Contents

The Magic Manual

Section 7

Sequences

A guide for fabricators and users to stations from the Magic Mathworks Travelling Circus

7. SEQUENCES

7.1 - 7.4 can all be derived from Pascal's Triangle, the subject of **7.5**.

These stations correspond vertically:

7.1.1	**7.2.1**	**7.3.1**
7.1.5	**7.2.2**	**7.3.2**

The experiments in the first row are 2-dimensional - and take place in the vertical plane; those in the second, 2- and 3-dimensional - the former take place in the horizontal plane. **7.1.1**, **7.2.1** and **7.3.1** may therefore be dispensed with, but are particularly suitable for young children, not least because they use an everyday, household object.

7.6 - 7.9 are connected conceptually only by the section heading, and pedagogically by the range of levels on which the tasks can be tackled. Take **7.7** (**LEAPFROG**) as an example:

Age	An able child of that age can ...
4	make legitimate moves.
7	interchange the frogs according to a consistent scheme.
10	find the optimal scheme.
13	enter the number of moves n for f = 1, 2, 3 frogs each side on a difference table and use it to deduce n for f = 4.
16	use explicitly the method of finite differences implicit in that table to find a general formula for n in terms of f.
19+	justify the formula - or derive it from scratch, arguing from the numbers of frog jumps and slides; look for the recursive structure suggested by the fact that the pattern of moves for f_k is embedded in the pattern for f_{k+1}.

Hence the many boxes checked for this series of stations on the section analysis.

7.1 TRIANGLE NUMBERS

c Triangle numbers, T_n, viz. numbers of the form $n(n + 1)/2$.

7.1.1 TRIANGLE NUMBERS

p This station introduces the topic.

7.1.2 HOW MANY PILLS?

p This application tests one's understanding of it.

7.1.3 HANDSHAKES

p This activity has two virtues:
(i) Younger children can enact the process.
(ii) There are several ways to approach or justify the general formula.
(Note that the number of handshakes for n people is T_{n-1}.)

7.1.4 CROSSINGS

p This is paired with **7.1.3** as a contrasting embodiment.

7.1.5 NUMBER-BUILDING 1

p As stated above, this is the 3-D analogue of **7.1.1**: it allows us to build triangle numbers of the 2nd order from those of the 1st.

7.2 SQUARE NUMBERS

c Square numbers, S_n, viz. those of the form n^2.

7.2.1 SQUARE NUMBERS

p This station introduces the topic.

7.2.2 NUMBER-BUILDING 2

p This does for squares what **7.1.5** does for triangles.

7.3 TRIANGLE & SQUARE NUMBERS

c The relation between **7.1** and **7.2**, viz. $S_n = T_n + T_{n-1}$.

7.3.1 TRIANGLE & SQUARE NUMBERS

p This station introduces the topic and provides an additional embodiment presenting the conflict between a triangular *shape* and a square *number*.

7.3.2 NUMBER-BUILDING 3

p (As **7.1.5**, **7.2.2**.)

7.4 FIBONACCI NUMBERS

c Numbers in the sequence $u_r = u_{r-1} + u_{r-2}$, $r \geq 2$, $u_0 = 0$, $u_1 = 1$.

7.4.1 THE PLANT POT COMPUTER

p This station introduces the topic by allowing children to enact the above recurrence relation.

7.4.2 THE FIBONACCI SLIDE RULE

p This station does the same but more concisely. It is therefore a suitable starting point for older visitors.

7.4.3 PATH-PAVING

p For the advanced student, the interesting question is how the Fibonacci sequence arises here (see *The Magic Mathworks Travelling Circus: Heuristics*). For others, the station imposes the disciplines of careful observation and systematic recording.

7.4.4 'PENTAGON' TRIANGLES

p The experimenter builds the Fibonacci sequence in terms of both areal measure and the numbers of triangles: 'thin', 'fat' and both.

7.4.5 PINE CONE SPIRALS

p A natural exemplar.

7.5 PASCAL'S TRIANGLE

c Pascal's Triangle.

7.5.1 THE PASCAL SCANNER

p The generator uses a cellular automaton to generate the Triangle. (This method requires only a single initial unit, not borders filled with them.)

7.5.2 BACK TO THE CAR
7.5.3 CHOPS

p Embodiments. Cf. **7.1.3** and **7.1.4** in relation to **7.1**. But the Pascal array is embedded in **7.5.3** at a deeper level than in **7.5.2**. As throughout the Circus, the investigations can be tackled on an empirical and a theoretical level. In the present section we can characterise these as follows:

Level 1: Identify a number pattern.
Predict the next term of the sequence.
Test the prediction.

Level 2: Discover intuitively why a relation holds.
Prove algebraically that it does so.

c In **7.5.2** the choice of routes at a junction models the addition performed by the cellular automaton in **7.5.1**.
In **7.5.3** the fact that the number of ordered partitions of the number matches the binomial coefficient at that point can only be explained by interpreting the coefficient as the number of combinations of n things (possible chops) r at a time (actual chops).

7.5.4 NUMBER-BUILDING 4

p In this 'ready reckoner', perspex overlays connect cell pairs and single cells on the Triangle itself and one derived from it, summarising all the numerical relations of **7.1 - 7.3** and **7.5**.

7.5.5 FIBONACCI & PASCAL

p This mechanical aid brings **7.4** within that ambit.

7.6 ONE MATHEMATICAL STORY

c Both exercises generate the same arithmetic progression, 8n + 5.

7.6.1 SLIDING SAM

c n = (board edge length - 2).

7.6.2 THE FERRY PROBLEM

c n = (number of men - 1).

p Both are so presented as to lead the investigators to expect an arithmetic progression and therefore to check their operational procedures if the numbers they produce do not constitute one. *Note however that one can obtain different , but still arithmetic, progressions by adopting schemes which, though not optimal, are nevertheless consistent.* When this happens, the helper must judge the student's need, i.e. whether s/he is to be applauded merely, or applauded and challenged to seek a better solution.

What was noted for **7.7** in the introduction to this section applies to **7.6.1**: the youngest children can at least perform the moves.

7.7 LEAPFROG

c The sequence m = n(n + 2); (n is the number of frogs of one colour, m the number of moves).

p It can be recognised more easily from the emerging values as $(n + 1)^2 - 1$ - though the difference table provided renders that unnecessary.

The alternating pattern of black and white stripes helps the visitor achieve the corresponding alternations of frog colour needed to achieve the optimal scheme. A helper can advise, for example, "Try to keep red frogs on black squares".

The accompanying chart is intended to reveal the recursive structure of the activity to advanced students. But even middle school pupils can appreciate the rotational symmetry of each figure.

7.8 THE HANOI PATTERNS

c The sequence characterised explicitly as $r_n = 2^n - 1$ and iteratively as $2r_{n-1} + 1$; (n is the number of storeys in the pagoda, r_n the number of moves).

The fractal structure of the pattern of moves.

7.8.1 THE HANOI PAGODA

p The board states for n = 1 through 3 are displayed in sequences of photographs. But even without these the young experimenter soon gets a feel for whether or not s/he is proceeding in the most economical way. The kinaesthetic way in which the mathematical structure asserts itself in a 6-year-old child is wonderful to behold.

Once older students are 'in the swing' and have solved the puzzle for all cases up to n = 4, an assistant can direct their attention to the board **USE YOUR RULER TO SOLVE THE HANOI PUZZLE.**

7.8.2 THE HANOI COMPUTER

p The device is an interactive version of that board.

7.9 WEIGHINGS

p The base 2 exercise is a useful preliminary to that involving base 3.

Both exercises are self-validating.

7.9.1 BASE 2 WEIGHINGS

c The right-hand pan sequence represents the numbers 1 through 15 in binary notation.

7.9.2 BASE 3 WEIGHINGS

c The sequence of pan differences represents the numbers 1 through 40 in ternary notation.

SECTION		AGE RANGE					
SEQUENCES		Appropriate point of entry - not necessarily to the task set by the caption - and levels on which extension activities generated (some to be pursued off-site)					
STATION							
NUMBER	NAME	4	7	10	13	16	19+
7.1	*TRIANGLE NUMBERS*						
7.1.1	TRIANGLE NUMBERS	*	*	*			
7.1.2	HOW MANY PILLS?			*	*	*	
7.1.3	HANDSHAKES		*	*	*	*	
7.1.4	CROSSINGS			*	*	*	
7.1.5	NUMBER-BUILDING 1		*	*	*	*	
7.2	*SQUARE NUMBERS*						
7.2.1	SQUARE NUMBERS		*	*			
7.2.2	NUMBER-BUILDING 2		*	*	*	*	
7.3	*TRIANGLE & SQUARE NUMBERS*						
7.3.1	TRIANGLE & SQUARE NUMBERS			*			
7.3.2	NUMBER-BUILDING 3				*	*	
7.4	*FIBONACCI NUMBERS*						
7.4.1	THE PLANT POT COMPUTER		*	*			
7.4.2	THE FIBONACCI SLIDE RULE			*	*	*	
7.4.3	PATH-PAVING			*	*	*	
7.4.4	'PENTAGON' TRIANGLES			*	*	*	
7.4.5	PINE CONE SPIRALS				*	*	
7.5	*PASCAL'S TRIANGLE*						
7.5.1	THE PASCAL SCANNER			*	*	*	
7.5.2	BACK TO THE CAR				*	*	
7.5.3	CHOPS				*	*	*
7.5.4	NUMBER-BUILDING 4				*	*	
7.5.5	FIBONACCI & PASCAL				*	*	*

	INSTRUCTION NEEDED				SUPERVISION NEEDED			SERVICING NEEDED		
→	Needs no explan-ation or caption	Caption enough for most people	Needs aural prompt	Visitors must be talked through stages	None	Benefits from a helper's input	Session must be directed	None	A little	Much
7.1.1		*				*		*		
7.1.2		*				*		*		
7.1.3		*				*		*		
7.1.4			*			*		*		
7.1.5			*			*		*		
7.2.1		*				*		*		
7.2.2			*			*		*		
7.3.1			*			*		*		
7.3.2			*			*		*		
7.4.1			*			*			*	
7.4.2		*			*			*		
7.4.3			*			*			*	
7.4.4		*				*		*		
7.4.5			*		*			*		
7.5.1			*			*		*		
7.5.2			*			*		*		
7.5.3			*			*		*		
7.5.4		*			*			*		
7.5.5		*				*			*	

