Goldbach activities

Goldbach's Conjecture is notorious. People are sure it's correct but, according to the late Paul Erdos, the mathematics needed to confirm it may not yet even exist. However, like many conjectures in advanced number theory, it's easy to state. Year 6 or 7 children will certainly like to list cases and may be intrigued that such a simple theorem has eluded the best brains in mathematics. The usual statement reads:

'Every even number greater than 2 is the sum of at least one pair of primes.'

But it may be rephrased:

'Every natural number greater than 1 is either prime itself or the mean of at least one pair of primes.'
(The 'or' is inclusive.) This brings out the symmetry of the situation.

Do not reveal the conjecture, or even the topic, till you have demonstrated what you announce as a 'Goldbach zip'. Make this as follows. Take an IKEA paper measuring tape and a hole punch and punch out all 25 primes under 100. (Make sure that all the children know what a prime is.) Tell the class that this is what you've done. Ask them to choose a number between 1 and 100. Say they choose 54. Using thumb and forefinger of one hand, pinch the tape at that position and drag the doubled length of tape between thumb and forefinger of the other hand. Squint at the tape as you drag it, announcing that you'll stop when you can see all the way through. (In the case of 54, this will of course happen when 47 and 61 coincide.) Tell the class that, whatever number they choose, it is believed that you'll always stop before you reach 0 on the doubled tape. Have the children working in groups to puzzle out what this means. To help them think, project the following chart. Ask them how the red, black and green numbers correspond to what happened with the doubled measuring tape.

2																						
	3																					
	3	4	5																			
	3		5		7																	
			5	6	7																	
	3				7				11													
	3		5			8			11		13											
			5		7		9		11		13											
	3				7			10			13				17							
	3		5						11						17		19					
			5		7				11	12	13				17		19					
	3				7						13						19		23			
			5						11			14			17				23			
					7				11		13		15		17		19		23			
	3										13			16			19			29		
	3		5						11						17				23	29	31	
			5		7						13				17	18	19		23	29	31	
					7												19				31	
	3								11						17			20	23	29		37

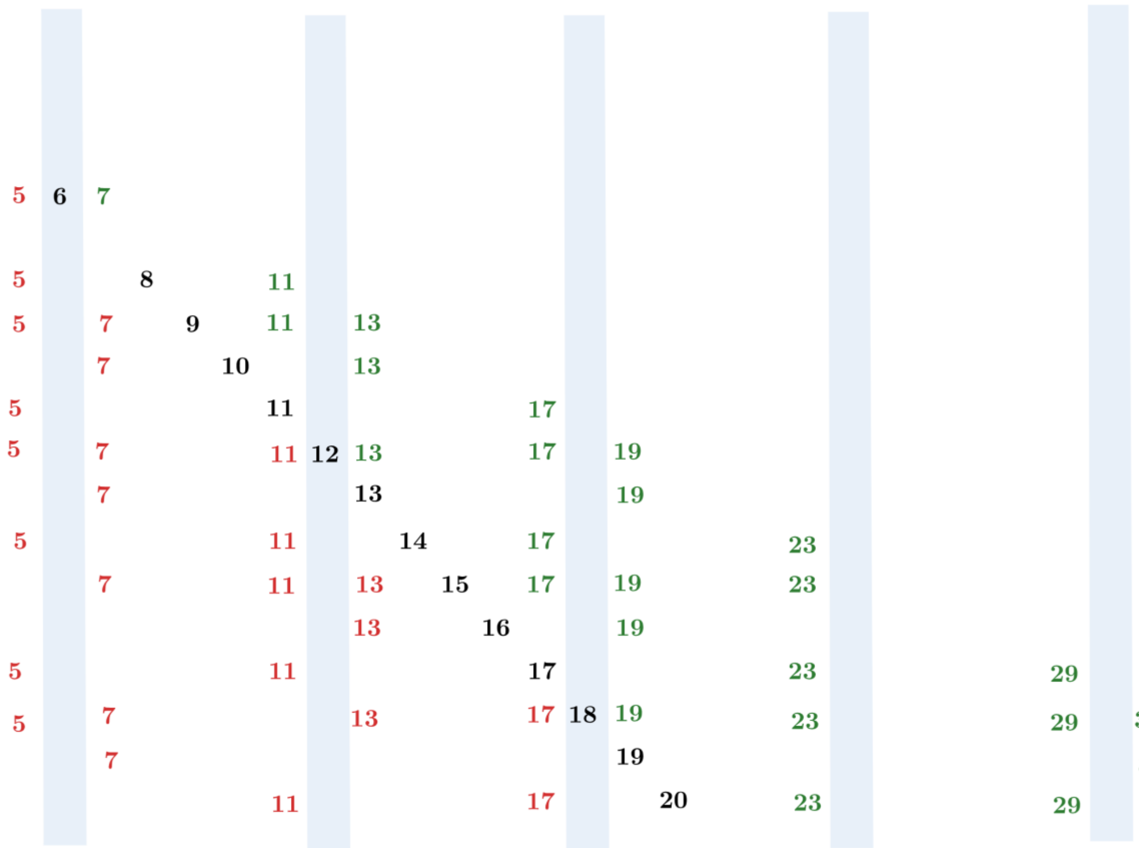
They may not have met the term *mean* but the tape experiment and the chart should force them to define it in their own terms. With suitable prompts, the pooled contributions of the class will lead to a statement of the Conjecture in the second form, to which you can now attach the name of the eighteenth century mathematician Christian Goldbach, hence the name 'Goldbach zip'. Tell the children that, in ordinary English, 'conjecture' just means 'guess' but in mathematics, it means a guess which is highly likely to be true.

