Contents

The Magic Manual

Section 8

Loci and Linkages

A guide for fabricators and users to stations from the Magic Mathworks Travelling Circus

8. LOCI & LINKAGES

8.1 - **8.4** concern loci;
8.5 - **8.6** concern loci and linkages;
8.7 concerns linkages.

p Whereas the linkage work is designed to teach the investigators specific properties, the locus work is intended to encourage the students to visualise the motions without formalising the geometry involved.

8.1

p As stated.

c The curves produced are as follows.
The ratios have been chosen to produce interesting special cases.

8.1.1 ROLLING CURVES 1

straight track: wheel:	central hole: straight line middle hole: trochoid outer hole: cycloid
drum: wheel:	central hole: circle middle hole: hypotrochoid outer hole: hypocycloid

8.1.2 ROLLING CURVES 2

hub: straight arm:	Archimedean spiral
hub: wheel:	central hole: circle middle hole: epitrochoid outer hole: epicycloid

8.2

p Children who fail to study or understand the instructions assume the task is **8.2.1** rather than **8.2.2**. To distinguish the second activity, **8.2.1** is therefore presented as an experiment in its own right.

8.2.1 THE FALLING LADDER

c The loci are circular arcs centred on the ladder's foot.

8.2.2 THE SLIDING LADDER

c The general case is the ellipse with axes aligned along the perpendicular guides. The midpoint of the ladder describes a circular arc - just as it does in **8.2.1**. The circle marks the point of transition at which the major and minor axes are equal.

8.3 THE TRAPPED ANGLE

c The model demonstrates the converse of the 'angles in same segment' property of the circle, viz. a point P which moves so that 2 fixed points subtend a constant angle at P traces a circular arc.

p One can change the angle of the sector and the spacing of the fixed points and observe the effect on the circle radius.

8.4 THE EQUIANGULAR SPIRAL

c This curve has the defining property that the tangent makes a constant angle with a line to a fixed point. One generates the curve as an envelope. The shorter the segments, the closer the approximation to the curve itself.

p Templates for different angles are provided.

8.5 THE PANTOGRAPH

c In Euclidean terms, and focussing on the linkage, we are concerned with similar triangles. In terms of transformation geometry, and focussing on the resulting locus, we are concerned with enlargements (dilatations).

p As stated.

8.6 'HOW TO DRAW A STRAIGHT LINE'

c In the steam age the conversion of rotary to linear motion was an engineering necessity. The model shows Peaucellier's solution to the problem. (The title is that of the book Kempe produced from his lecture series on the topic.)

p For the younger experimenters the device is a curiosity. Older students can use the geometry of the linkage to prove that the resulting locus is indeed a straight line.

8.7

c Complex as link motions in real machines can be, particularly in animal skeletons, they are built from a handful of simple elements.
Here we single out 2 jointed polygons: the general parallelogram and a special case, the rhombus. The properties of the linkages illuminate the properties of the static polygons because their definitions, encompassing as they must the general case, necessarily treat the polygons as if they are jointed.

8.7.1 LINKAGES 1

p In the form of a line drawing the investigators are given a special case which they must achieve by flexing the jointed model.

8.7.2 LINKAGES 2

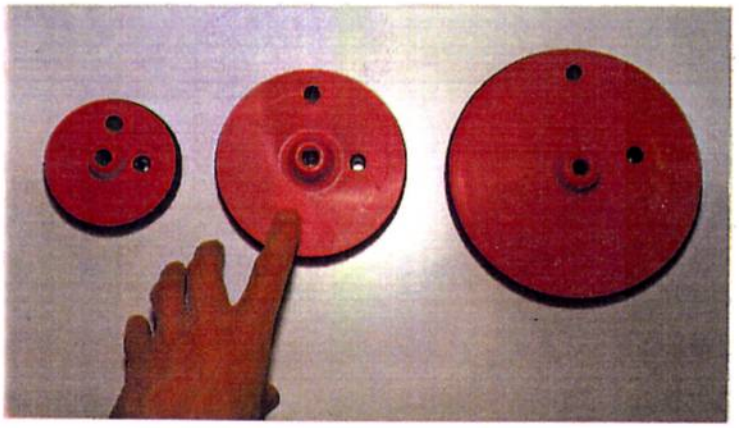
p In **LINKAGES 1** we animate a static diagram; in **LINKAGES 2** we locate the diagram in a real embodiment.

SECTION		AGE RANGE					
LOCI & LINKAGES		Appropriate point of entry - not necessarily to the task set by the caption - and levels on which extension activities generated (some to be pursued off-site)					
STATION							
NUMBER	NAME	4	7	10	13	16	19+
8.1.1	ROLLING CURVES 1		*	*	*		*
8.1.2	ROLLING CURVES 2		*	*	*		*
8.2.1	THE FALLING LADDER		*	*	*		
8.2.2	THE SLIDING LADDER		*	*	*	*	
8.3	THE TRAPPED ANGLE		*	*	*	*	
8.4	THE EQUIANGULAR SPIRAL					*	*
8.5	THE PANTOGRAPH				*	*	
8.6	'HOW TO DRAW A STRAIGHT LINE'			*	*	*	
8.7.1	LINKAGES 1		*	*	*		
8.7.2	LINKAGES 2			*	*		

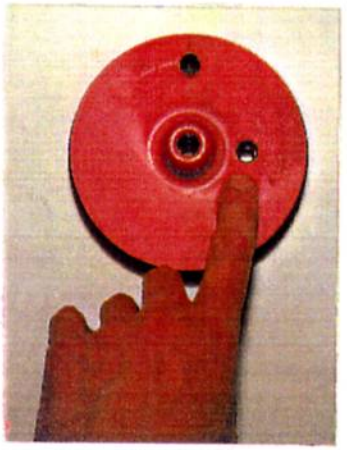
	INSTRUCTION NEEDED				SUPERVISION NEEDED			SERVICING NEEDED		
→	Needs no explanation or caption	Caption enough for most people	Needs aural prompt	Visitors must be talked through stages	None	Benefits from a helper's input	Session must be directed	None	A little	Much
8.1.1		*				*			*	
8.1.2		*				*			*	
8.2.1		*				*			*	
8.2.2			*			*			*	
8.3		*				*			*	
8.4			*			*			*	
8.5			*			*			*	
8.6		*			*			*		
8.7.1		*			*			*		
8.7.2		*			*			*		

	NUMBER	TITLE
GROUP	8	LOCI & LINKAGES
STATION	8.1.1	ROLLING CURVES 1
TOPIC		Cycloids, trochoids

