5
Steinhaus' Toy Cart

www.magicmathworks.org/geomlab5

A) Put a pen in the cart and set it at *P*. Stretch the string to *Q* and move your hand along the black line. The result is a tractrix.

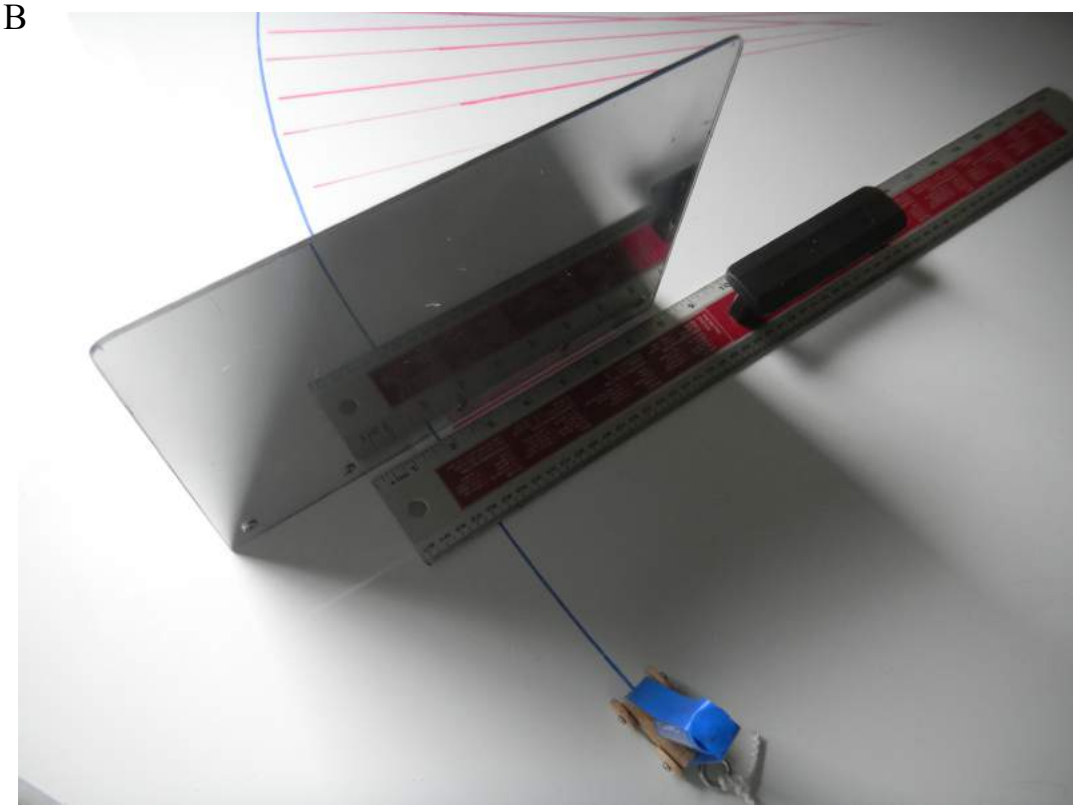
B) Use the 'mira' to draw normals to the curve, then the set square to draw tangents. Each time make a mark on the set square where it cuts the black line. You should find the marks coincide, i.e. the length of each tangent from the tractrix to the black line, the curve's asymptote, is the same, the distance *l* from *P* to *Q*. By Mamikon's Sweeping Tangent Theorem the area under the curve is exactly that of a quarter-circle, radius *l* . If you extended the normals and drew enough, you would see a catenary emerge as the envelope curve, as shown on the next page.

