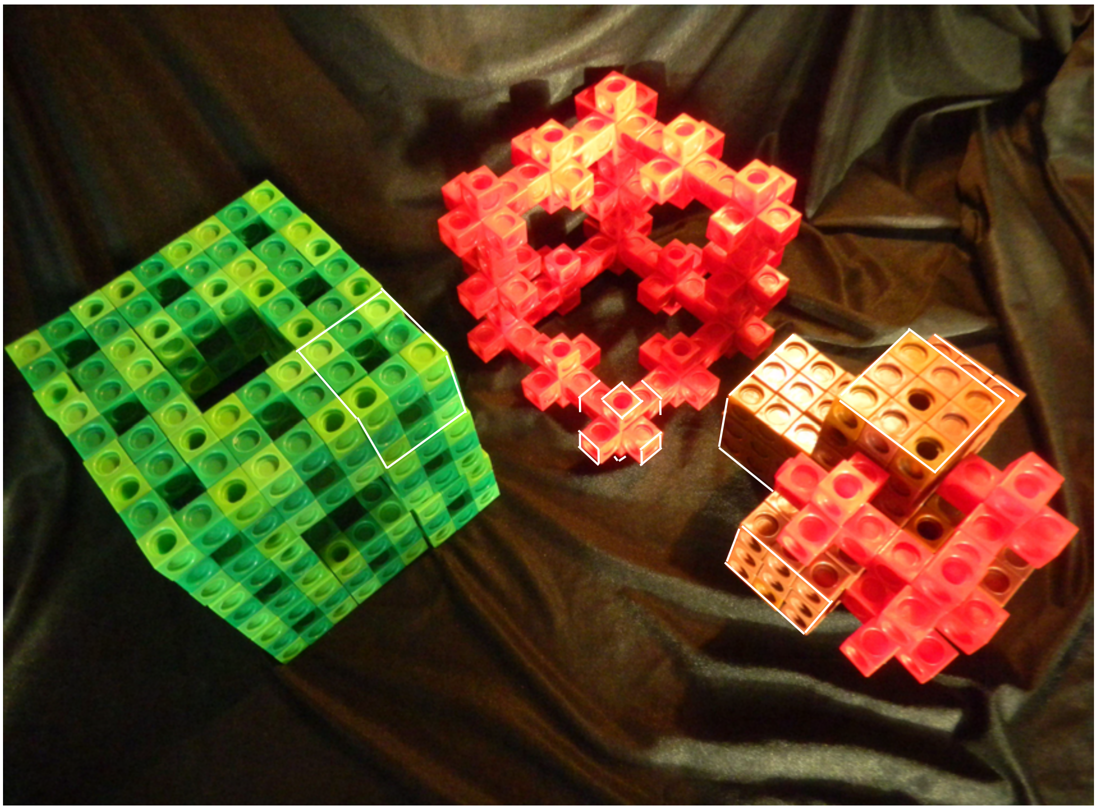
My level 2 model uses Multilink, dark green for edge sites, light green for corners. I've picked out a level 1 in white.

Consider two questions:

1. *What happens to the volume of a Menger Sponge relative to the cube containing it as the number of iterations increases?*

The level 1 Sponge occupies 20 of the 27 possible 1 x 1 x 1 cubes. Its volume, v_1, relative to the solid is therefore $\frac{20}{27}$. The level 2 Sponge occupies 20 of the possiblle 3 x 3 x 3 cubes, but each of those occupies only $\frac{20}{27}$ of its space. Its volume relative to a solid 9 x 9 x 9 cube is

therefore $\left(\frac{20}{27}\right)^2$. Moving up the levels in this way, $v_k = \left(\frac{20}{27}\right)^k$. As $k \to \infty$, $v_k \to 0$.
We can derive the same result by considering the negative of the Menger Sponge, the Menger Hole. This is shown centre and right, where it is seen to consist of a frame of 20 3-dimensional Greek crosses (red) enclosing one 3^3 times the volume (brown).



The relation between the relative volumes of a level *k* Menger Hole, h_k, and a level (*k* - 1) is:

$$h_k = \frac{20h_{k-1} + 7}{27} \Rightarrow h_k - h_{k-1} = \frac{7(1 - h_{k-1})}{27} .$$

Since $h_{k-1} < 1$, $h_k - h_{k-1} > 0$, i.e. the hole grows at each iteration, so, as $k \to \infty$, $h_k \to 1$, so $v_k \to 0$.

2. *What happens to the interior surface area of a Menger Sponge relative to the exterior surface area as the number of iterations increases?*

The level 1 Sponge has 2 x 12 = 24 inner faces (the number of overprinted cards needed). It has 8 x 6 = 48 outer faces. So the relative surface area of a level 1 Sponge, s_1, = $\frac{24}{48} = \frac{1}{2}$.

Readers may like to show that $s_k = s_{k-1} + \frac{5^{k-1}}{2^k}$. Since there is an increase with each iteration, as $k \to \infty$, $s_k \to \infty$.

In surface catalysis two chemicals are made to react by their mutual attraction for a substrate. A Menger Sponge, where the reagents are admitted to the centre of and diffuse outwards, would provide an ideal structure. However, there are natural 'molecular sieves' to hand. For example, imagine a packing of truncated octahedra. This is solid. The shapes fill space. But now, keeping the shapes the same size, expand the space and insert cubes as spacers between the square faces of the shapes. You now have the open structure adopted by certain alumino-silicate minerals.

The Patent Office still awaits someone with a Menger Sponge and a Good Idea.