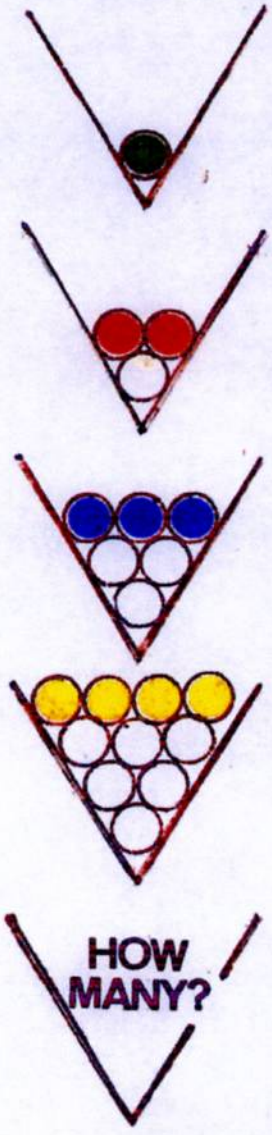
SECTION		AGE RANGE					
		Appropriate point of entry - not necessarily to the task set by the caption - and levels on which extension activities generated (some to be pursued off-site)					
STATION							
NUMBER	NAME	4	7	10	13	16	19+
7.6	*ONE MATHEMATICAL STORY*						
7.6.1	SLIDING SAM		*	*	*	*	*
7.6.2	THE FERRY PROBLEM			*	*	*	*
7.7	LEAPFROG		*	*	*	*	*
7.8	*THE HANOI PATTERNS*						
7.8.1	THE HANOI PAGODA		*	*	*	*	*
7.8.2	THE HANOI COMPUTER					*	*
7.9	*WEIGHINGS*						
7.9.1	BASE 2 WEIGHINGS			*	*	*	
7.9.2	BASE 3 WEIGHINGS				*	*	*

	INSTRUCTION NEEDED				SUPERVISION NEEDED			SERVICING NEEDED		
→	Needs no explanation or caption	Caption enough for most people	Needs aural prompt	Visitors must be talked through stages	None	Benefits from a helper's input	Session must be directed	None	A little	Much
7.6.1			*			*			*	
7.6.2		*				*			*	
7.7			*			*			*	
7.8.1			*			*			*	
7.8.2			*			*		*		
7.9.1		*				*			*	
7.9.2			*			*			*	

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.1.1	TRIANGLE NUMBERS
TOPIC		Triangle numbers

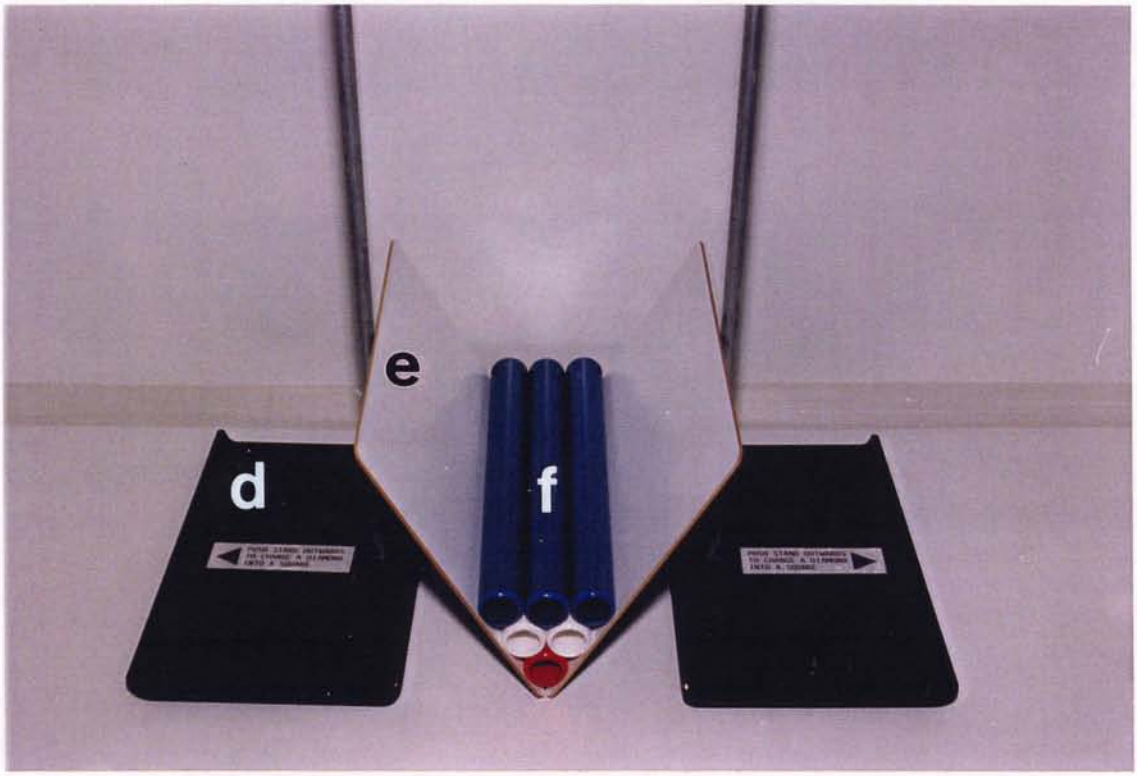
TRIANGLE NUMBERS

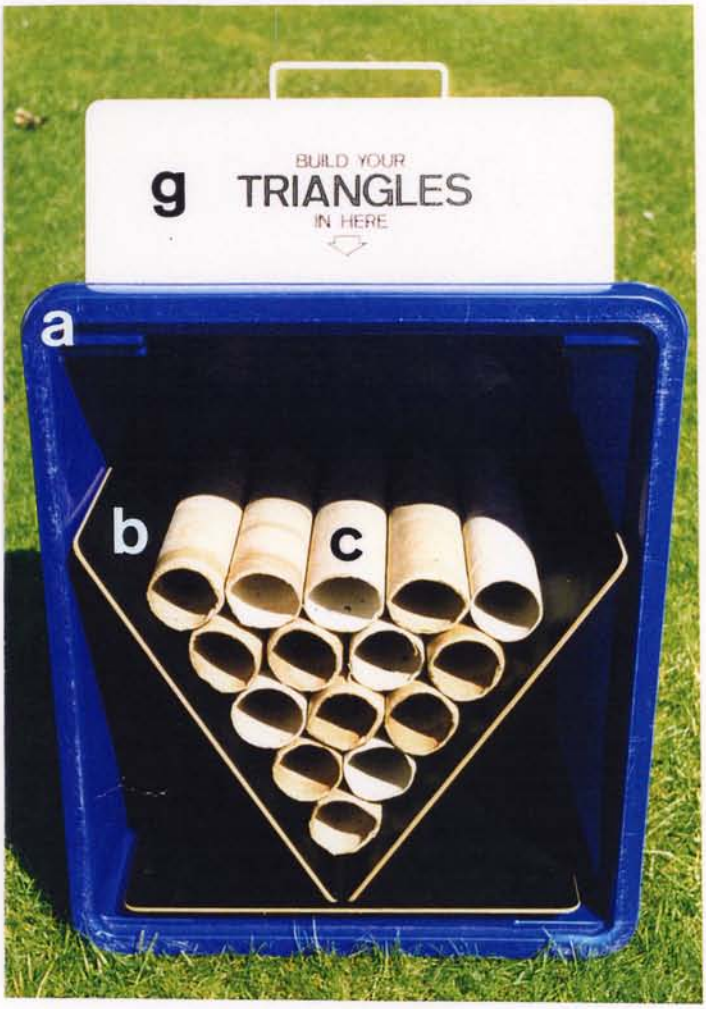
- **Build these number-patterns in the 'V' with the paper towel rolls:**