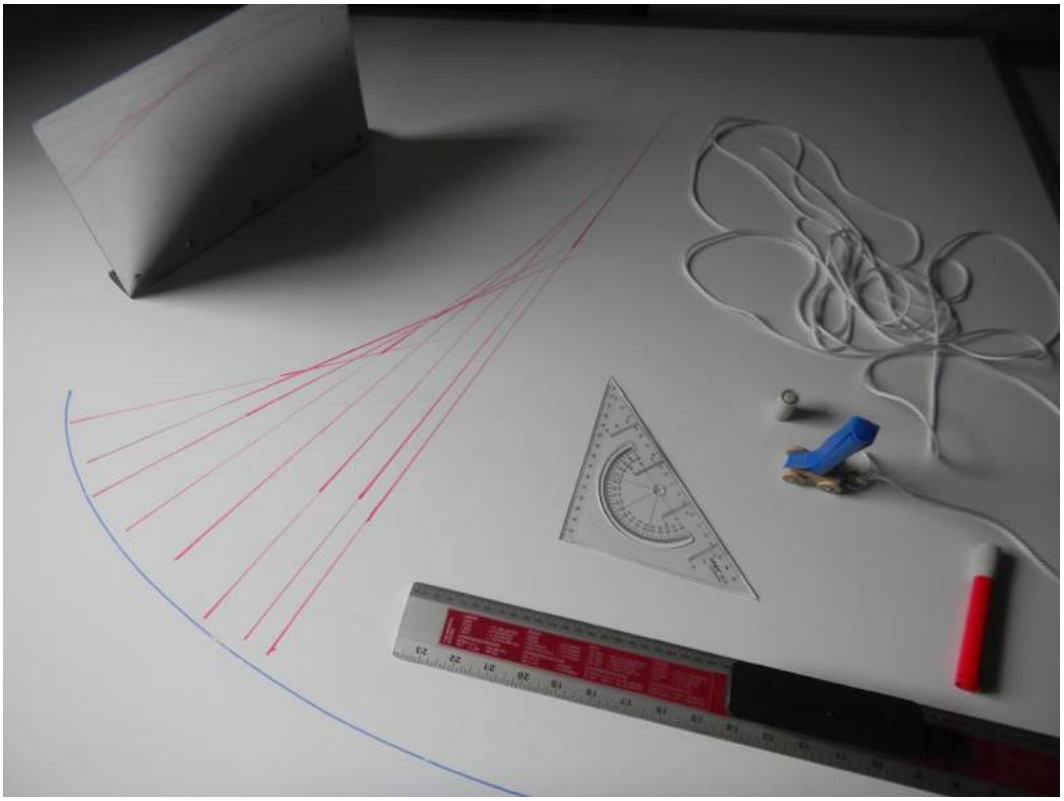
In place of the string, you can picture a bicycle frame as the cyclist turns through a right angle, or the leash as you and your dog turn a corner, (hence the German name for the tractrix, *Hundkurve)* ... or Claude Perrault's watch chain (see hom.wikidot.com/the-tractrix).

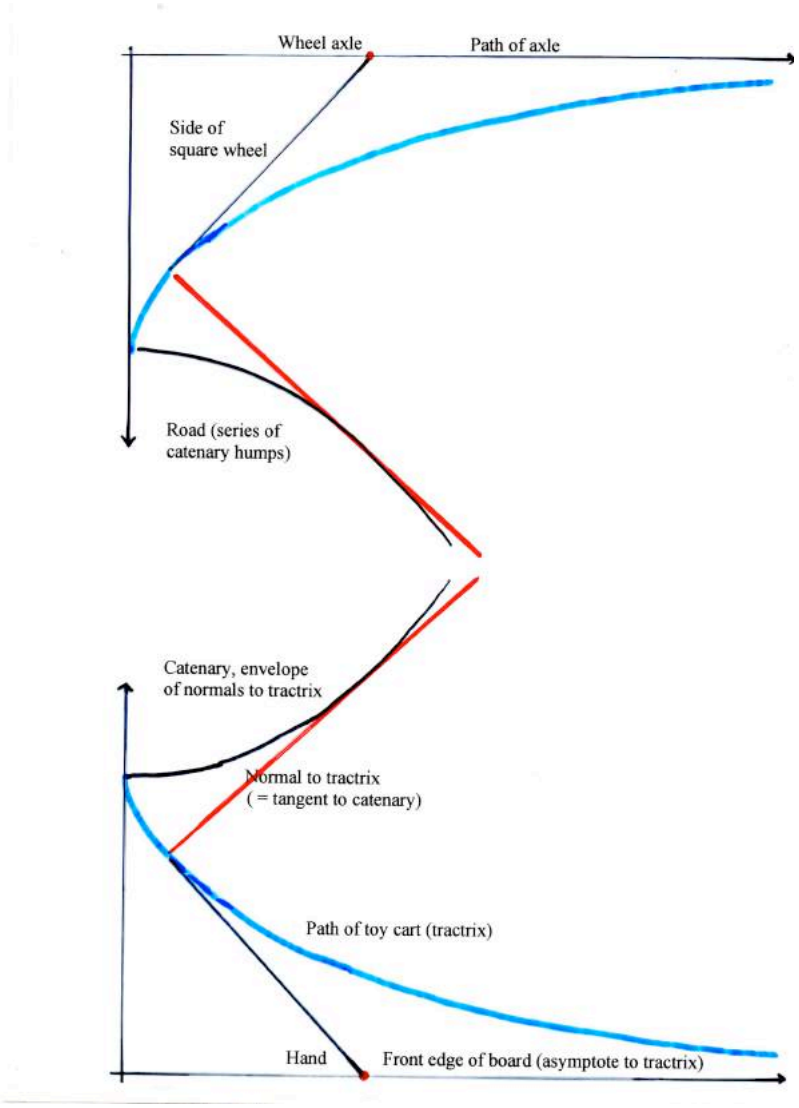
The bottom half of the diagram shows the result of **(A)** & **(B)**. The top half turns that upside down to explain why square-wheeled vehicles are possible.

