

# Courant's Shrinking Bubble

[www.magicmathworks.org/geomlab24](http://www.magicmathworks.org/geomlab24)

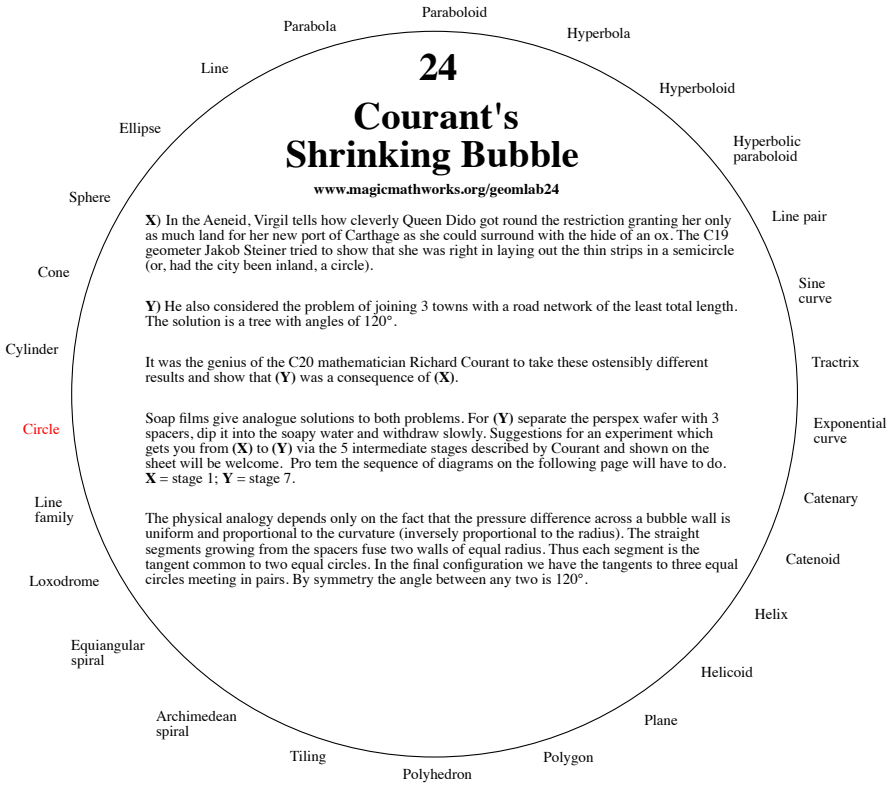
**X)** In the Aeneid, Virgil tells how cleverly Queen Dido got round the restriction granting her only as much land for her new port of Carthage as she could surround with the hide of an ox. The C19 geometer Jakob Steiner tried to show that she was right in laying out the thin strips in a semicircle (or, had the city been inland, a circle).

**Y)** He also considered the problem of joining 3 towns with a road network of the least total length. The solution is a tree with angles of  $120^\circ$ .

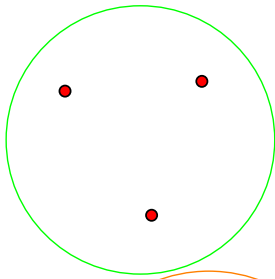
It was the genius of the C20 mathematician Richard Courant to take these ostensibly different results and show that **(Y)** was a consequence of **(X)**.

Soap films give analogue solutions to both problems. For **(Y)** separate the perspex wafer with 3 spacers, dip it into the soapy water and withdraw slowly. Suggestions for an experiment which gets you from **(X)** to **(Y)** via the 5 intermediate stages described by Courant and shown on the sheet will be welcome. Pro tem the sequence of diagrams on the following page will have to do. **X** = stage 1; **Y** = stage 7.

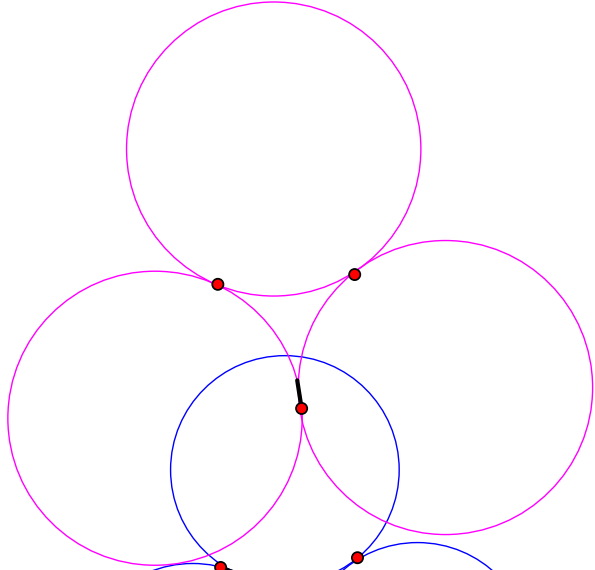
The physical analogy depends only on the fact that the pressure difference across a bubble wall is uniform and proportional to the curvature (inversely proportional to the radius). The straight segments growing from the spacers fuse two walls of equal radius. Thus each segment is the tangent common to two equal circles. In the final configuration we have the tangents to three equal circles meeting in pairs. By symmetry the angle between any two is  $120^\circ$ .



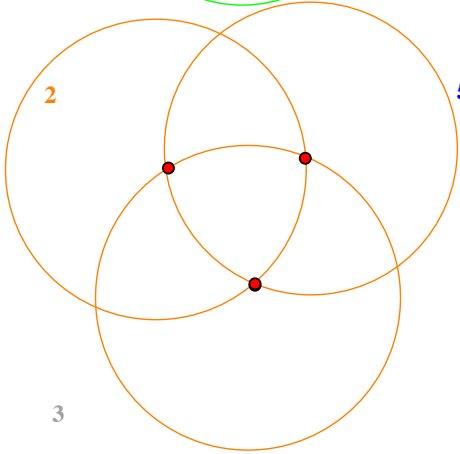
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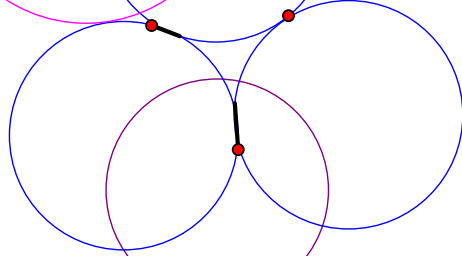
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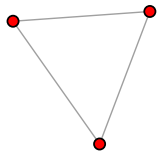
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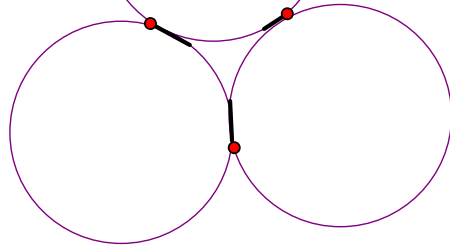
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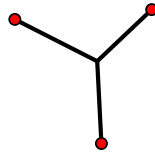
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6



7



**Stage 1**



**Stage 7**