ROLLING CURVES 1

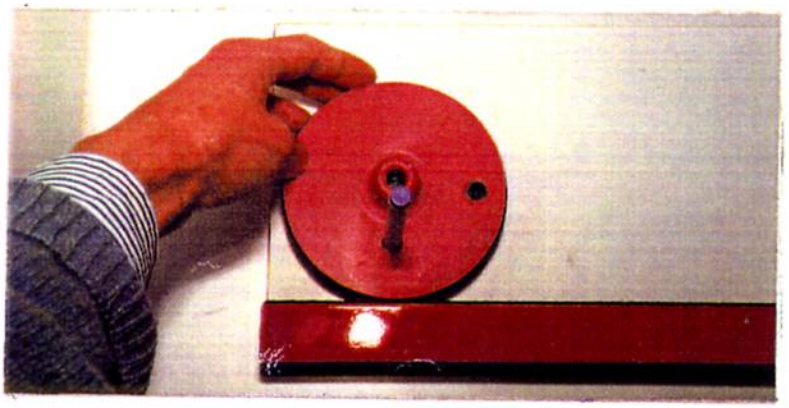


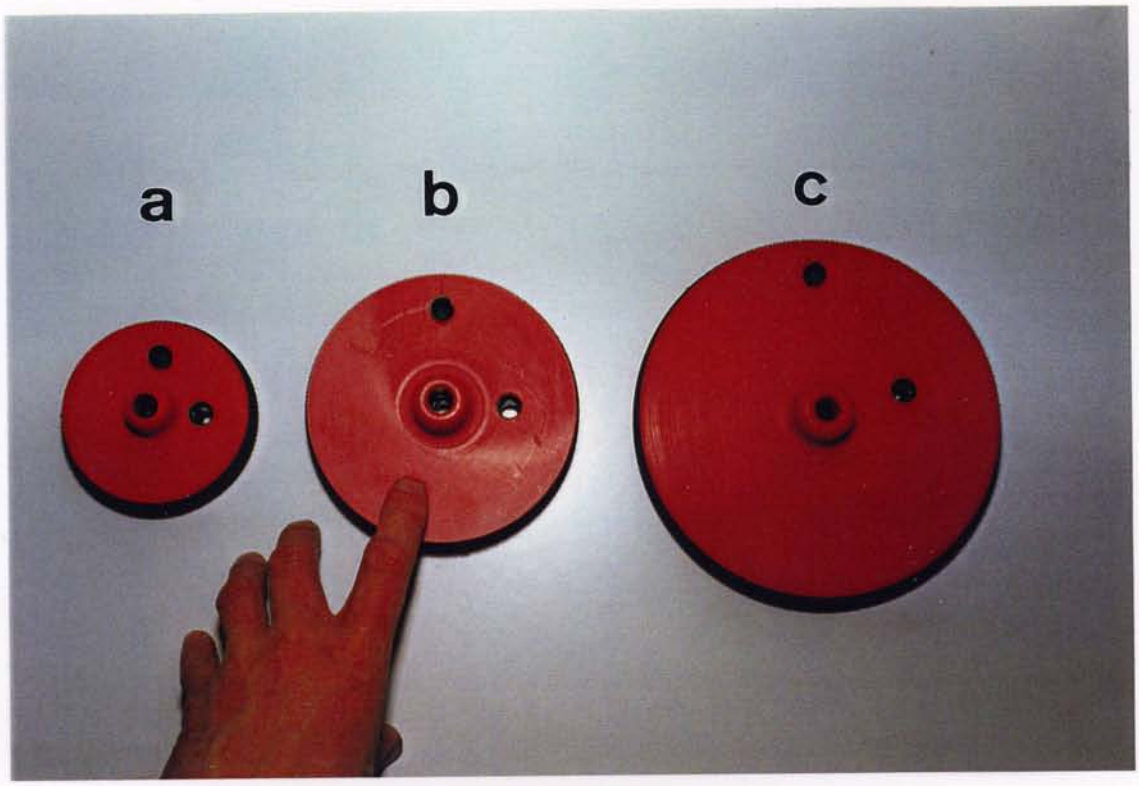
- Choose a wheel.

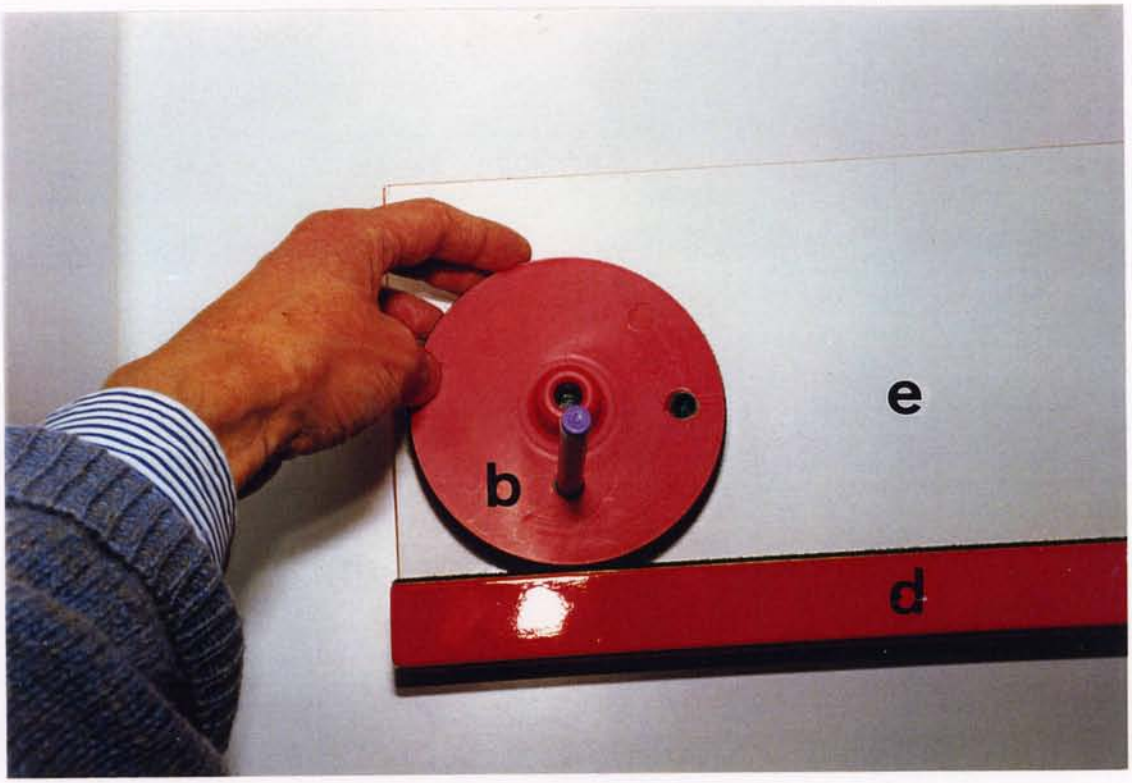


- Choose a hole.
- Click in a pen.

- Draw with your finger what you think the pen will draw.
- Roll the wheel.



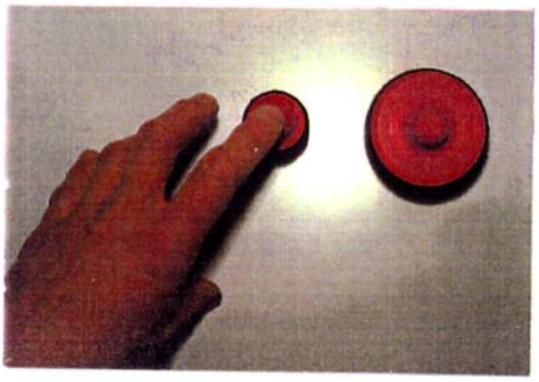

a
b
c

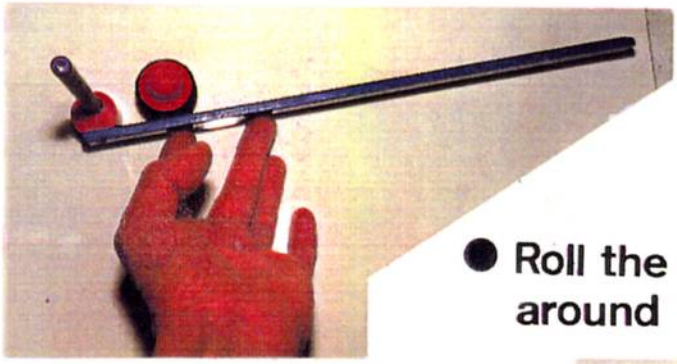

b
e
d

	NUMBER	TITLE
GROUP	8	LOCI & LINKAGES
STATION	8.1.2	ROLLING CURVES 2
TOPIC		Archimedean spirals; epitrochoids, hypotrochoids epicycloids, hypocycloids

ROLLING CURVES 2

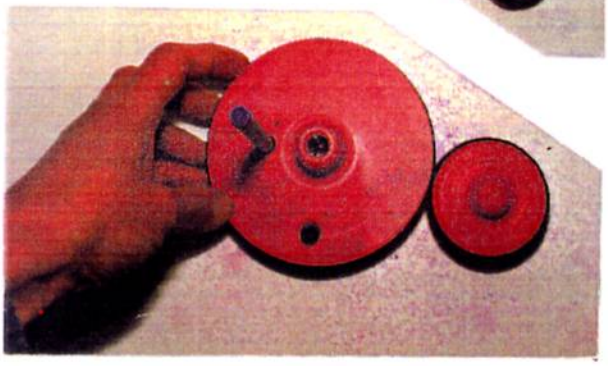
- Choose a hub.
- Seat it on the pegs at the centre of the board.