TRIANGLE NUMBERS


a
b
c

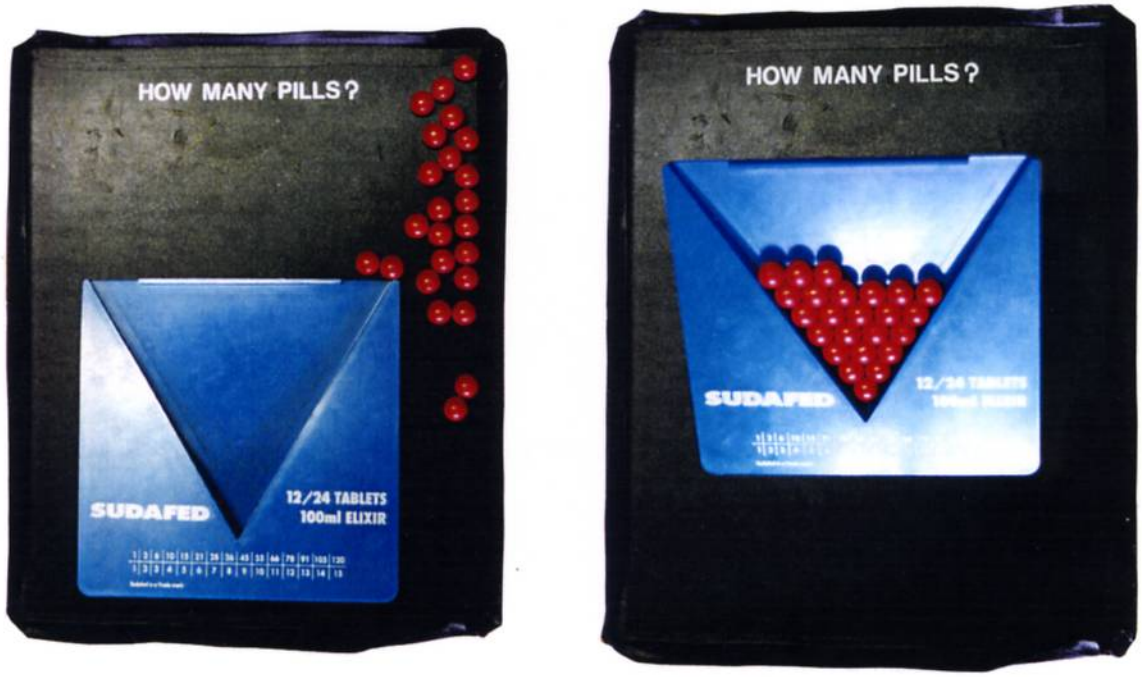

e
d
f

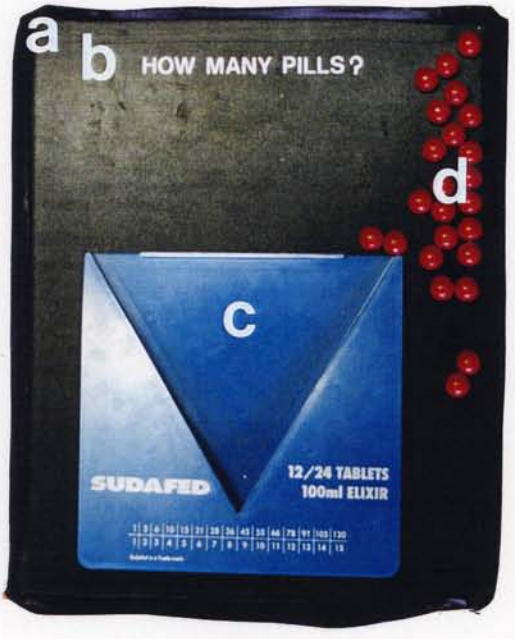

g
BUILD YOUR
TRIANGLES
IN HERE
a
b
c

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a,b,c / d,e,f	These alternative arrangements differ in that **d,e,f** can also be used for **7.2.1**, to which the instruction on each retort stand base applies: **PUSH STAND OUTWARDS TO CHANGE A RHOMBUS INTO A SQUARE.** Parts correspond as follows: **a** is equivalent to **d**. **b** is equivalent to **e**. **c** is equivalent to **f** and interchangeable with it.		
a - c			
a	Addis Module 2000 unit 4		(see **THE STORAGE SYSTEM**)
b	caption boards as described, hinged with tape by their shorter edges to the centre of a piece of caption board 200 mm (wide) x 250 mm (deep)		
c	paper towel rolls, varnished, conveniently stored in a unit like **a**		local
d - f			
d	retort stand base, cast iron, rectangular, 200 mm (wide) x 315 mm (deep), used inverted,	Griffin catalogue: STA-522-000E	Griffin Education (address above)
	with plastic grips to increase friction,	Sticky Feet	local
	with steel rod to fit, 500 mm long, 10 mm diameter	Griffin catalogue: STA-800-030U	Griffin Education (address above)
e	caption boards as described, hinged with tape by their longer sides		
f	relay batons	Hope catalogue: W 3917/001	Hope Education Ltd (address above)
g	caption board as described, halved lengthways, bearing the caption: **BUILD YOUR TRIANGLES IN HERE**		

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.1.2	HOW MANY PILLS?
TOPIC		As 7.1.1

HOW MANY PILLS?




a
b
HOW MANY PILLS?
d
c
SUDAFED
12/24 TABLETS
100ml ELIXIR

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	tape wall, 10 mm high, round		
b	caption board as described		
c	device supplied to dispensing chemist by pharmaceutical firm by way of advertisement		local
d	spheres; those shown are from a binostat:	NES Arnold catalogue: SX 479/7	NES Arnold Ltd (address above)
	but any will do, e.g. ball bearings or marbles, (indeed the instrument is useful precisely because its reading does not depend on the size of the spheres being counted)		local

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.1.3	HANDSHAKES
TOPIC		As 7.1.1

HANDSHAKES

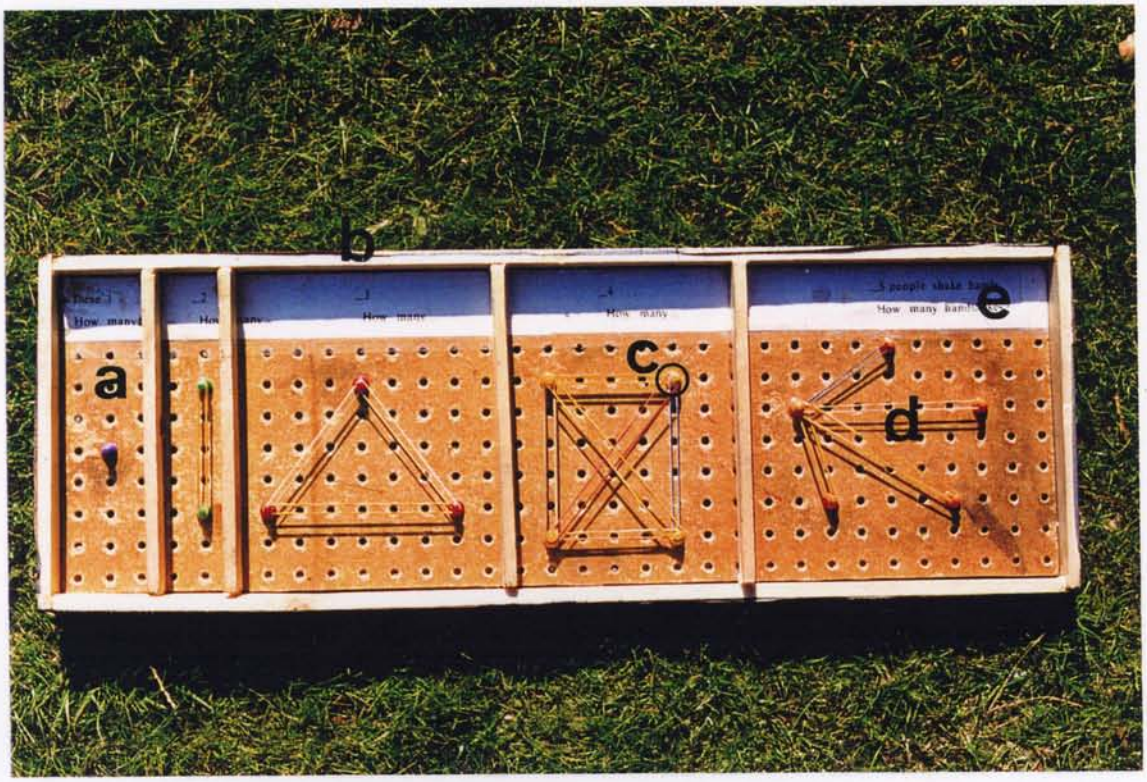
- On this board the pegs are people.
- A different number meet in each room.
- They shake hands.

- Join 2 pegs with a rubber band to show 2 people shaking hands.

- How many handshakes will take place when ...

1 person 'meets'(!)?	0	
2 people meet?	1	
3 people meet?	3	
4 people meet?	☐	Predict.
5 people meet?	☐	Predict.
⋮ v	⋮ v	⋮ v

- Test your predictions with the rubber bands.


b
a
c
d
e


a
d
c
b

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	pegboard, grid of 20 mm squares, holes 4 mm diameter, 660 mm x 220 mm		local
b	frame of wood, 25 mm x 12.5 mm		local
c	pegs, stuck in, those in each compartment a different colour	pin pegs NES Arnold catalogue: SX 049/0	NES Arnold Ltd (address above)
d	rubber bands Buy from an office stationer packs of bands of a standard size which are under light tension when stretched to 60 mm and extend to twice that length without breaking.		local
e	This text, shown here on a prototype, duplicates the caption and may be omitted.		

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.1.4	CROSSINGS
TOPIC		As 7.1.1

CROSSINGS

- Lay down 1 green strip at a time.
- Count the crossings (dark diamonds).

▼

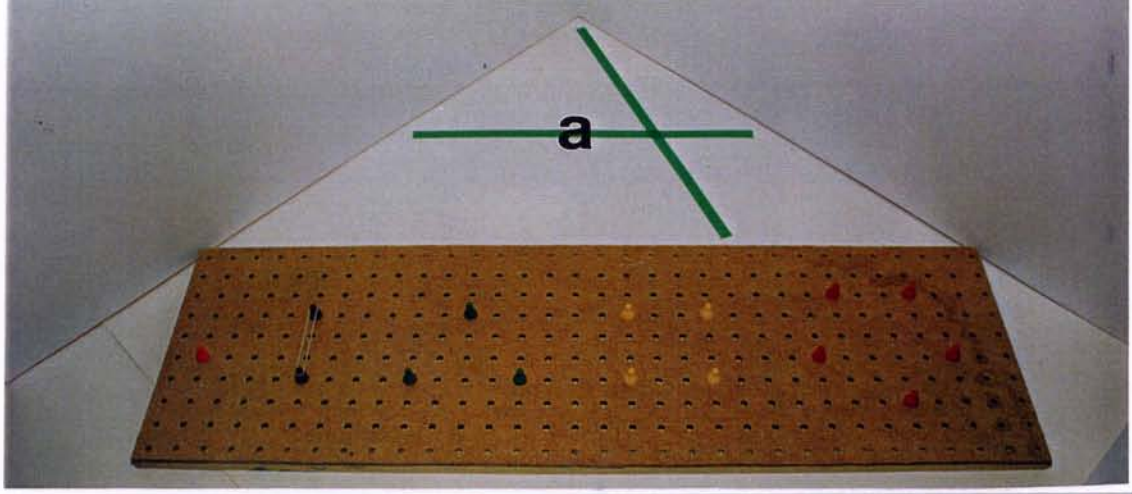
▶ Rules:
1. The new strip must cross all the others.
2. It must not pass through an old crossing.

