Fibonomino formulas

0	F_0	$+0^2$	-0^2							
1	F_1	$+1^2$		$+1^2$	-1^2					
1	F_2		$+1^2$	$+1^2$		$+1^2$	-1^2			
2	F_3				$+2^2$	$+2^2$		$+2^2$	-2^2	
3	F_4						$+3^2$	$+3^2$		
5	F_5								$+5^2$	
⋮	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	…
	0	1	1	2	3	5	8	13	21	…

Generalising from the table, we have:

$F_{2n-1} = F_{n-1}{}^2 + F_n{}^2$.
$F_{2n} = -F_{n-1}{}^2 + F_{n+1}{}^2 = (F_{n+1} + F_{n-1})(F_{n+1} - F_{n-1}) = (F_{n+1} + F_{n-1})\, F_n$.

When we add the two to get F_{2n+1}, we see how the term which cancels corresponds to the 'peg' of a sum-of-squares fibonomino fitting the 'hole' of a difference-of-squares fibonomino.

It is less clear why the result of this sum:

$F_{2n} = -F_{n-1}{}^2 + F_{n+1}{}^2$
$F_{2n+1} = F_n{}^2 + F_{n+1}{}^2$

is a difference of squares. We combine the difference-of-two-squares identity with the defining Fibonacci identity as above :

$F_{2n} = F_{n+1}{}^2 - F_{n-1}{}^2 = (F_{n+1} + F_{n-1})(F_{n+1} - F_{n-1})$
$= (F_{n+1} + F_{n-1})F_n = F_n F_{n+1} + F_{n-1} F_n$

The sum is now:

$F_n F_{n+1} + F_{n-1} F_n$
$F_n{}^2 + F_{n+1}{}^2$

Total:

$F_n(F_{n-1} + F_n) + F_{n+1}(F_n + F_{n+1}) = F_n F_{n+1} + F_{n+1} F_{n+2}$
$= F_{n+1}(F_n + F_{n+2}) = F_{n+2}{}^2 - F_n{}^2$, which is the expected value of F_{2n+2}.

(The expressions in blue are analogous to those in red.)