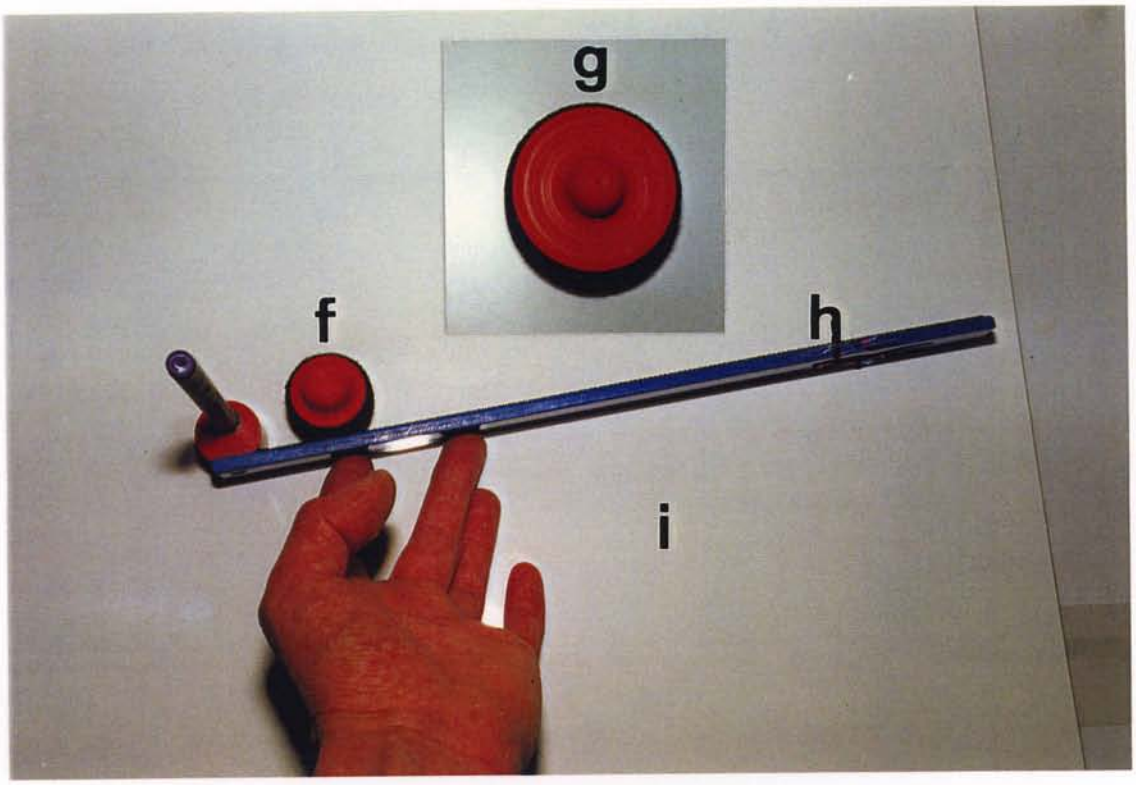


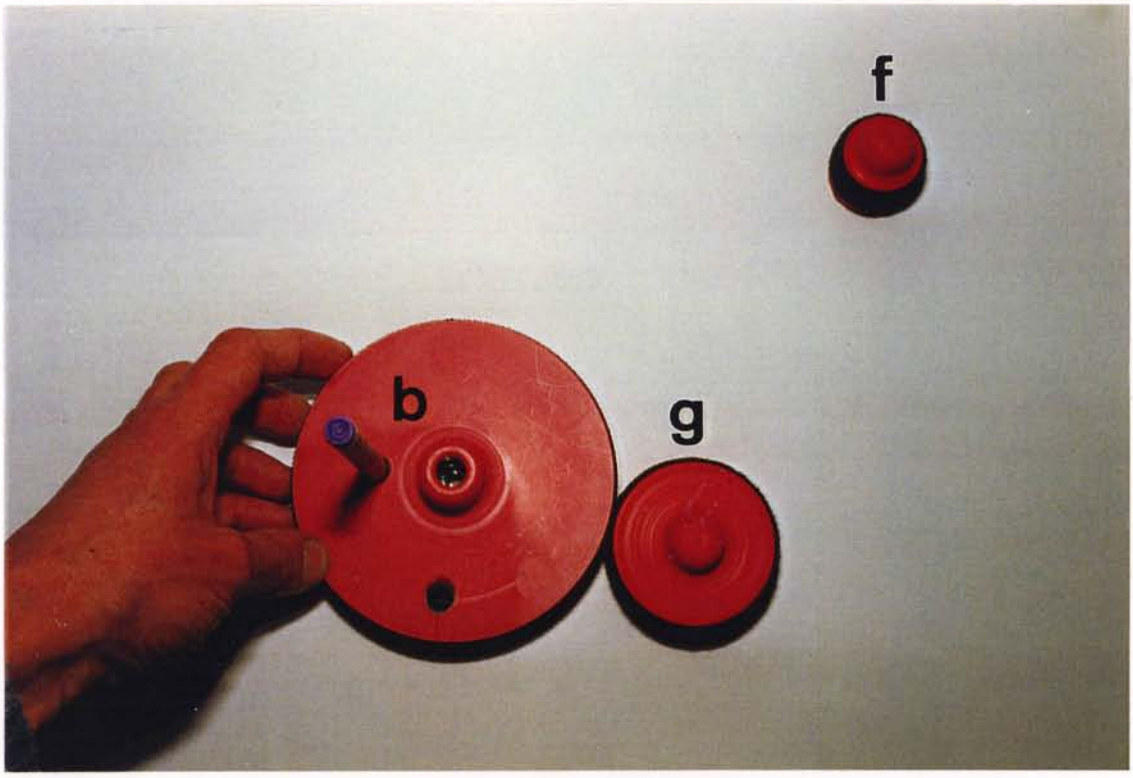
- Draw with your finger what you think the pen will draw.
- Roll the stick around the peg.

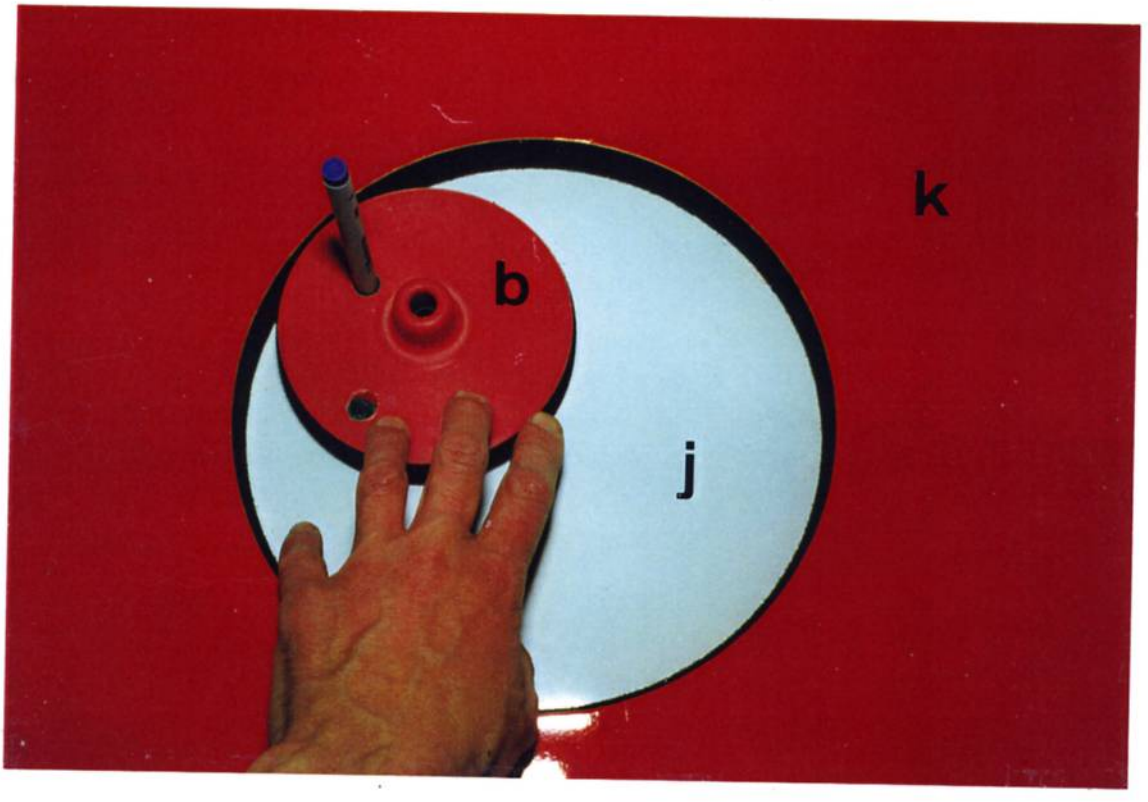
- Try a wheel.