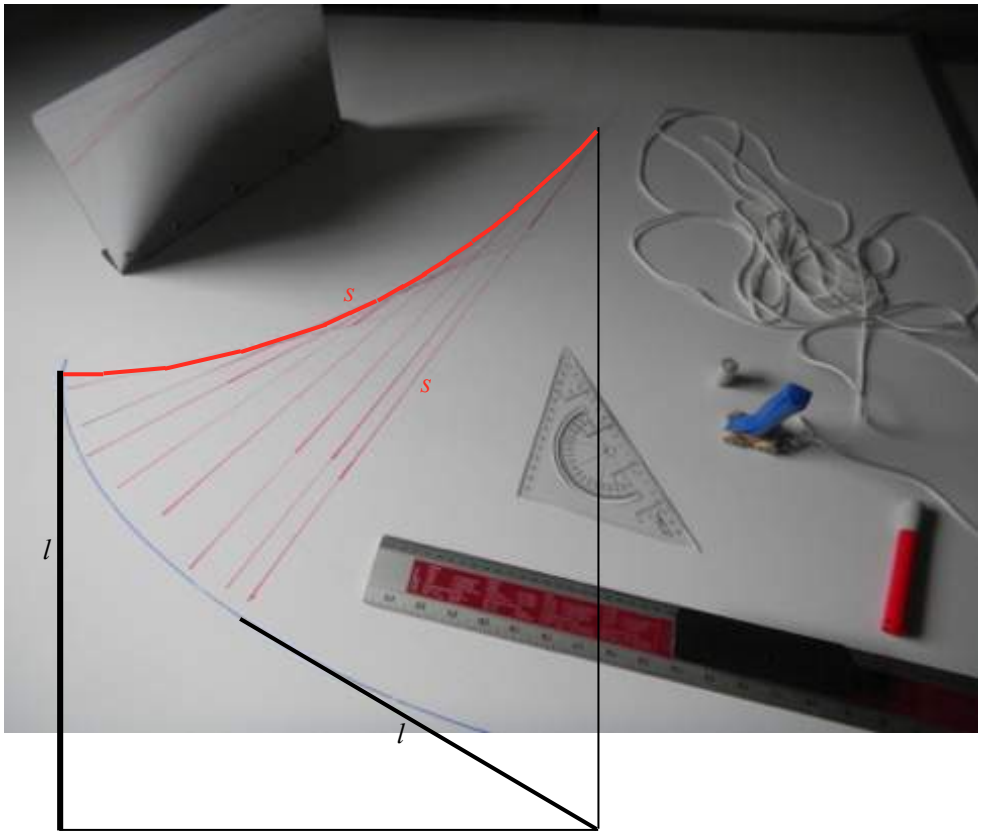
Parabola
Hyperbola
Line
Hyperboloid
Ellipse
Hyperbolic paraboloid
Sphere
Line pair
Cone
Sine curve
Cylinder
Tractrix
Circle
Exponential curve
Line family
Catenary
Loxodrome
Catenoid
Helix
Equiangular spiral
Helicoid
Archimedean spiral
Plane
Tiling
Polygon
Polyhedron

5 B






Wheel axle
Path of axle
Side of
square wheel
Road (series of
catenary humps)
Catenary, envelope
of normals to tractrix
Normal to tractrix
(= tangent to catenary)
Path of toy cart (tractrix)
Hand
Front edge of board (asymptote to tractrix)



Using Mamikon's method of sweeping tangents it can be shown that the area under the red curve is *sl*.