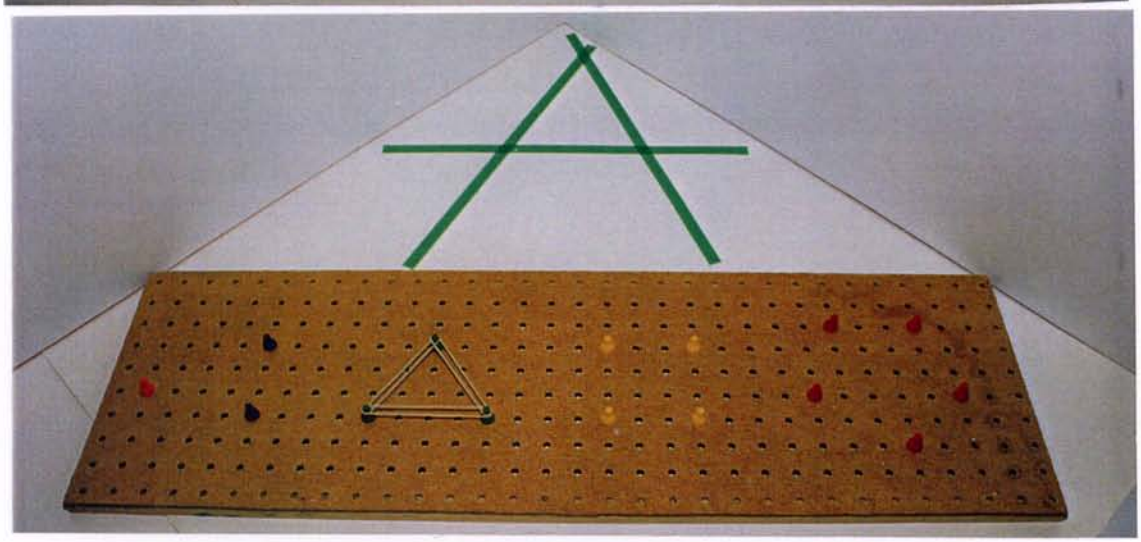
0

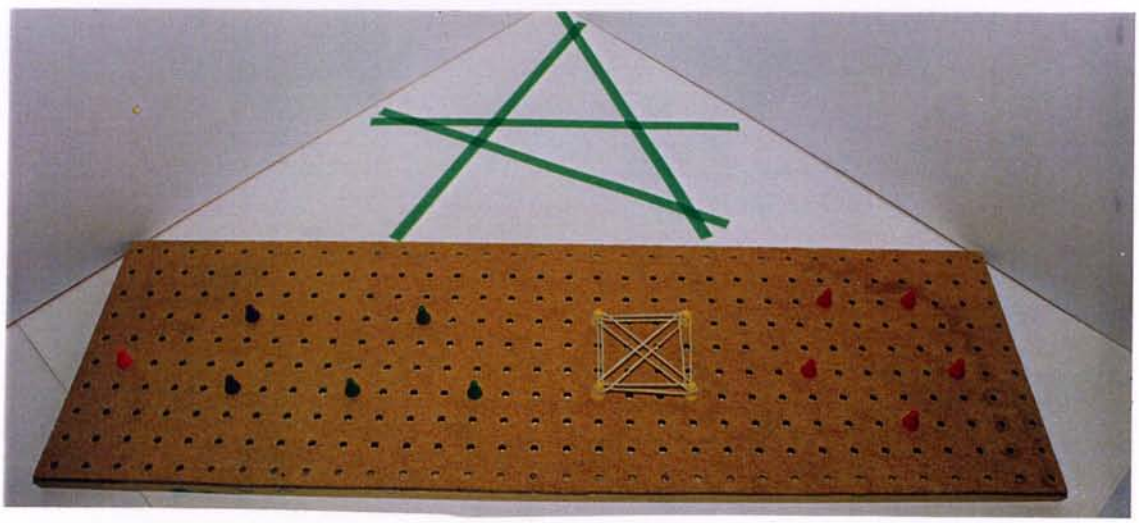
1

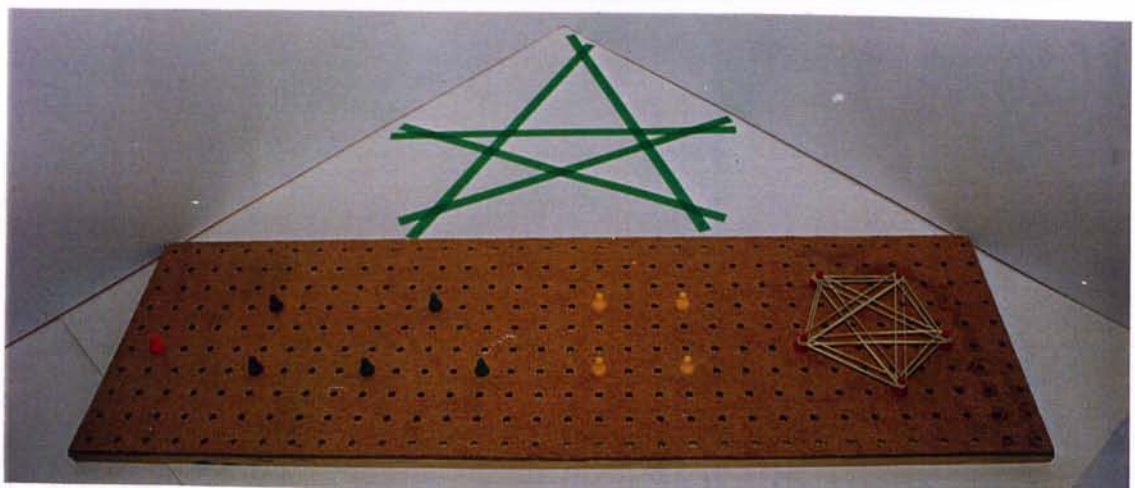
3

- Make your predictions for 4 strips, 5 strips, ...>


a



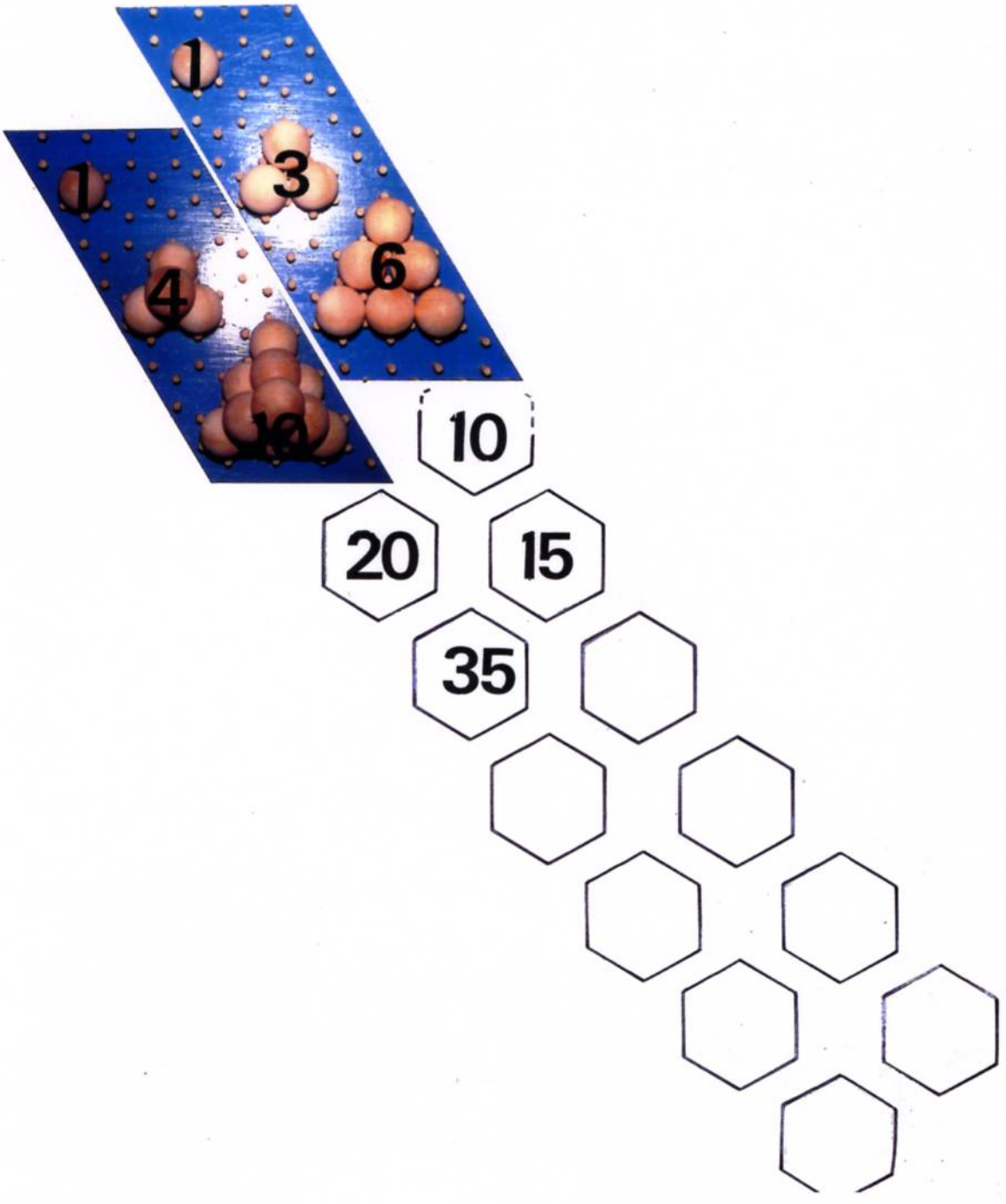




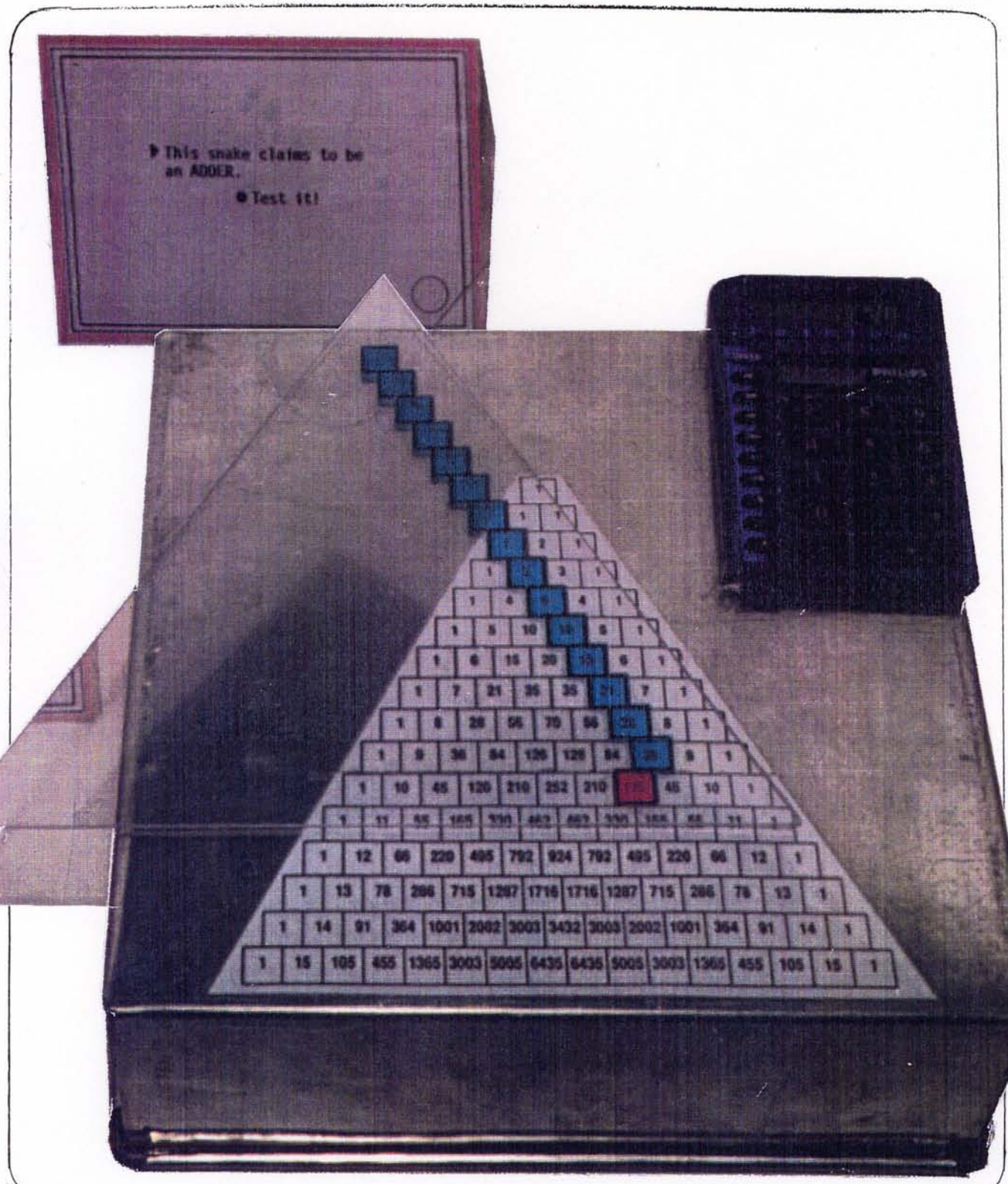
PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	strips, 297 mm x 10 mm, in green 200μ acetate The photograph sequence shows corresponding stages in investigations **7.1.3** and **7.1.4**.		local