- Try it in the drum.


g
f
h
i


f
b
g


k
b
j

l


LOCI Stick a object to a red object. Heat a pen in a hole. Roll it - but roll it in your mind first!

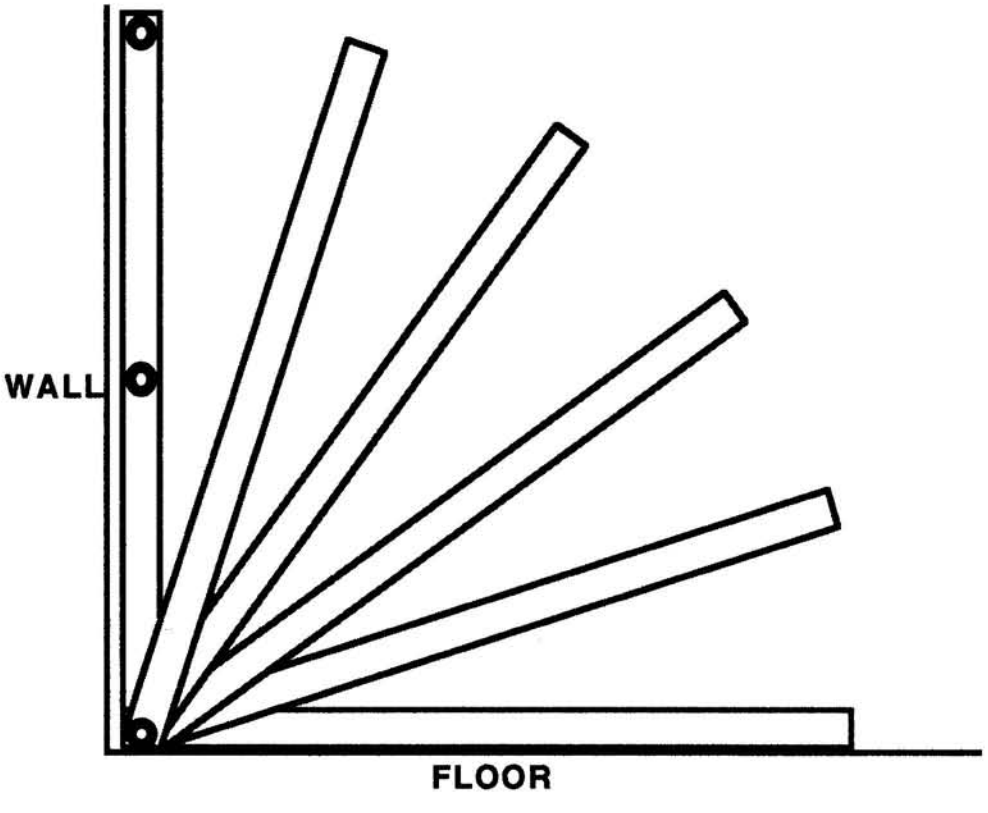
PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
	The apparatus for **8.1.1** and **8.1.2** shown was built by: from whom complete specifications should be sought. However, essential details are given below.		Techniquest Enterprises Ltd Stuart Street Pier Head Cardiff CF1 6BW T +44 1222 475475 F +44 1222 482517
	<u>general details</u>		
a,b,c,d, f,g,h,k	working face 15 mm wide, with central band, 10 mm wide, of:		
a,b,c,h	'hook' velcro		
d,f,g,k	'loop' velcro		
e,i,j	baseboards in 20 mm MDF, faced with Glodex		
d,k	in 15 mm MDF, painted to match:		
a,b,c, f,g,h	in PVC (**h** blue only in prototype shown)		
	<u>radii</u>		
	pen tip in middle hole: / pen tip in outer hole: / overall*:		
	f — 15 mm **g** — 30 mm **a** 20 mm = overall 40 mm **b** 30 mm = overall 60 mm **c** 40 mm = overall 80 mm **k** — 120 mm		
	*must include velcro in compressed state		
	<u>apparatus common to **8.1.1** and **8,1.2**</u>		
a,b,c	'wheels', holes drilled to accommodate dri-wipe pens#		(see **THE CAPTION BOARDS**)
	To bring the outer pen tip to the wheel circumference the outer hole must be drilled at the requisite angle.		
	<u>apparatus specific to **8.1.1**</u>		
d,e	track, **d** stuck on **e** where shown, length at least 1510 mm, i.e. 3 x circumference of largest 'wheel'; overall width: 200 mm, i.e. that of **d** + exposed **e**:		
	d: 35 mm, **e**: 165 mm, i.e. at least diameter of largest wheel		
	baseboard (**e**) therefore: 1510 mm (wide) x 200 mm (deep)		
	track strip (**d**) therefore: 1510 mm (wide) x 35 mm (deep)		

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
	apparatus specific to **8.1.2**		
f,g	'hubs'		
	These are located on 2 steel pins, protruding from the centre of **i**.		
h	'stick', 300 mm x 10 mm, with attached collar, drilled to hold dri-wipe pen#		
	The pen tip is 15 mm from the working face, giving an accurate Archimedean spiral for the smaller 'hub'. For all practical purposes, however, the collar might as well terminate the 'stick' symmetrically.		
i	baseboard, 600 mm square		
j,k	'drum'		
k	'drum' proper, cut out of board 600 mm square		
	# The pen shown on the captions and in the photographs is not of the type now used: **Pilot WBMA-TM.** **These are cylindrical, dri-wipe pens, which rest in the holes under their own weight.**		
l	**Another change:** **All moving items - 'wheels' and 'sticks' - are made from clear PVC. As the object is transparent, the experimenters can observe their progress and stop when the trace is complete.**		

	NUMBER	TITLE
GROUP	8	LOCI & LINKAGES
STATION	8.2.1	THE FALLING LADDER
TOPIC		The circle

THE FALLING LADDER

- **The foot of the ladder sticks in the corner. The top of the ladder falls backwards.**
- **Choose a rung - red, yellow or green. Rest a pen in it.**
- **What will the pen draw as the ladder falls?**



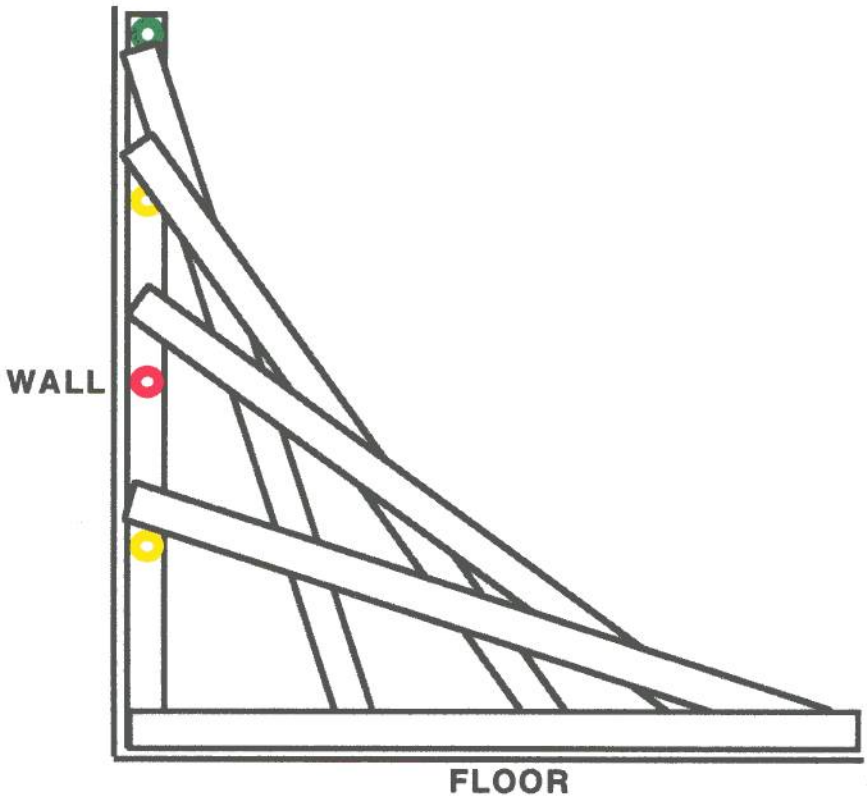
	NUMBER	TITLE
GROUP	8	LOCI & LINKAGES
STATION	8.2.2	THE SLIDING LADDER
TOPIC	The ellipse - with the circle as a special case	

