Fibonomino (= Fibonacci) **identities**

If we accept that fibonominoes characterise Fibonacci numbers correctly, we can write all the following equations with capital Fs. The proofs are proofs-without-words. In cases **3** and **4** a complete proof would proceed by induction and we would have to justify the inductive step by showing that the fibonominoes add in the way assumed.

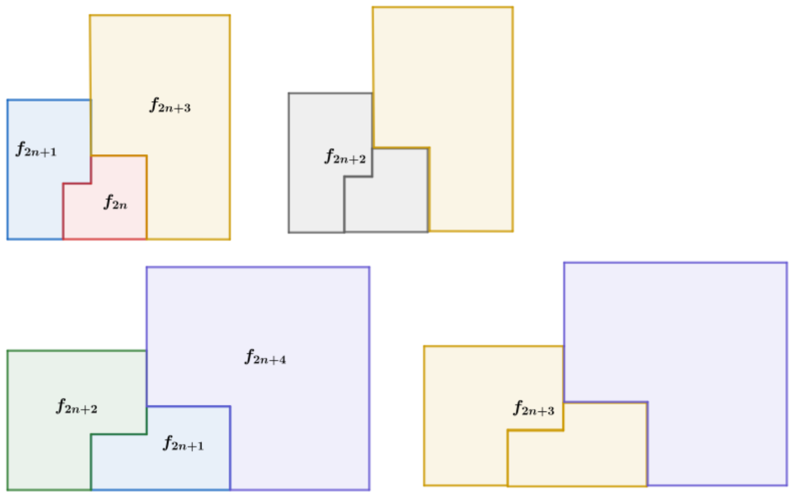
1. $f_n + f_{n+1} + f_{n+3} = f_{n+4}$

The proof follows immediately from the defining equation:

$(f_n + f_{n+1}) + f_{n+3}$
$f_{n+2} + f_{n+3}$
f_{n+4}

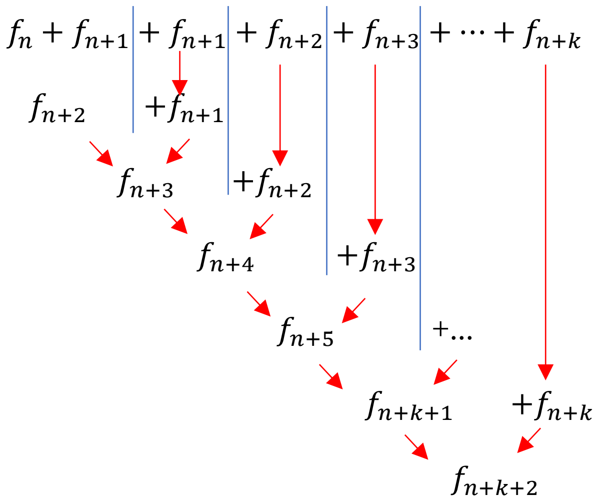
The upper figures show the case where the smallest fibonomino is even.

The lower figures show the case where the smallest fibonomino is odd.

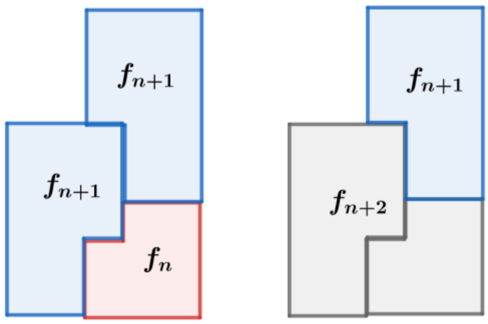


2. $f_n + 2f_{n+1} + f_{n+2} + f_{n+3} + \cdots + f_{n+k} = f_{n+k+2}$

The algebra look like this:



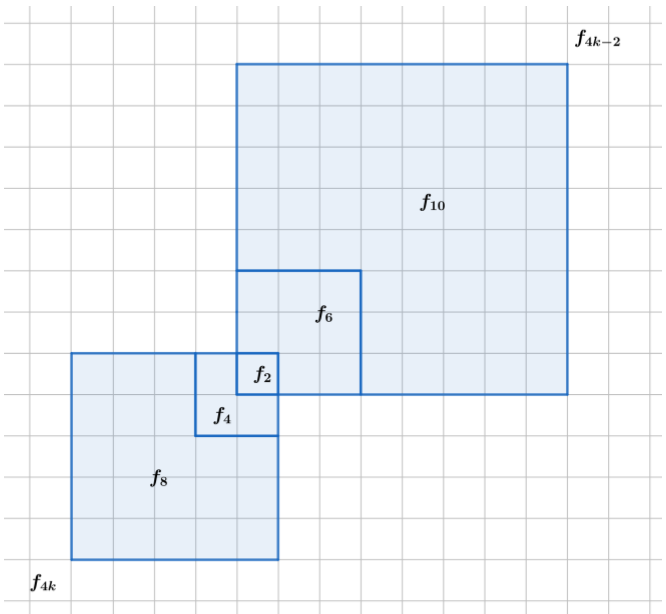
The geometry looks like this:



As you see, by duplicating f_{n+1}, we create the grey block, f_{n+2}, giving the sequence starting $f_{n+1}, f_{n+2}, \dots$, which will continue to produce new fibonominoes by the addition of consecutive blocks.

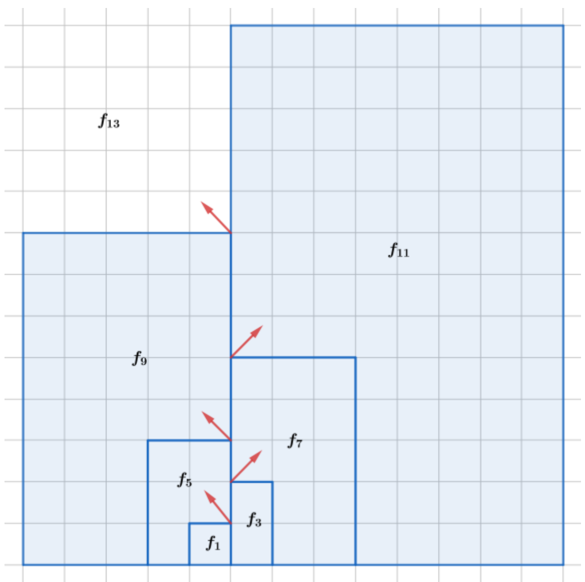
By duplicating f_{n+1} we get the fibonomino two beyond the last one added, f_{n+3} in the figure.

3. $\sum_{j=1}^{j=n} f_{2j} = f_{2n+1} - 1$



The terms added are gnomons to existing squares. Consecutive terms add alternately to the north-east and south-west square. The two squares belong to the fibonomino f_{2n+1}.They overlap in a single cell. Thus the sum is f_{2n+1}-1.

4. $\sum_{j=1}^{j=n} f_{2j-1} = f_{2n}$



The arrows are there to show how each block added completes a difference of two squares, that is, the addition of f_{2n-1} completes f_{2n}.