

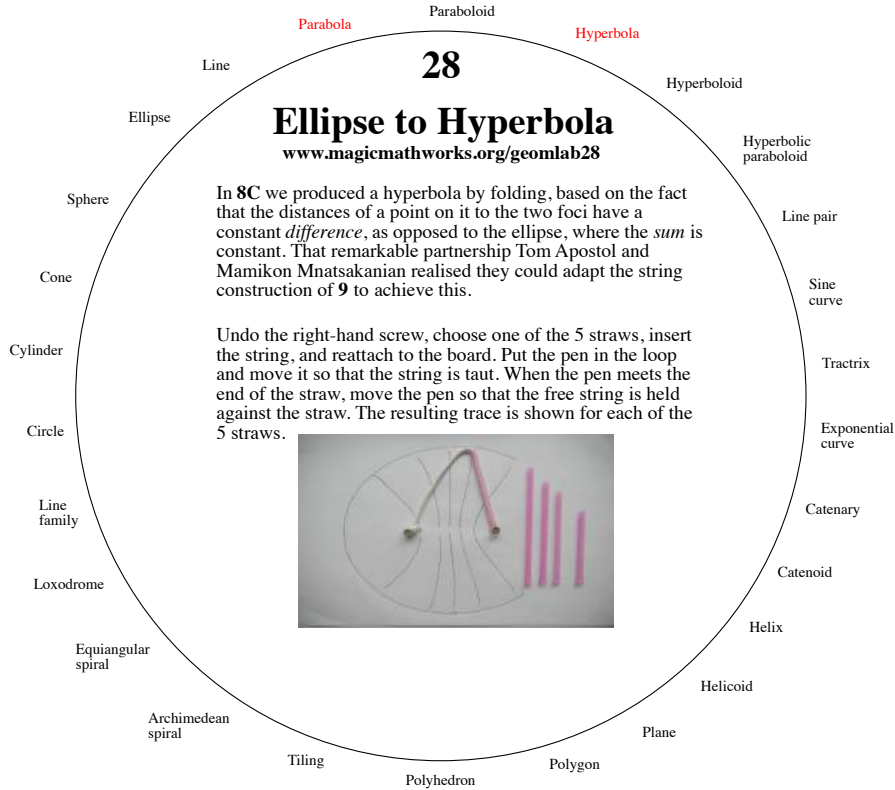
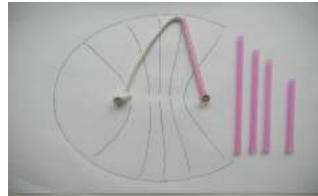
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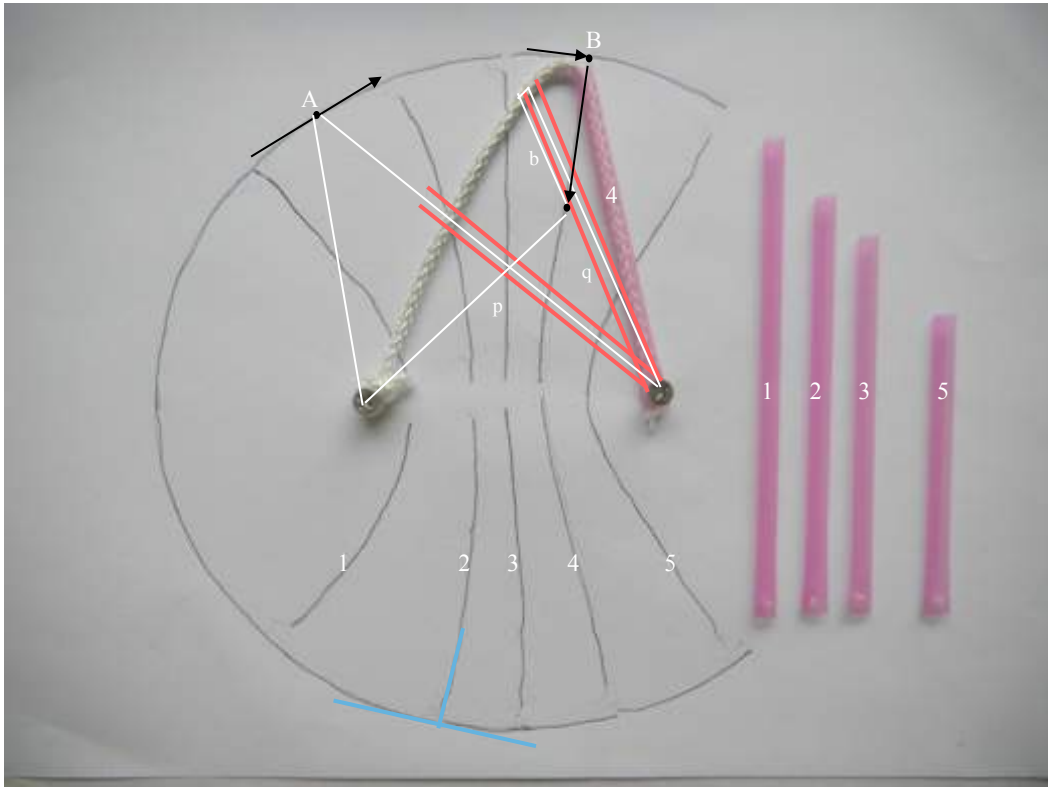
Ellipse to Hyperbola

www.magicmathworks.org/geomlab28

In **8C** we produced a hyperbola by folding, based on the fact that the distances of a point on it to the two foci have a constant *difference*, as opposed to the ellipse, where the *sum* is constant. That remarkable partnership Tom Apostol and Mamikon Mnatsakanian realised they could adapt the string construction of **9** to achieve this.

Undo the right-hand screw, choose one of the 5 straws, insert the string, and reattach to the board. Put the pen in the loop and move it so that the string is taut. When the pen meets the end of the straw, move the pen so that the free string is held against the straw. The resulting trace is shown for each of the 5 straws.





- A: As long as it stays clear of the straw's end, the pencil follows an ellipse.
- B: When it reaches the end, it turns abruptly through a right angle (as shown in blue).
- C: Thereafter it follows a hyperbola, sharing the ellipse foci.

The paths for 5 lengths of straw, 1 - 5, are shown.

We can show that path C is a hyperbola as follows.

$p + 2b + q = c$, the length of the string,
 i.e. $p - q = c + 2(b + q)$.
 But $(b + q) = d$, the length of the straw, also constant.
 therefore $p - q$ is constant, defining a hyperbola.