THE SLIDING LADDER

- **Choose a rung - red, yellow or green. Ask a partner to rest a pen in it as you operate the ladder.**

- **With your left hand keep the top of the ladder pressed against the wall. With your right hand slide the foot of the ladder out across the floor from the corner.**

- **What will the pen draw as the ladder slips?**

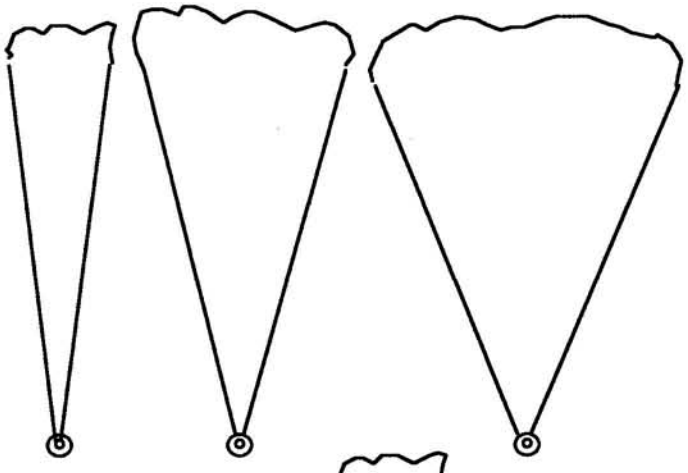


	NUMBER	TITLE
GROUP	8	LOCI & LINKAGES
STATION	8.3	THE TRAPPED ANGLE
TOPIC		Circle properties

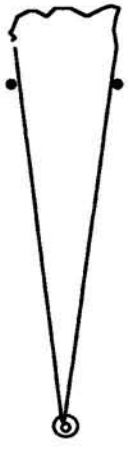
THE TRAPPED ANGLE

- **Choose 2 of the small holes.**
  Insert 2 pegs from the back of the board:

- **Choose a sector:**



- **Sit it between the 2 pegs:**



- **Rest a pen in the hole.**
  Move it so that the sector never leaves the pegs.

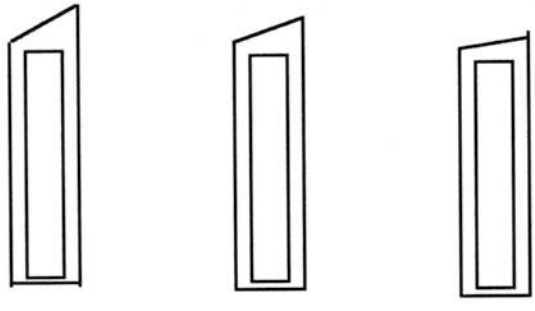
■ **The curve you have drawn is a special one.**
If you trace part of it on a clear sheet
and slide your tracing along, it fits everywhere.

What have you drawn?

	NUMBER	TITLE
GROUP	8	LOCI & LINKAGES
STATION	8.4	THE EQUIANGULAR SPIRAL
TOPIC		The equiangular spiral

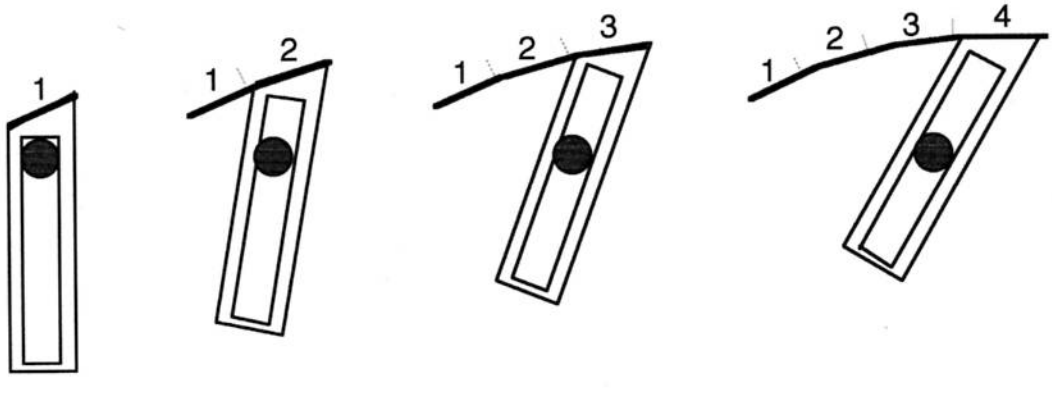
THE EQUIANGULAR SPIRAL

- **Choose a slider:**



- **Peg it to the board with a magnet:**

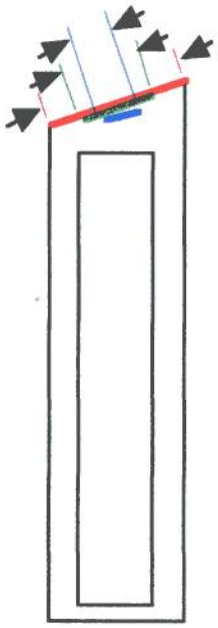
- **Proceed like this:**



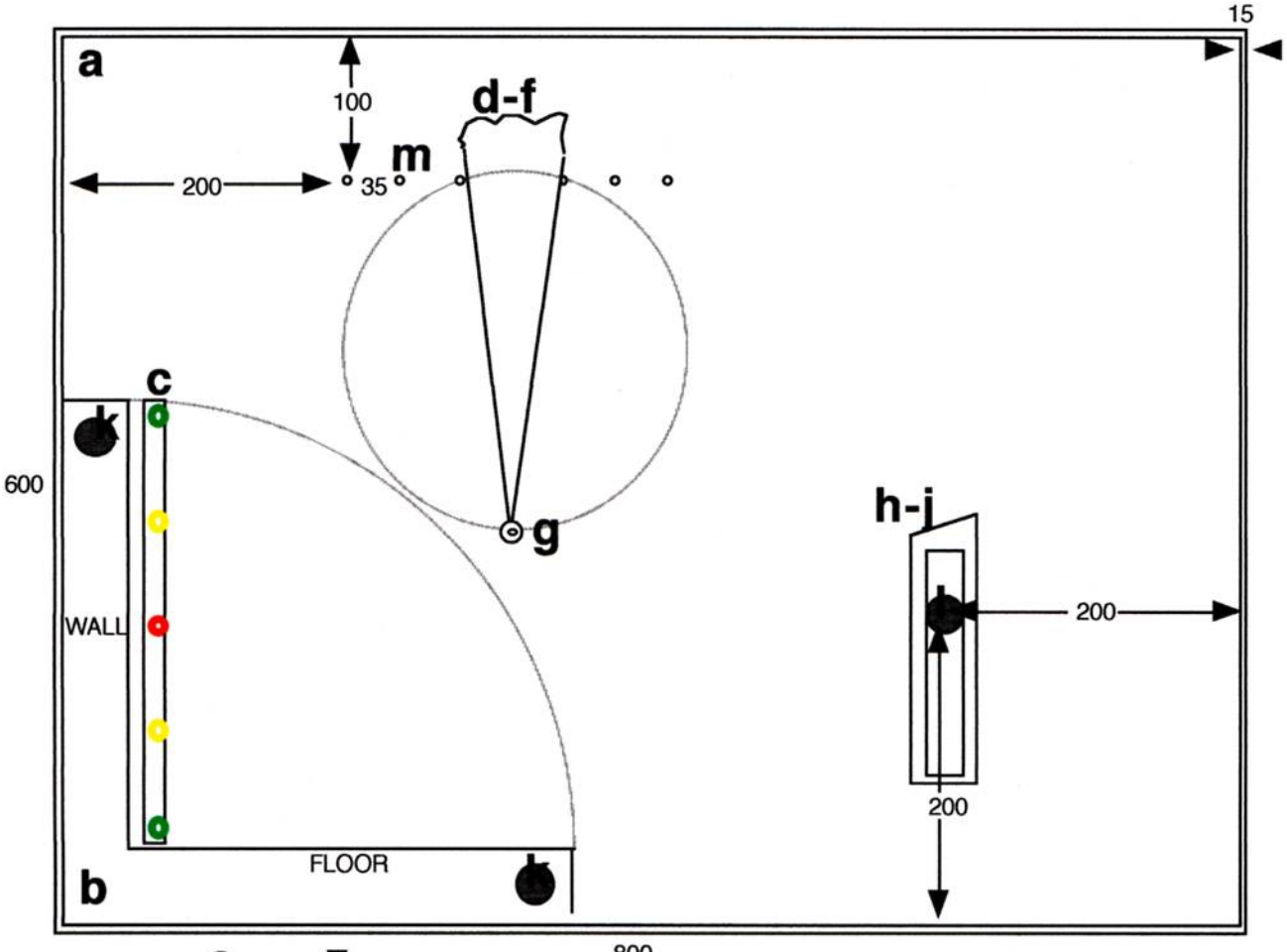
	NUMBER	TITLE
GROUP	8	LOCI & LINKAGES
STATION	8.4	THE EQUIANGULAR SPIRAL
TOPIC		The equiangular spiral (contd.)

