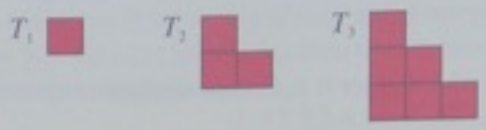
ASSEMBLING TRIANGULAR NUMBERS

From their name you will realise that we can build the triangular numbers by joining '*Multilink*' cubes like this:

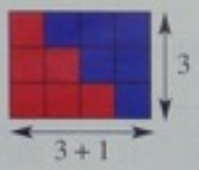


... and so on.

We'll assemble copies of such models in different ways.

When you make a physical model, as we are doing, you're always taking a *special* case. You hope, however, that it will lead you to the *general* case, expressed with the help of algebra. Alongside each diagram I'll write an algebraic equation. These equations are in fact true for all values of the variable, n. Such an expression is called an *identity*. It will be your task to work out how I got from the diagram to the identity.

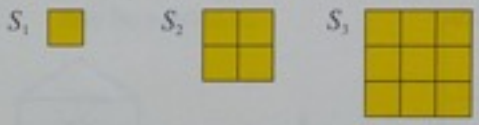
(1)



$$T_n = \frac{n(n+1)}{2}$$

(2)

Here are the first 3 square numbers:

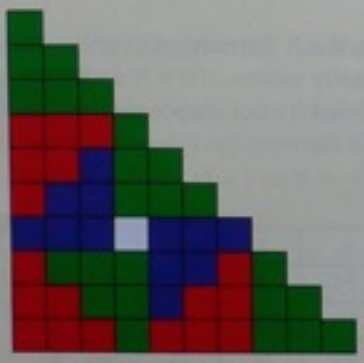


$$S_n = T_n + T_{n+1}$$

(3)

$$S_{2n+1} = 8T_n + 1$$

(4)



$$T_{3n+1} = 9T_n + 1$$

(5) How could you alter the last model to show this:

$$T_{3n+1} = S_{2n+1} + T_n ?$$

Build models to show these results:

(6)

$$T_{2n} = 3T_n + T_{n-1}$$

(7)

$$T_{2n+1} = 3T_n + T_{n+1}$$

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The Magic Mathworks Travelling Circus