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.1.5	NUMBER-BUILDING 1
TOPIC		Building tetrahedral numbers from triangle numbers

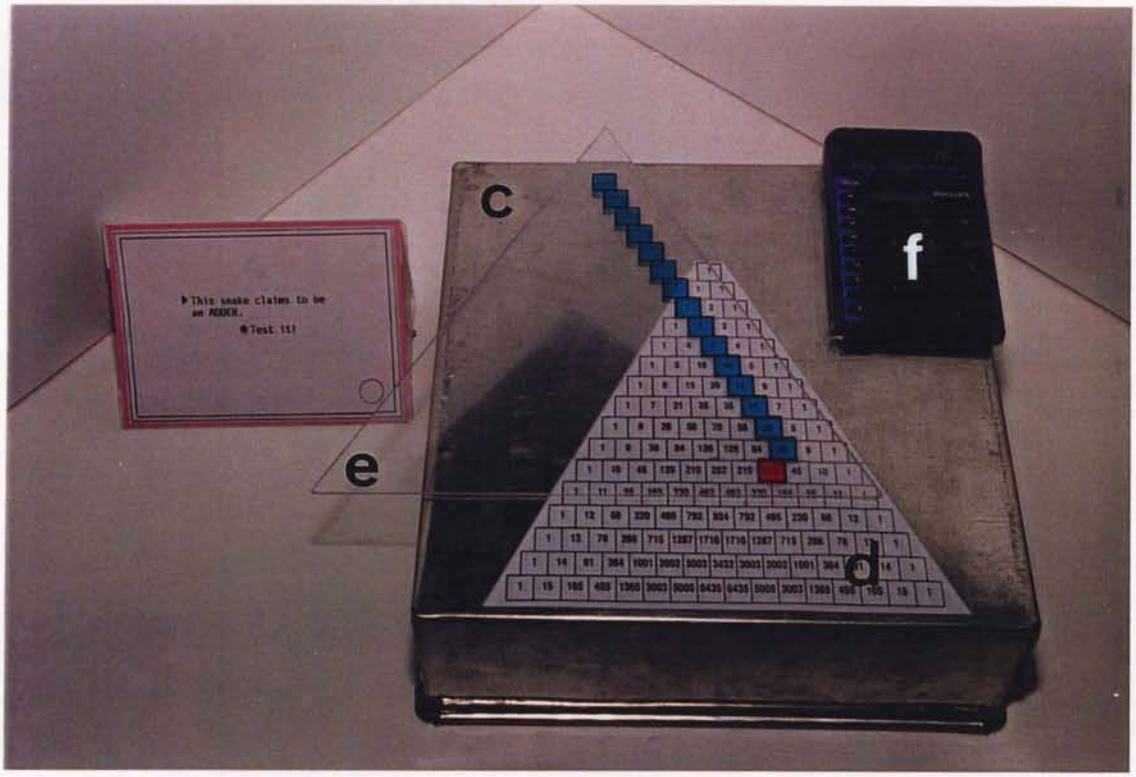
NUMBER-BUILDING 1

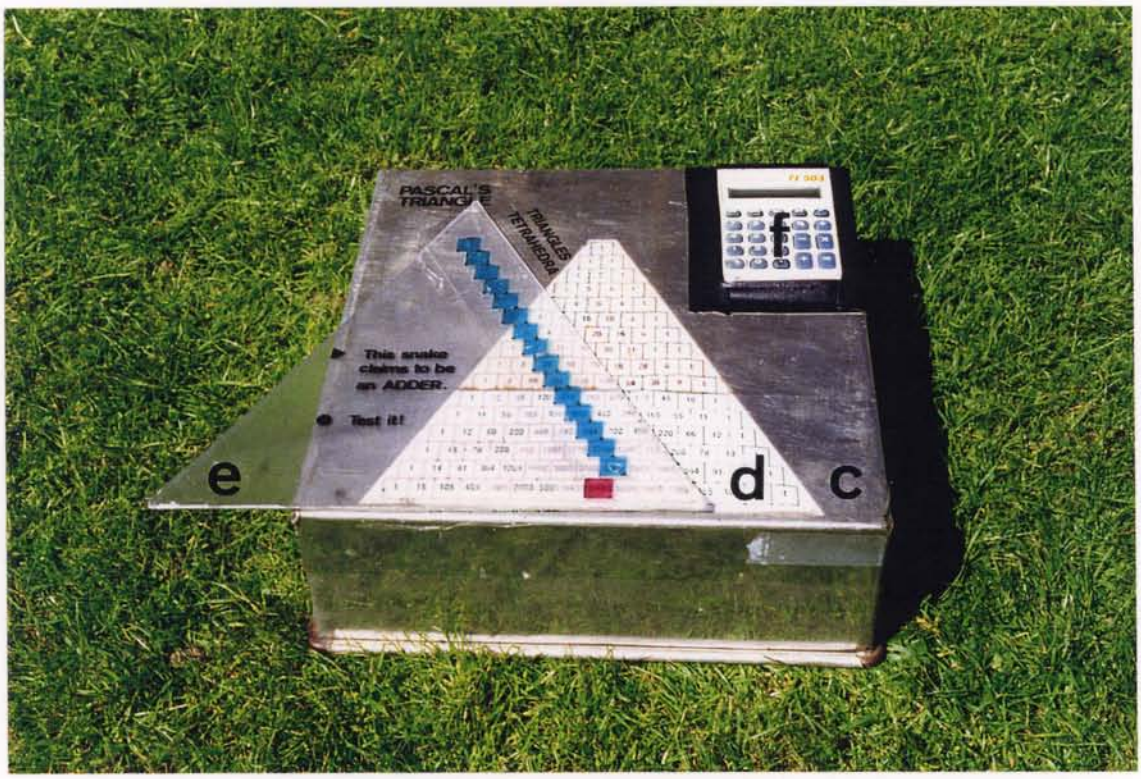


	NUMBER	TITLE
GROUP		
STATION		(Above continued)
TOPIC		7.1.5 read from Pascal's Triangle




a
b


c
d
e
f
This snake claims to be an ADDER.
Test it!

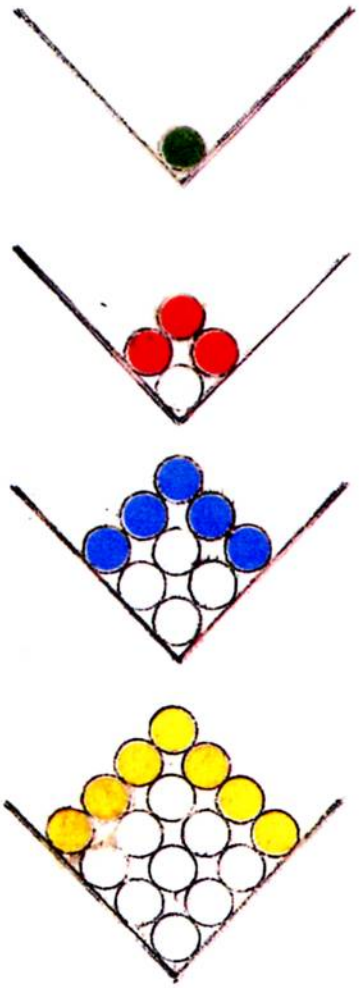

PASCAL'S TRIANGLE
TRIANGLES
TETRAHEDRA
This snake claims to be an ADDER.
Test it!
e
d
c
f

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a,b	as **4.4**		
c	cake tin, 200 mm square x 100 mm (high), used inverted,* bearing		
d	Pascal's Triangle down to row 15, reproduced from:	'Pascal's Triangle' Tony Colledge	Tarquin Publications (address above)
	with 'triangle' and 'tetrahedron' diagonals indicated as shown, all covered with Transpaseal		local
e	equilateral triangle, 185 mm edge, in Glodex, carrying this caption: ● **This snake claims to be an ADDER.** ● **Test it!**		
	and squares in coloured acetate as shown		local
f	pocket calculator		local
	*A board would serve almost as well. The tin is part of an earlier prototype where the acetate squares of **e** were embedded in a magnetic strip in place of the Glodex triangle. But it does have the advantages of: 1. standing clear of **b** on a crowded table and 2. catching the eye for that reason.		

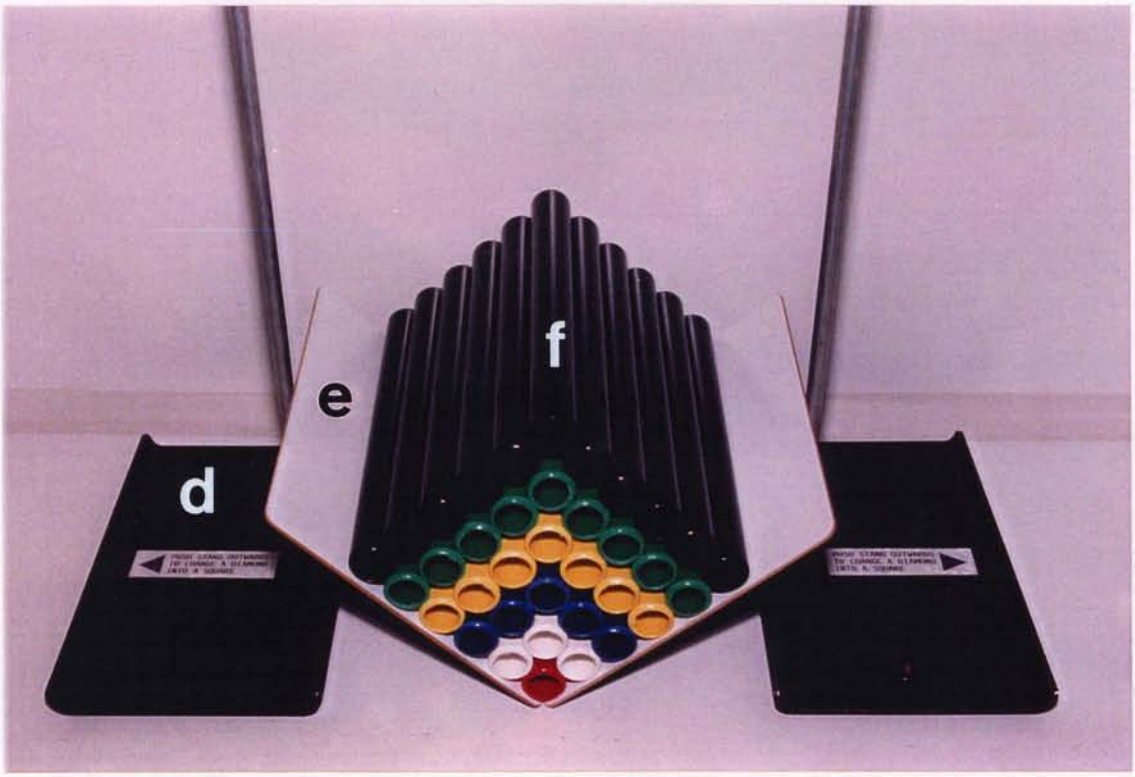
	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.2.1	SQUARE NUMBERS
TOPIC		Square numbers