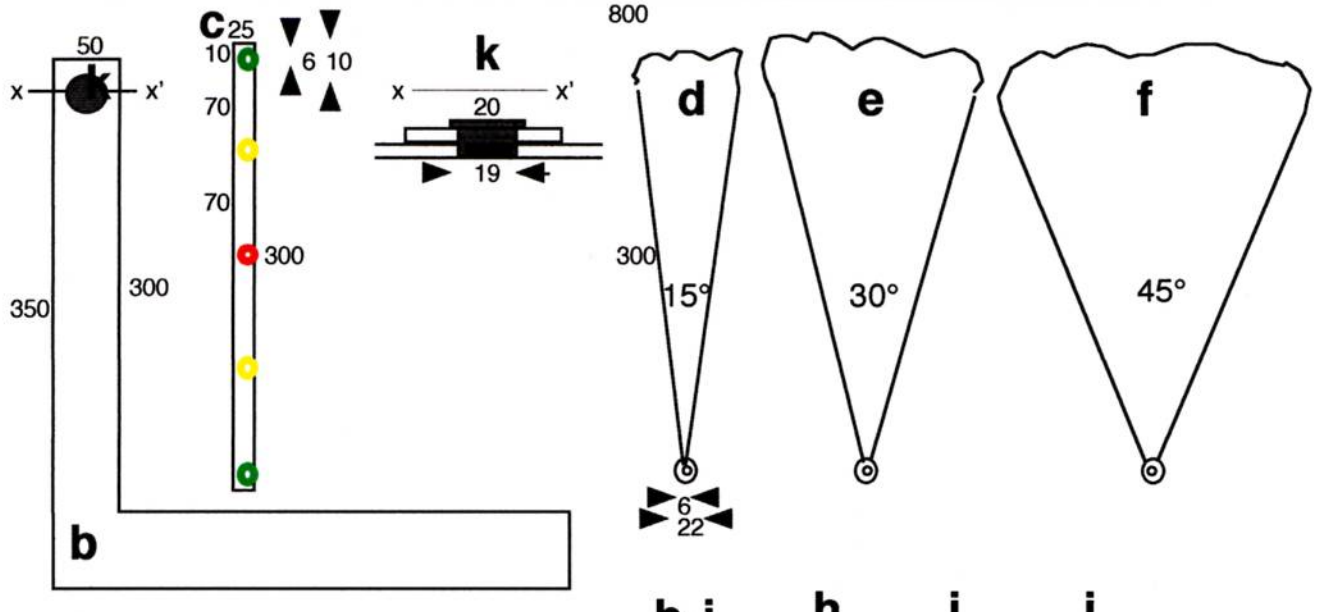
THE EQUIANGULAR SPIRAL

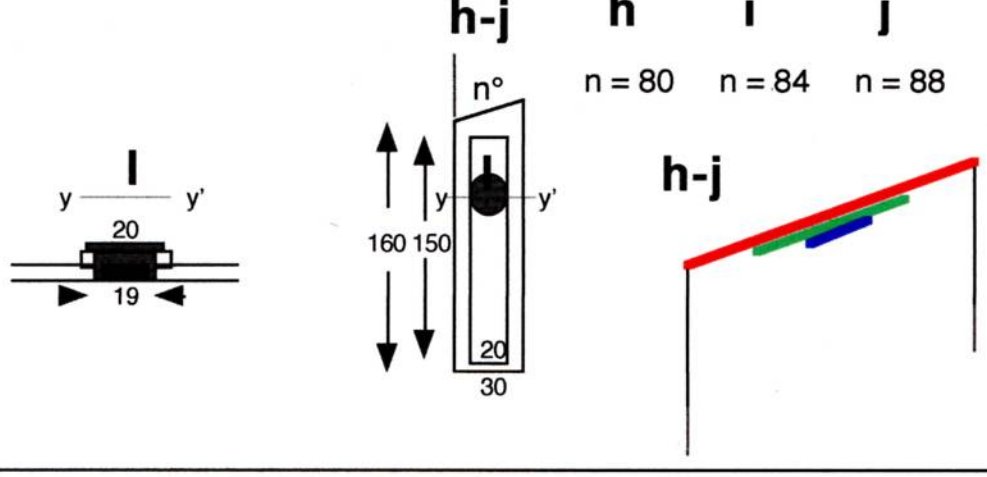
- **Choose also the length of your line segment - red, green or blue:**



- **After you've tried one colour, predict the effect of a different one.**

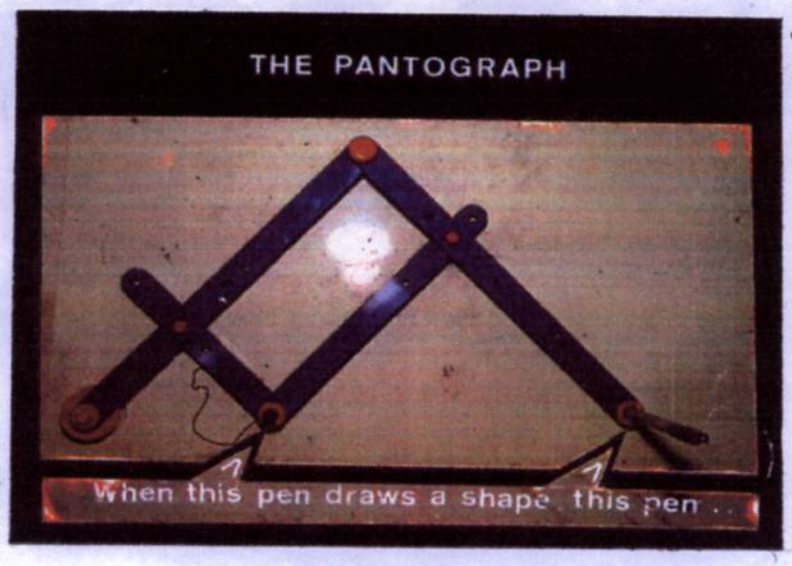

a
15
100
d-f
m
200
35
c
600
g
h-j
200
WALL
200
FLOOR
b


50
x
x'
c
25
10
6
10
70
k
x
x'
20
19
70
300
350
300
b
800
d
e
f
300
15°
30°
45°
6
22

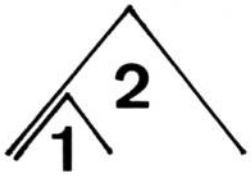

h-j
h
i
j
n°
n = 80
n = 84
n = 88
l
y
y'
20
19
160
150
y
y'
20
30
h-j

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	magnetboard, 800 mm x 600 mm, with raised border, faced with 2 mm Glodex, drilled to locate magnets and pegs where shown		local IKEA
b	guide for **8.2.1-2**, 3 mm Glodex, marked as shown, drilled to accommodate:		local
k, l	magnet	Nobo 20 mm	local Staples or manufacturer, Acco: www.accoeurope.com
c	ladder, 3 mm Glodex, marked and drilled as shown		local
c, g	The hole size is chosen to accommodate the standard dri-wipe pen, Pilot WBMA-M.		
c, h-j	Beneath are tiny squares of felt, 1 mm thick or less, located at corners to act as spacers.		local
d-f	sectors for **8.3** in 2 mm Glodex, attached with epoxy resin to		local
g	large steel washer		local
m	moveable pegs as guides, nylon 5 mm long, 4 mm diameter The magnetboard itself is drilled through. The pegs are inserted from the back and have collars to prevent them popping out under pressure. The diameter of the emergent part is 4 mm.		local
h-j	templates for **8.4** in 2 mm Glodex, marked as shown in inset The dotted lines show the maximum extent of the drawing area in cases **8.2.1-2**, **8.3**. With holes cut around magnets and pegs, sheets of paper may be interposed for a permanent record.		local

	NUMBER	TITLE
GROUP	8	LOCI & LINKAGES
STATION	8.5	THE PANTOGRAPH
TOPIC		Similar triangles; enlargement



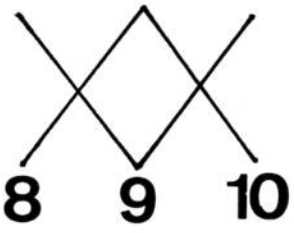
The linkage articulates 2 isosceles triangles, **1.**, **2.**, so as to preserve their similarity.