Discuss the chart. Tell the children to use the phrase 'prime pair' for a matching red and green prime. The children may notice these two things:

8 is the mean not just of 5 and 11, but also of 3 and 13, and so on. Hence the phrase '*at least* one pair of primes'.

3 is on its own but 5 is the mean of 3 and 7. Discuss the two ways in which we use the word 'or'. "It's either day or night" means it can only be one or the other. "In the hotel bathroom you can use either the bar of soap or the liquid soap." If you wanted to, you could use both. In the Conjecture, we use 'or' this second way. 5 is a prime itself and it's also the mean of two primes. Mathematicians always use the word 'or' this way. They call it 'inclusive or' because it includes the two possibilities.

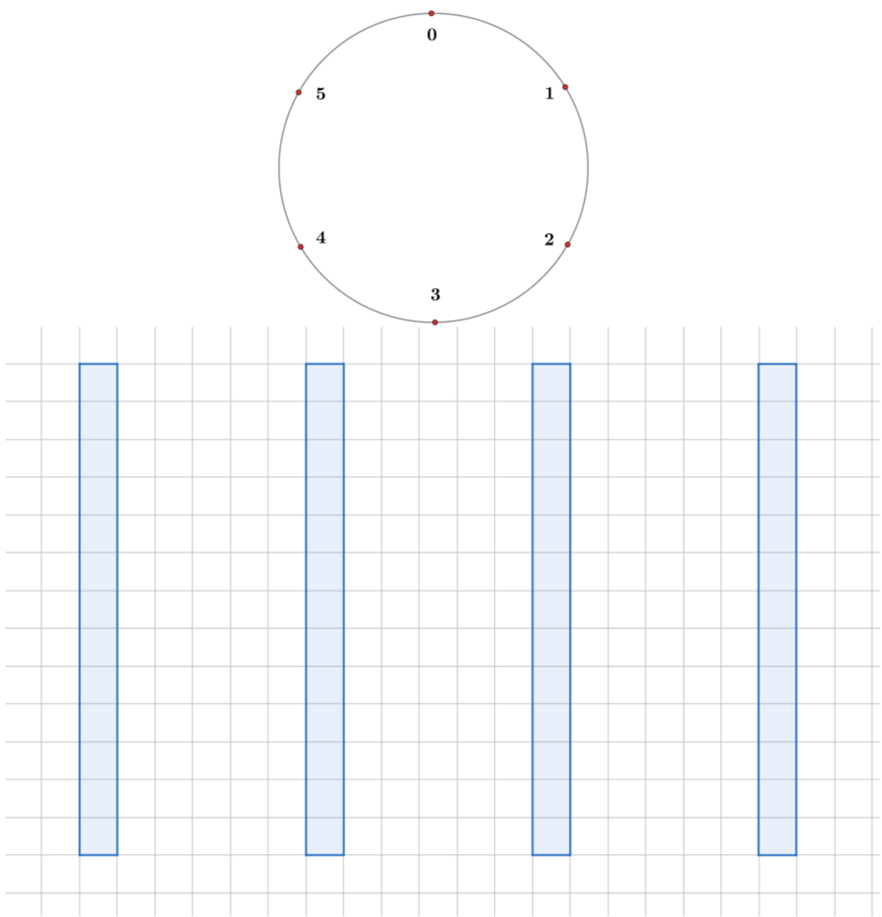
Send the children home with the following edited chart. Tell them that you've left out all prime pairs containing the number 3 because of its special status, and that the blue numbers are multiples of 6. What patterns can they find among the red, black, green and blue numbers? Can they prove that their patterns must occur?

