## How long is the rope? Children's estimates

In what follows $t$ is the thickness of the rope, $n$ the number of coil turns, $r$ the radius of the complete spiral, $A$ its area, $L$ the length sought.

There were two basic approaches to the problem. Children treated the coil either:

**1.** as an arithmetic sequence of concentric circles
or
**2.** as just so much rope material.

**1.** These children in turn fell into one of two groups:

**A)** They multiplied the common difference, $2\pi t$, by the triangular number $\frac{n(n+1)}{2}$.

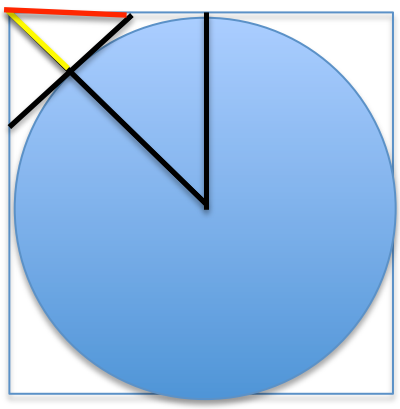
They realised that $t = \frac{r}{n}$.

So their formula was $L = \frac{n(n+1)}{2}.2\pi.\frac{r}{n} = (n+1)\pi r$.

**B)** They multiplied the mean of the smallest coil (length 0) and the largest coil (length $2\pi r$) by $n + 1$, thus arriving at the same formula (though some erroneously used $n$ for the number of coils, thus reducing their estimate by around 4%).

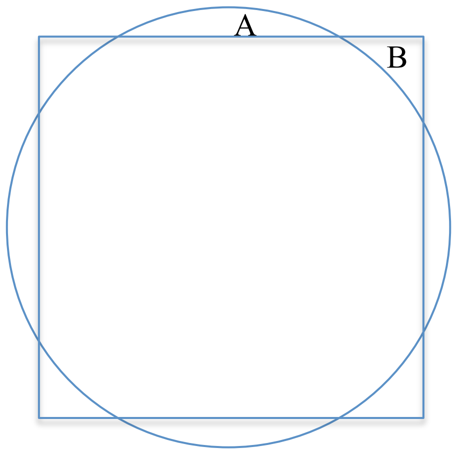
**2.** Here it was a matter of dividing the area of the complete coil by the area of a representative patch, and multiplying by the combined lengths of the rope strands found within it.

Students familiar with the area formula for a circle used it. A Y8 child who had not met $\pi$ approximated the area by enclosing it within a square then cutting off the corners by means of right-angled isosceles triangles, so that it was circumscribed by an octagon. Interestingly, this is exactly the method recorded in the Rhind Papyrus, copied from earlier sources by an Egyptian scribe named Ahmes around 1,650 BC. Here is the figure:



Taking unit radius, the yellow length is $\left(\sqrt{2}-1\right)$, the red length therefore $\sqrt{2}\left(\sqrt{2}-1\right)$, whence the area of each of the 4 isosceles right triangles which, subtracted from that of the square, gives $8\left(\sqrt{2}-1\right)$, a value too high by around 5%, but quite good enough for the present purpose.

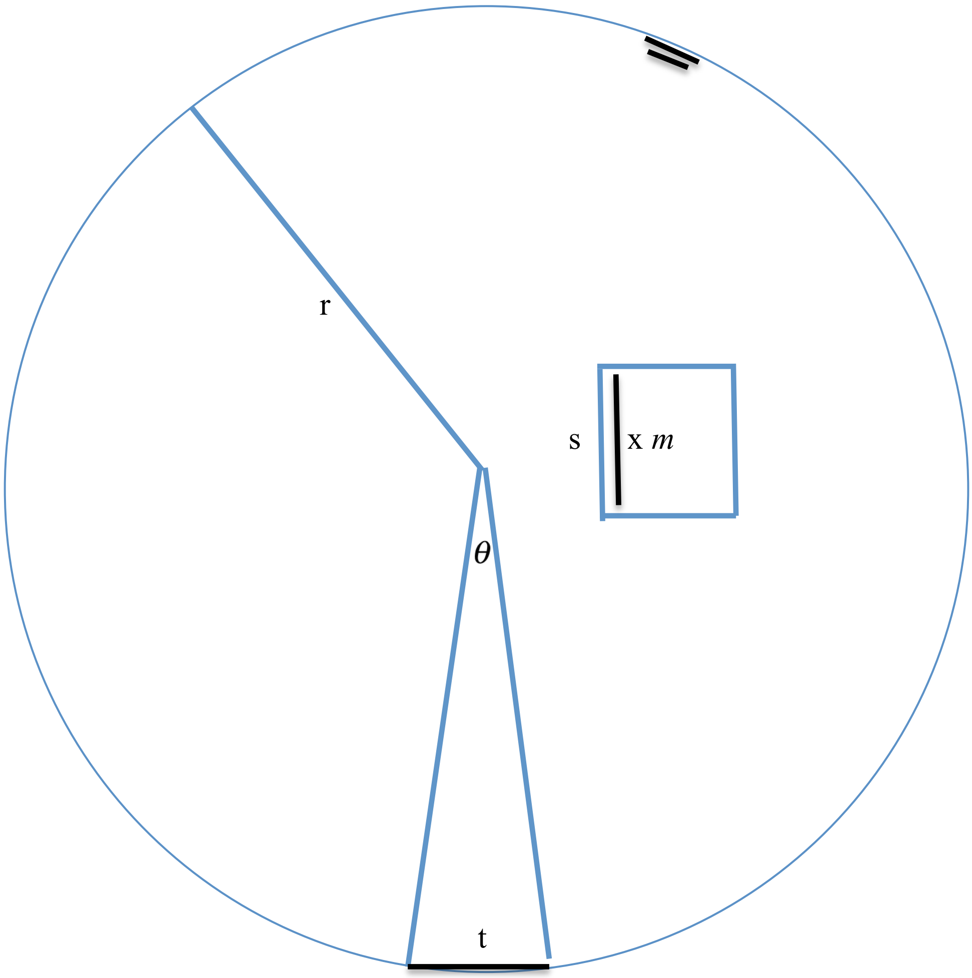
An approach not used was to find the square with the same area as the circle by shrinking a square until, judged by eye, part *A* is equal to part *B*:




The obvious patch was a square of convenient size – but where to take it? Towards the centre? Towards the edge? Somewhere in between? If the square side is *s* and you count *m* strands in it, *assuming the strands to be straight*, you have a combined length of *ms* within the square, and this formula:

$$L = \frac{A}{s^2}.ms = \frac{Am}{s}.$$

An alternative, not used, was to treat the coil as a regular polygon with *p* sides and take an isosceles triangle with its base on a side, of measured length *l*, and its apex at the centre. If the apical angle is $\theta$, the term $\frac{A}{s^2}$ is replaced by $\frac{2\pi}{\theta}$, and the term *ms* by $\frac{n+1}{2}l$, so that $L = \frac{2\pi}{\theta}.\frac{(n+1)l}{2} = \frac{(n+1)\pi l}{\theta}$.



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