It does so by means of the jointed parallelogram, **3.**, connecting them.

The proportions of **3.**, and thus the relative scaling of **1.** and **2.**, can be altered by changing the locating holes connecting arms **4.**, **5**, and **6.**, **7.** respectively.

Further, the arm ends, **8.**, **9.**, **10.**, can be fitted with interchangeable terminations according to their respective functions as pivot, **p**; master, **m**; slave, **s**.



This allows the following scale factors, k, to be achieved:

	p	m	s	k
8.	*			2, 3, 4
9.		*		
10.			*	

	p	m	s	k
8.	*			1/2, 1/3, 1/4
9.			*	
10.		*		

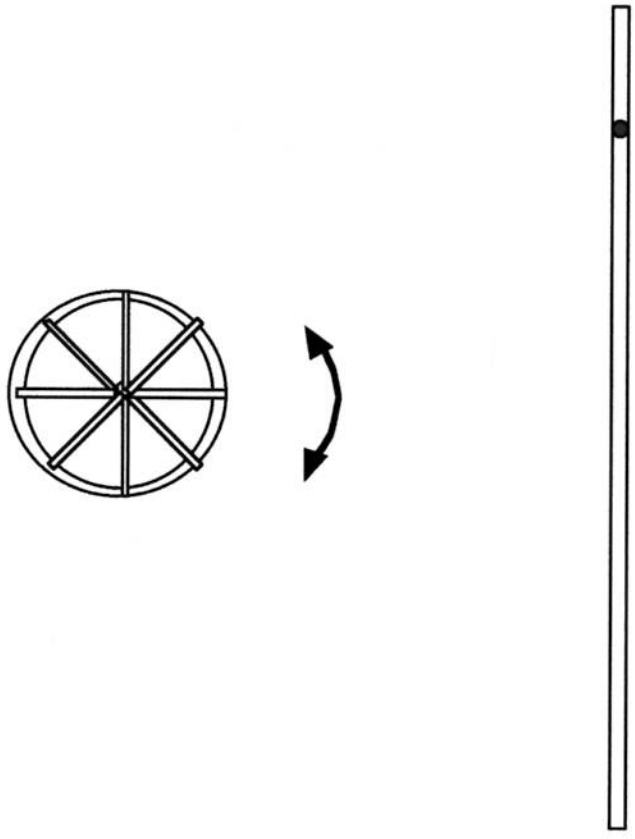
	p	m	s	k
8.		*		-2, -3, -4
9.	*			
10.			*	

	p	m	s	k
8.			*	-1/2, -1/3, -1/4
9.	*			
10.		*		

	NUMBER	TITLE
GROUP	8	LOCI & LINKAGES
STATION	8.6	'HOW TO DRAW A STRAIGHT LINE'
TOPIC	The geometric transformation 'inversion in a circle'	

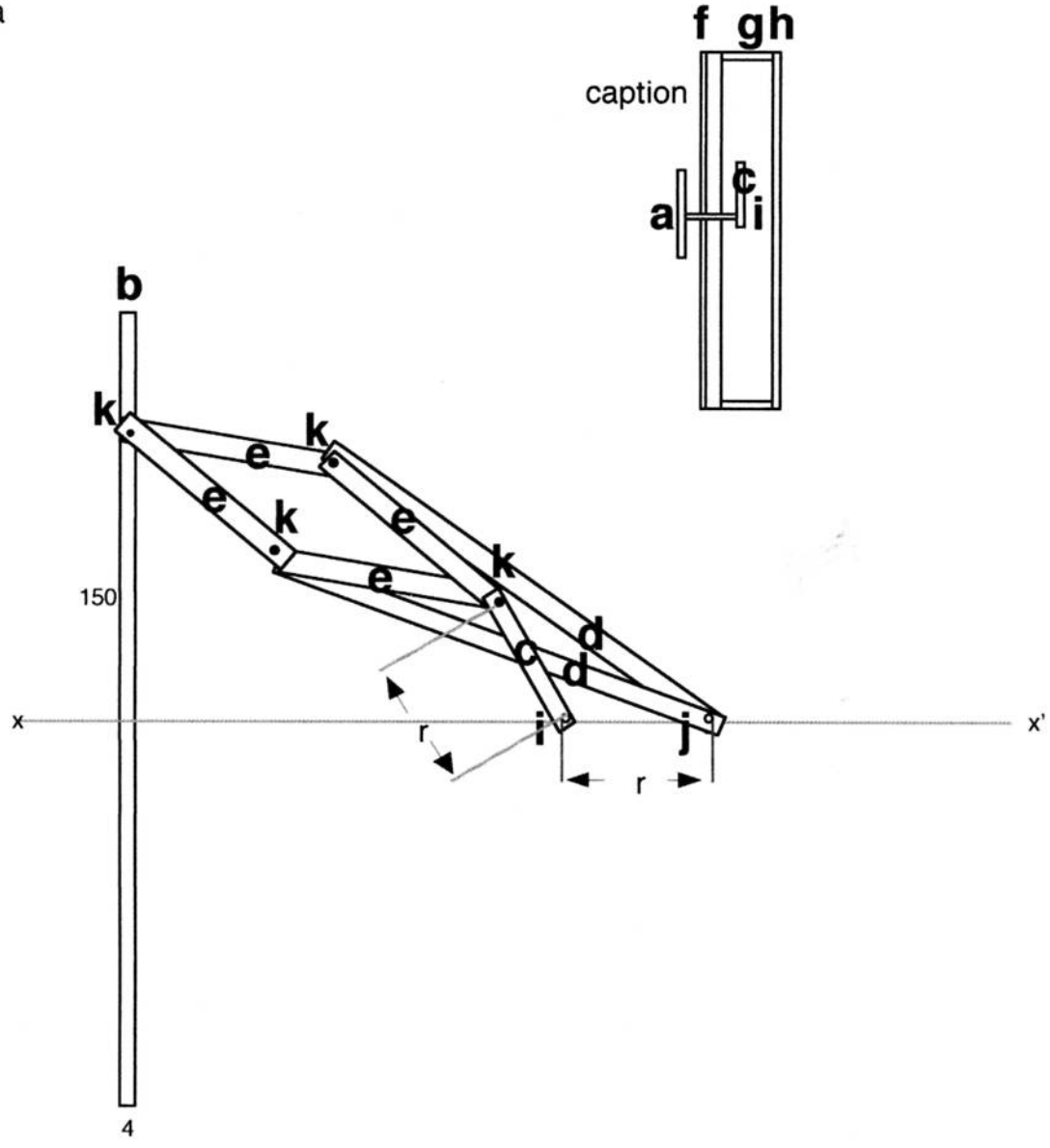
'HOW TO DRAW A STRAIGHT LINE'