SQUARE NUMBERS

- **Build these number-patterns in the 'V' with the paper towel rolls:**



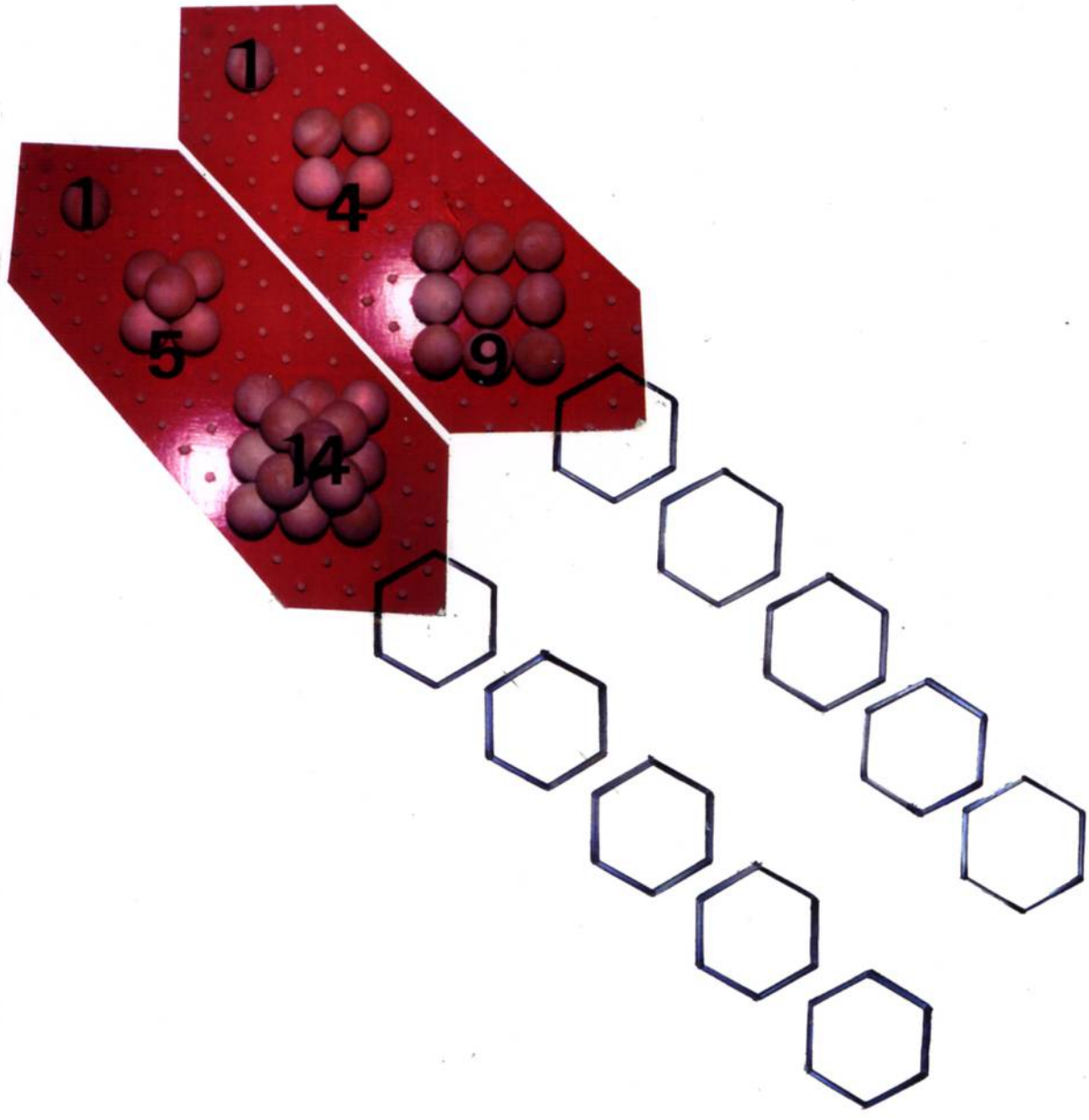

g
BUILD YOUR
SQUARES
IN HERE
a
c
b


f
e
d

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	Addis Module 2000 unit 5		(see **THE STORAGE SYSTEM**)
b	2 caption boards as described, taped by their longer edges to the centre of a 3rd board, similarly orientated		
c	as **7.1.1**		
d,e,f	as **7.1.1**		
g	caption board as described, halved lengthways, bearing the caption: **BUILD YOUR SQUARES IN HERE**		

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.2.2	NUMBER-BUILDING 2
TOPIC	Building pyramidal numbers from square numbers	

NUMBER-BUILDING 2

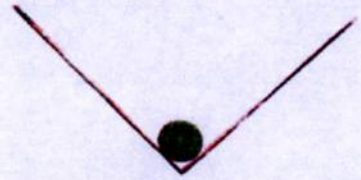


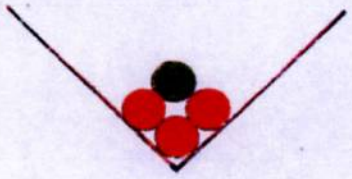
PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
	as **4.4 a,e**		

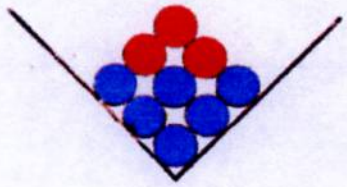
	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.3.1	TRIANGLE & SQUARE NUMBERS
TOPIC		A square as the sum of consecutive triangle numbers

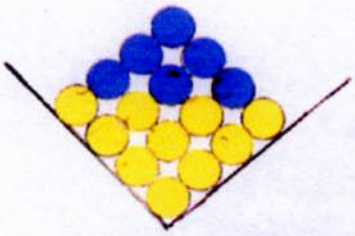
TRIANGLE & SQUARE NUMBERS

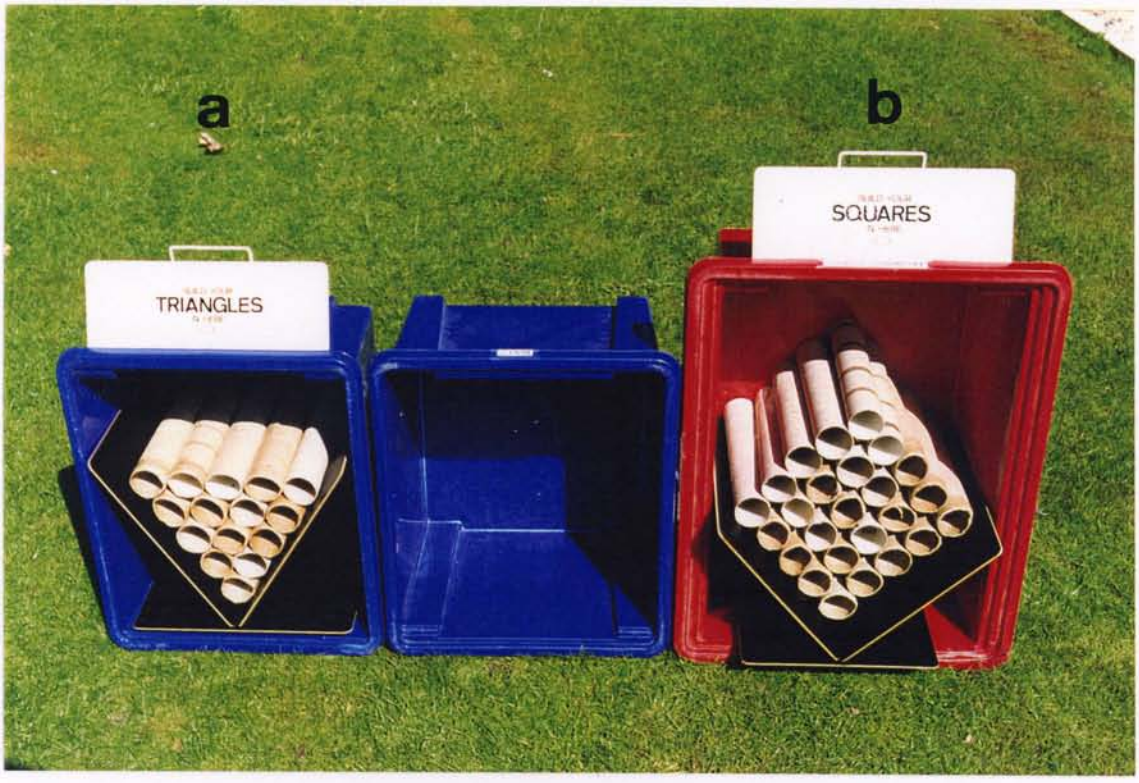
- **Build these number-patterns in the 'V' with the paper towel rolls:**