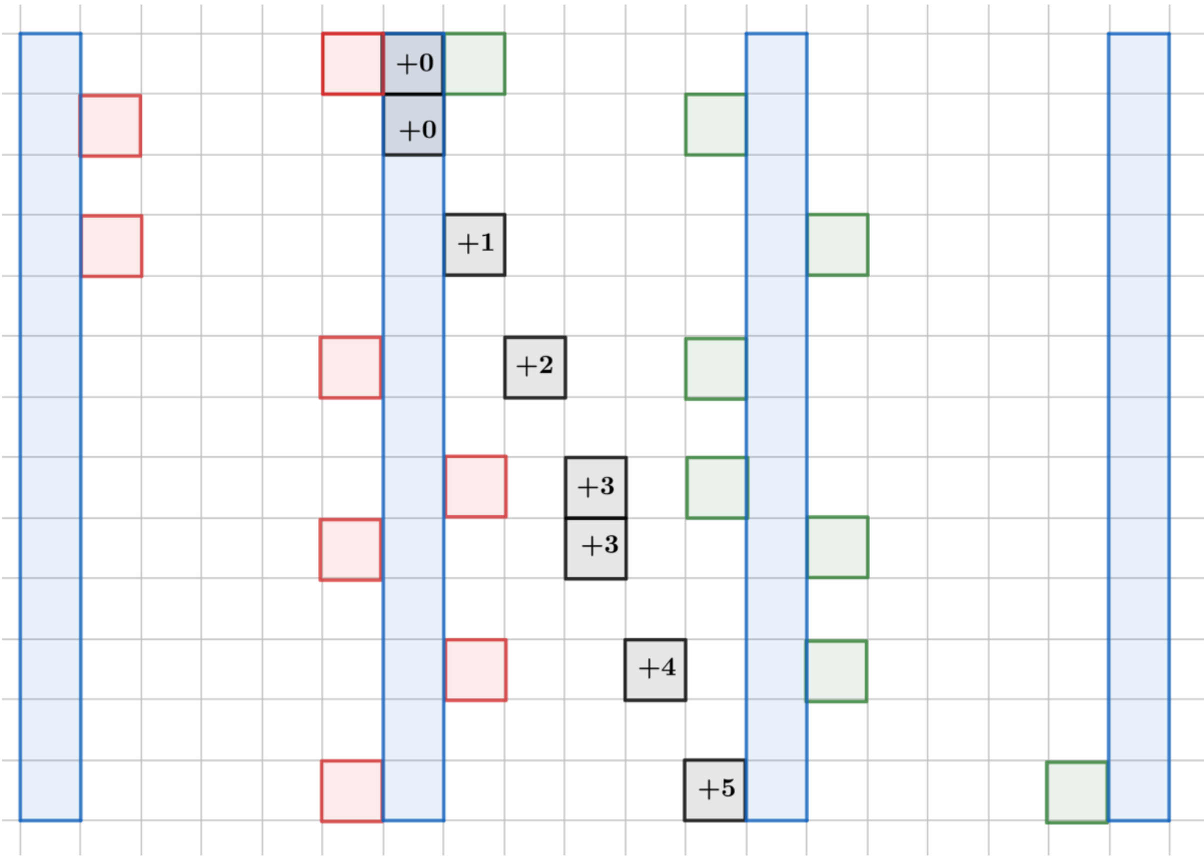


Here is a summary of what could emerge next lesson from their individual observations:

1. Apart from 2 and 3, all primes lie either side of a multiple of 6; that is to say, they're either 1 more than a multiple of 6 or 5 more. It's worth devoting time to this. From the children's contributions you hope to be able to distil the following argument. If they were 2 more than a multiple of 6, they would divide by 2; if they were 3 more, they would divide by 3; if they were 4 more, they would divide by 2. That leaves just the two possibilities found.

