Fibonominoes

On p.17 of mt215 (September 2009) we drew your attention to an NRICH environment in which you can manipulate polyominoes based on Fibonacci numbers. Here I analyse the pieces themselves, which depend on the fact that Fibonacci numbers are alternately sums and differences of squares. You may find it helpful to build the shapes with Multilink, as we did in mt215.

The Fibonacci number $F_n = \dfrac{\phi^n - (-1)^n \phi^{-n}}{\sqrt{5}}$, where $\phi = \dfrac{\sqrt{5}+1}{2}$.

Substitute in the left-hand sides to confirm the following identities:

$$F_{k+1}^2 - F_{k-1}^2 = F_{2k}, \quad \textbf{(1)}$$
$$F_{k+1}^2 + F_k^2 = F_{2k+1}, \quad \textbf{(2)}$$

characterising alternate terms in the sequence.

Diagram **(1)** as a square with a small square cut symmetrically from a corner [FIG. 1A].
Diagram **(2)** as a like square, an adjacent smaller square sharing a side [FIG. 1B].
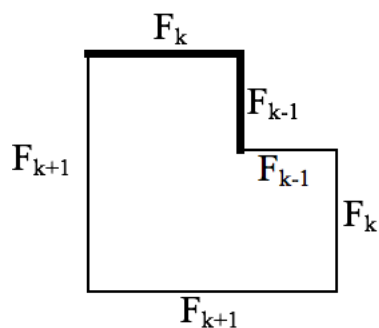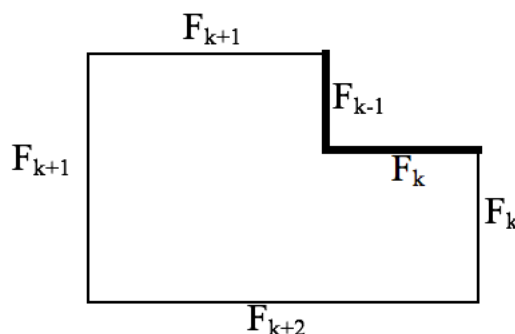
Mark lengths:

FIGURE 1A   FIGURE 1B

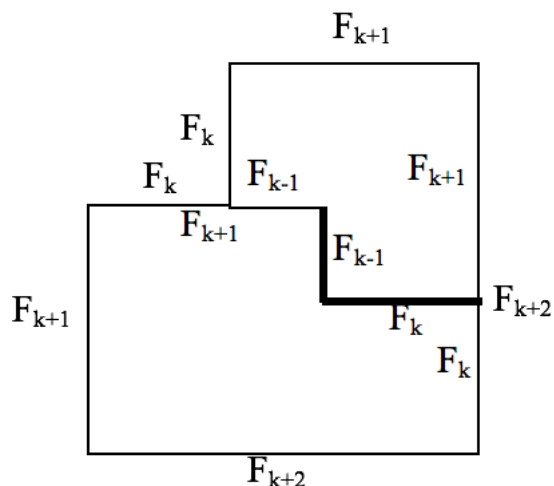Join the two by the matching parts picked out in bold [FIG. 2]:

FIGURE 2

You obtain the next term in the alternating sequence in a geometrical form matching **(1)**.

I leave you to identify matching pairs of coincident edges on FIG. 1B and FIG. 2 and repeat. You will not be surprised to obtain a form matching **(2)**.

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