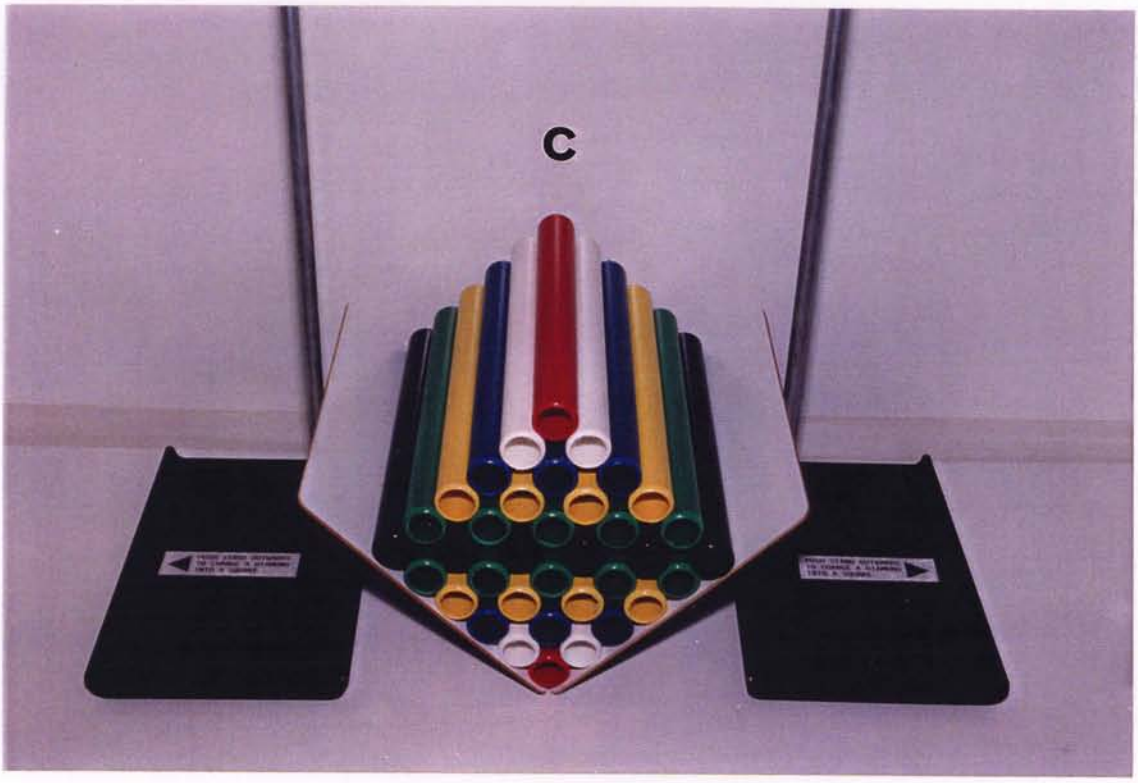






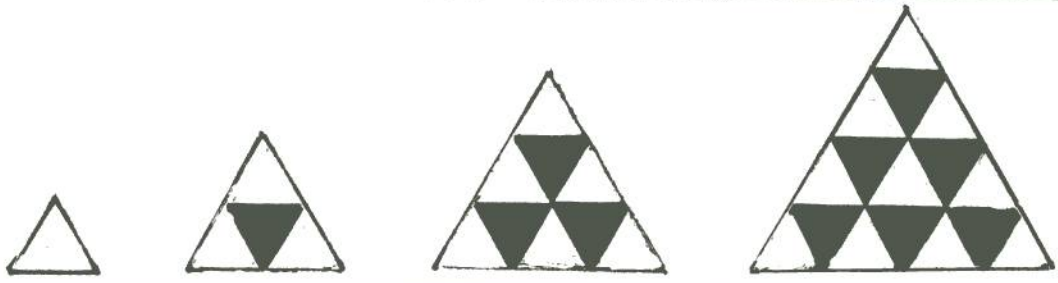



a
b
TRIANGLES
SQUARES

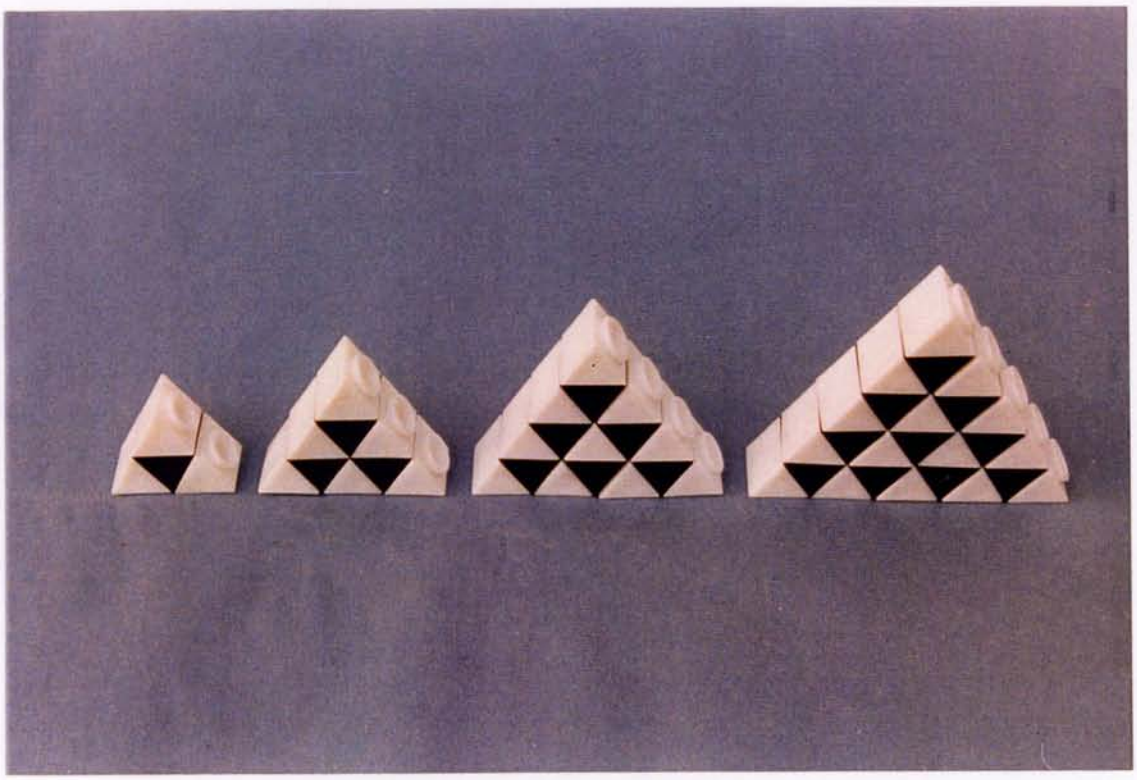

c

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	as 7.1.1 a,b,c,g		
b	as 7.2.1 a,b,c,g		
c	as 7.1.1 d,e,f		

	NUMBER	TITLE
GROUP		
STATION		(Above continued)
TOPIC		7.3.1 differently embodied



	COUNT			
BLACK ▲s				
WHITE △s				
TOTAL				



	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.3.2	NUMBER-BUILDING 3
TOPIC	7.3.1 extended: a pyramid as the sum of consecutive tetrahedra	

NUMBER-BUILDING 3

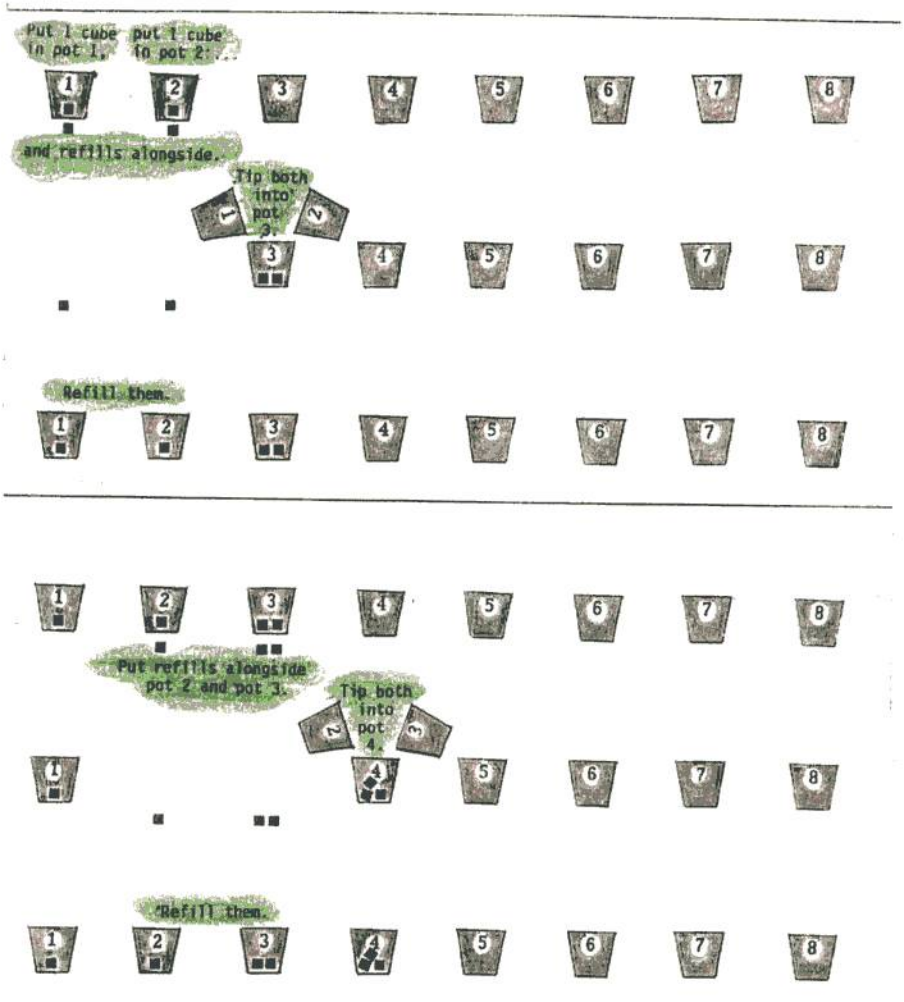
■ **Can you find the adding rule?**

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
	as **4.4 a,b,e**		

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.4.1	THE PLANT POT COMPUTER
TOPIC		The Fibonacci sequence

THE PLANT POT COMPUTER

- **Follow these instructions:**



- **How many cubes will end up in pot 8?**

Keep going, tipping the last 2 pots into the next and refilling them, till you reach ..... pot 8.