All the following properties are required by the equal red-black and black-green spacings on the number line, which we can diagram as follows. The chart takes each type of black number: multiples of 6, multiples of 6 + 1, multiples of 6 + 2, ... , through to multiples of 6 + 5, and shows where the red and green numbers must lie. If the children have met modular ('clock') arithmetic, point out that they're adding and subtracting equal numbers modulo 6. If they haven't, this is a good opportunity to introduce it. Project a template on which only the blue bars are marked and add the children's observations as they occur. Above it put the clock they need.



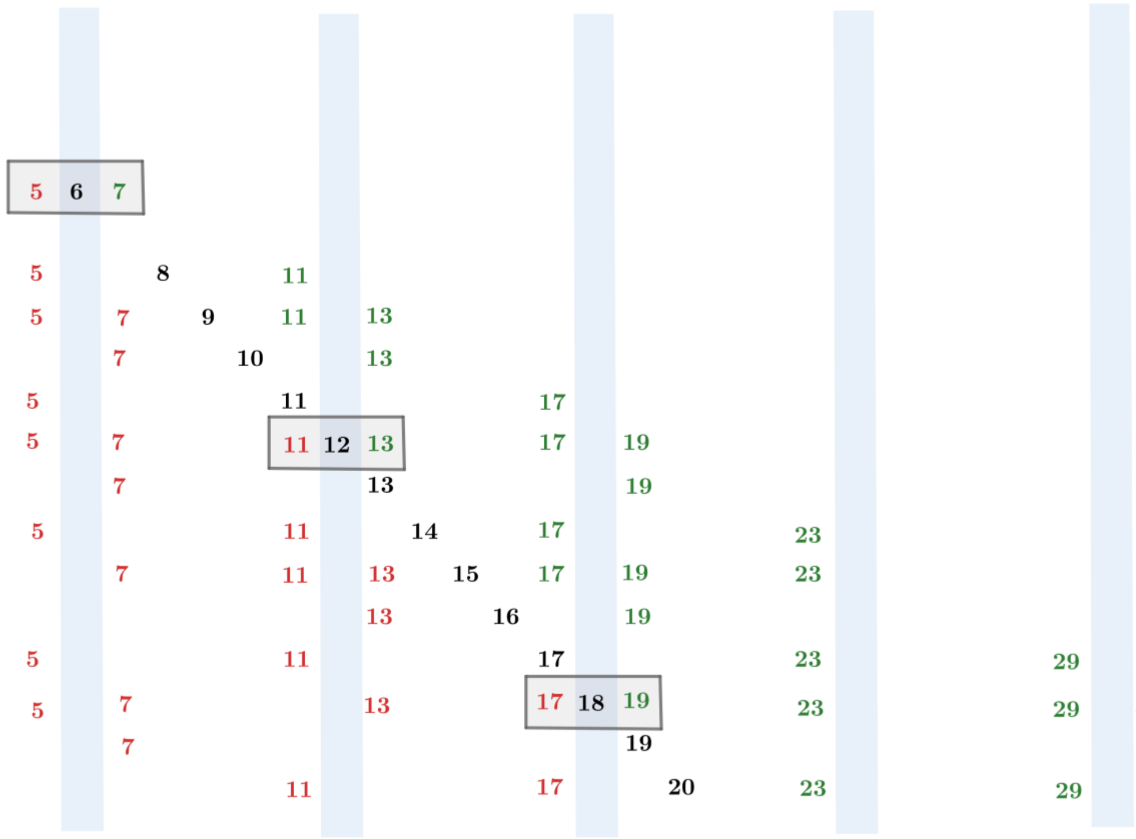


2. If the black number is a multiple of 6, or 3 more than a multiple of 6, either the red number is 1 less than a multiple of 6 and the green number 1 more, or vice versa.

3. If the black number is 1 or 4 more than a multiple of 6, the red and green numbers are 1 more than a multiple of 6.

4. if the black number is 2 or 5 more than a multiple of 6, the red and green numbers are 1 less than a multiple of 6.

At the end of the lesson, project the chart they took home with them but annotated further with the black boxes:



Ask what is special about the boxes. (Beyond 2 and 3, the primes either side of a multiple of 6 are as close together as they could possibly be.) Tell the children that prime pairs which lie either side of a multiple of 6 are called ‘twin primes’ and that mathematicians are now very near proving that there is an infinite number of these.

Tell them that the Ancient Greeks already knew, and proved, that the number of primes is itself infinite. The children will often learn on the news that a prime of record-breaking size has been discovered but the search can never end!