- **Turn the wheel backwards and forwards and watch the red spot.**

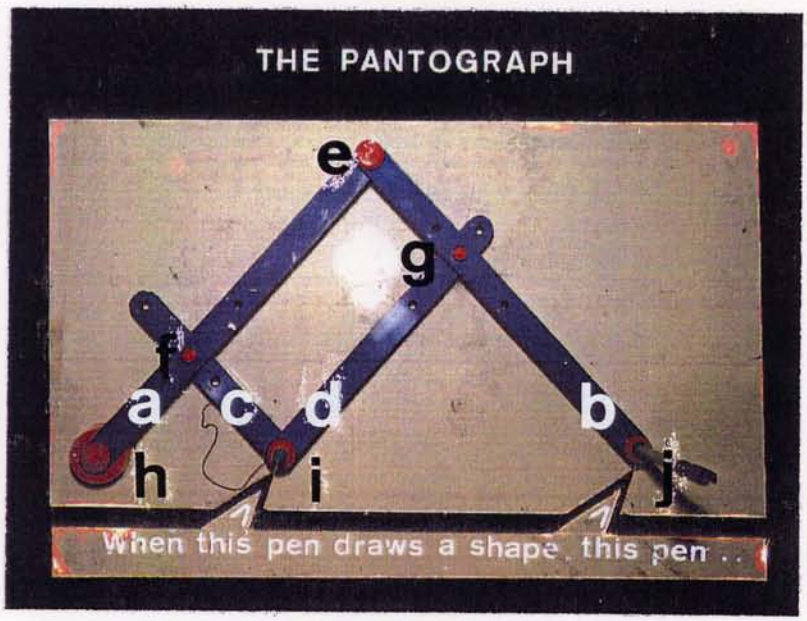


- **Now turn the board over to see Peaucellier's famous linkage.**

caption versa



PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
	The apparatus is symmetrical about the line x-x'.		
	The lengths marked 'r' should be equal to fair accuracy.		
a	'steering wheel'	Meccano	Everything Meccano 4 Greys Road Henley-on-Thames Oxon RG9 1RY T +44 1491 572436 F +44 1491 571175 www.metalworld.com/trade/aa645699
b	slot		
c	'crank', 1 1/2"	Meccano	
d	'strip', 3 1/2"	Meccano	
e	'strip', 2 1/2"	Meccano	
f	standard Glodex-faced caption board		
g	tubular spacers, nylon, 25 mm		local
h	Glodex, 2 mm		local
fgh	secured with bolts 40 mm long		local
	There are 2 fixed points, **I** and **j**. The pivots are of 3 different kinds:		
i	**a** and **c** are bolted to a 3/4" rod and spaced with washers from the caption board.	Meccano	
j	A 1/2" rod and collar with washers as spacers is set into the caption board.	Meccano	
k	1/2" bolts pass through the joints from the back and are 'lock-nutted', i.e. held in place by pairs of nuts tightened against each other.	Meccano	

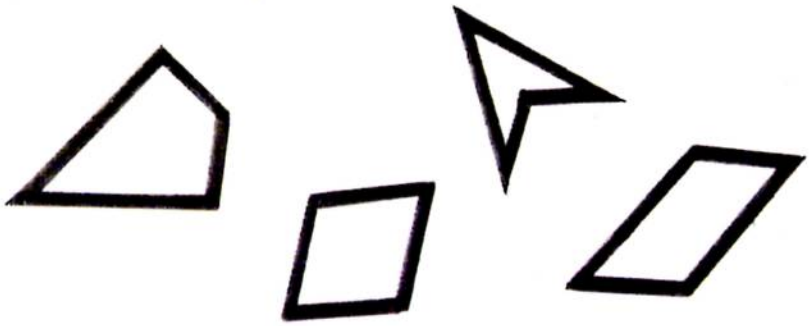

THE PANTOGRAPH
e
g
f
a
c
d
b
h
i
j
When this pen draws a shape, this pen ..

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
	The design is that of a standard pantograph, adapted to the purpose.		
	That shown was built by: from whom detailed specifications should be sought. However, essential details are on the preceding page and below.		Techniquest Enterprises Ltd (address above)
a,b,c,d	arms in PVC, cross-section: 15 mm (broad) x 3 mm (thick); lengths:		
	a,b: 240 mm, **c**: 120 mm, **d**: 180 mm;		
	arms drilled at intervals:		
	To locate holes for drilling, set arms in position so that:		
	1) k, as given by the ratio of lengths **8-10**: **8-9**, is as required, (2,3 or 4), 2) **8,9,10** are in line, 3) **5** ∥ **6**, **4** ∥ **7**,		
	and mark.		
e	fixed pivot		
f,g	moveable pegs		
h	pivot, carrying sucker at least 30 mm in diameter		
i,j	master/slave pens		(see **THE CAPTION BOARDS**)
	The pen shown is not of the type now used.		
k	baseboard, (larger than that shown), in 20 mm MDF, 600 mm square, faced with Glodex		

	NUMBER	TITLE
GROUP	8	LOCI & LINKAGES
STATION	8.7.1	LINKAGES 1
TOPIC	Transformations of standard quadrilaterals in linkages	

LINKAGES 1

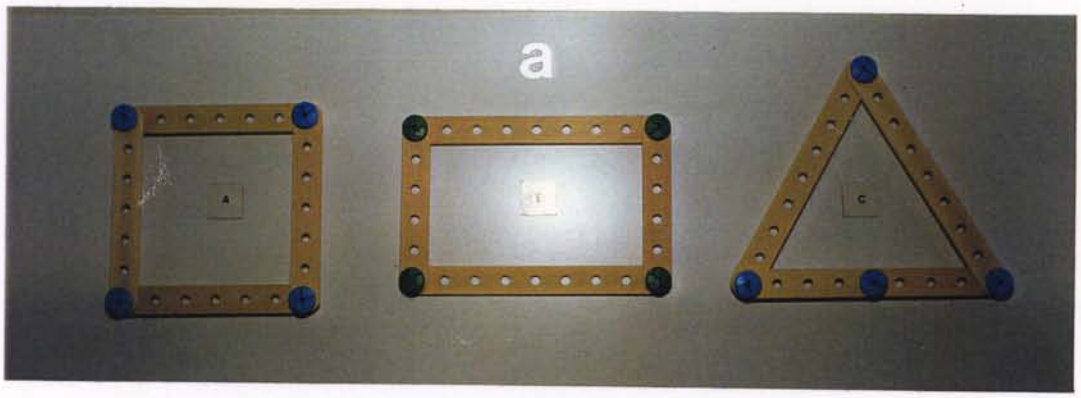
■ **Can you make these:**



...from these: **?**

● **Join opposite corners with rubber bands to show diagonals.**

■ **What can you discover?**


a


b
c

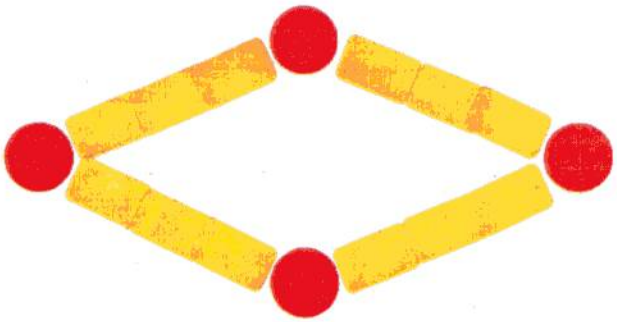
PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	perforated strips and pivots for same	Brio Mec Starter Set NES Arnold catalogue: NB 3170/7	NES Arnold Ltd (address above)
	The caption takes **a** to be already constructed as shown. However, youngsters prefer to construct the linkages for themselves. To this end, simply provide stocks of:		
b	Brio Mec,	v.s.	v.s.
c	rubber bands		local
	These should be lightly tensioned when stretched to 100 mm and stretch to 3 x that length without breaking.		
	See note to **7.1.3 d.**		
	The following Addis Module 2000 containers are suitable:		(see **THE STORAGE SYSTEM**)
	b: strips: unit 2 pivots: unit 1		
	c: unit 1		

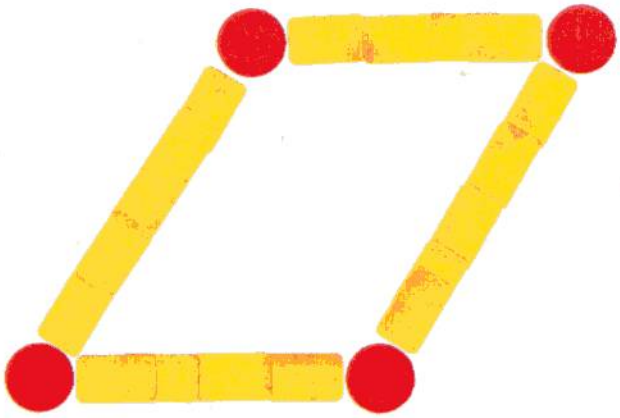
	NUMBER	TITLE
GROUP	8	LOCI & LINKAGES
STATION	8.7.2	LINKAGES 2
TOPIC		4-bar linkages identified in some common devices

LINKAGES 2

- A toolbox,
  a jack,
  a coat rack,
  an umbrella,
  a pantograph.

- What 4-sided shapes can you find in these mechanisms?





PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
	For the **pantograph** see **8.5**. The photograph shows the other items listed on the caption. Note that the relevant parts have been outlined to correspond with the quadrilaterals shown on the caption, i.e. with sides taped in yellow, vertices identified with red stickers, varnished		local