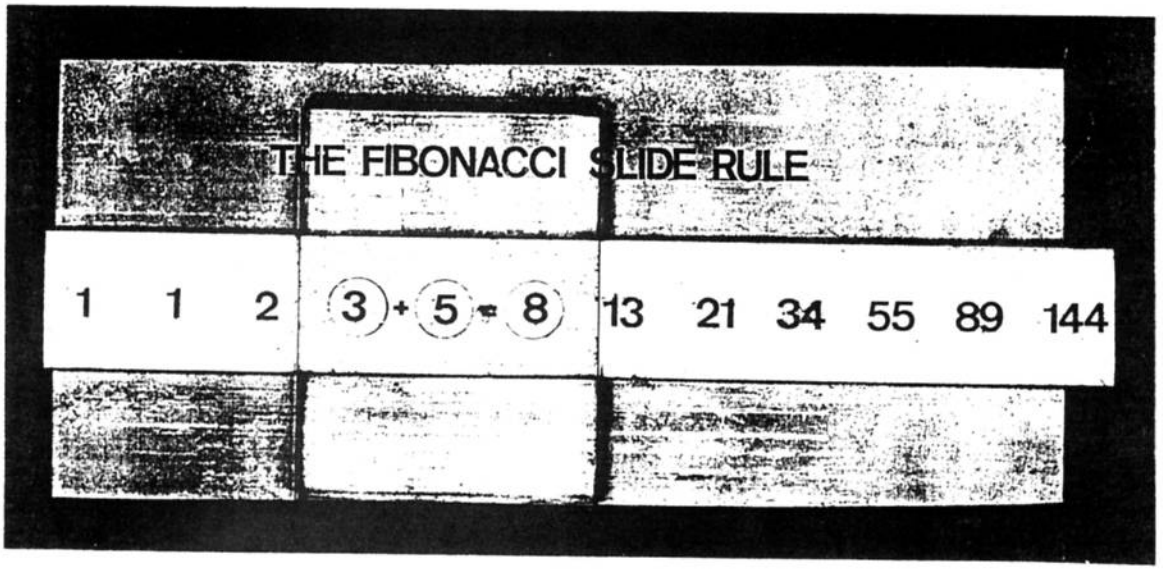
- **Tip out the cubes and count them!**

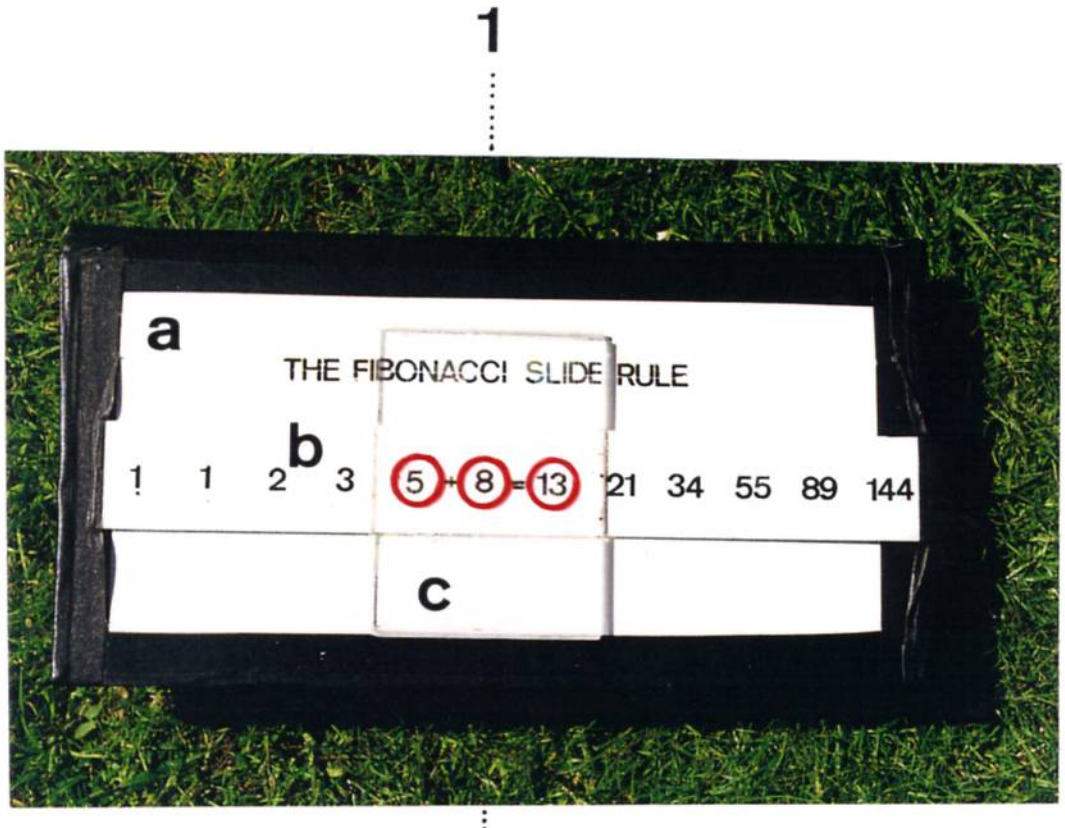
PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	suitable receptacle, e.g., (unlike that shown here), Addis Module 2000 unit 1, containing		(see **THE STORAGE SYSTEM**)
b	chunky counters, here:	Multilink: SY 007/9	NES Arnold Ltd (address above)
c	plastic plant pots, 120 mm diameter at open end, 120 mm high, numbered **1** to **8** from left to right, as shown on caption		
	For younger children a pocket calculator should be available.		local

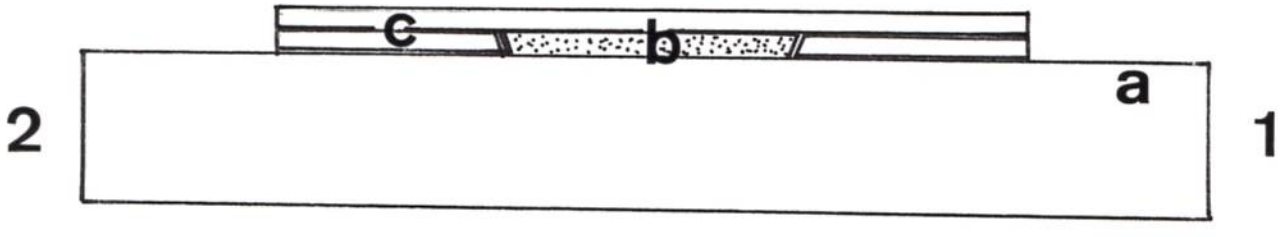
	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.4.2	THE FIBONACCI SLIDE RULE
TOPIC	As 7.4.1	

THE FIBONACCI SLIDE RULE

- **Use this slide rule to show you how the Fibonacci sequence grows:**




1
a
THE FIBONACCI SLIDE RULE
b
1 1 2 3 5 + 8 = 13 21 34 55 89 144
c
2


c
b
a
2
1

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	20 mm white-faced contiboard base, 300 mm (wide) x 150 mm (deep), marked as shown		local
b	plastic ruler, inverted, stuck along **a**'s longer symmetry axis and numbered as shown		local
c	slider, marked as shown, 75 mm (wide) x 100 mm (deep), made from 3 mm perspex as shown in section **1 - 2**, (drawn to scale, at size) note bevel so that **b** retains **c**		local

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.4.3	PATH-PAVING
TOPIC		As 7.4.1

PATH-PAVING

- Pave the paths with the red slabs.

- Here, for example, are different ways of paving the path 3 slab-widths long:

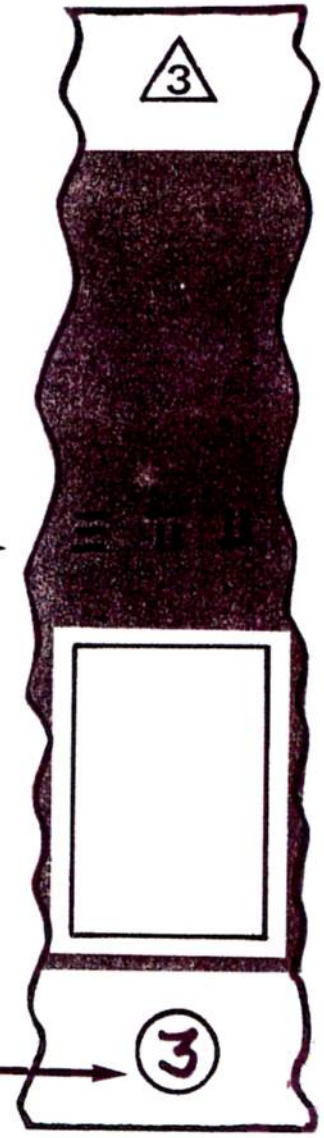
- Draw stick pictures on the grey board: →

- Have we got them all? If a picture $\overline{11}$ can be inverted $\underline{11}$, have we got the pair?
- Check you haven't drawn the same one twice:

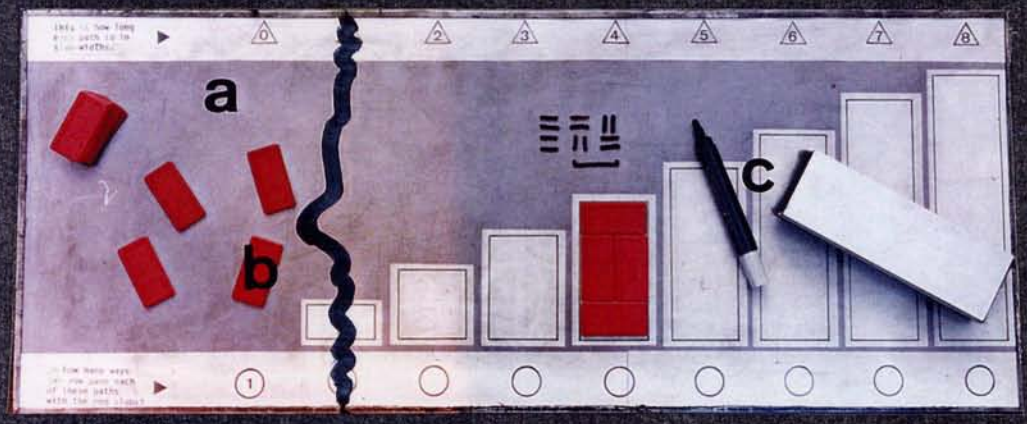
$\equiv \quad \overline{11} \quad \underline{11} \quad (\overline{11})$

- Once you're sure, write their number in the circle: →

- Predict the next. Test your guess. ...



This is how long each path is in slab-widths.



In how many ways can you pave each of these paths with the red slabs?

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	design shown, made from strips of white paper stuck on a sheet of dark sugar paper, 750 mm (wide) x 300 mm (deep), sandwiched between 2 similar Glodex sheets		local
b	dominoes, here plastic tiles, 50 mm x 25 mm, from a box of money tokens designed to be used with board and card games If dominoes of other dimensions are available, **a** may be scaled up accordingly.		local
c	dri-wipe pen and duster shown in use		(see **THE CAPTION BOARDS**)

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.4.4	'PENTAGON' TRIANGLES
TOPIC		As 7.4.1

'PENTAGON' TRIANGLES

▶ You can enlarge any triangle to produce the sequence of squares: 1, 4, 9, 16, ...

● Fill the first row of cutouts with red, "thin" 'pentagon' triangles:
and the second with blue, "fat" 'pentagon' triangles:

▶ But you can also enlarge 'pentagon' triangles to produce the Fibonacci sequence.
In this, each term sums the two before:
1, 1, 2, 3, 5, 8, 13, ...

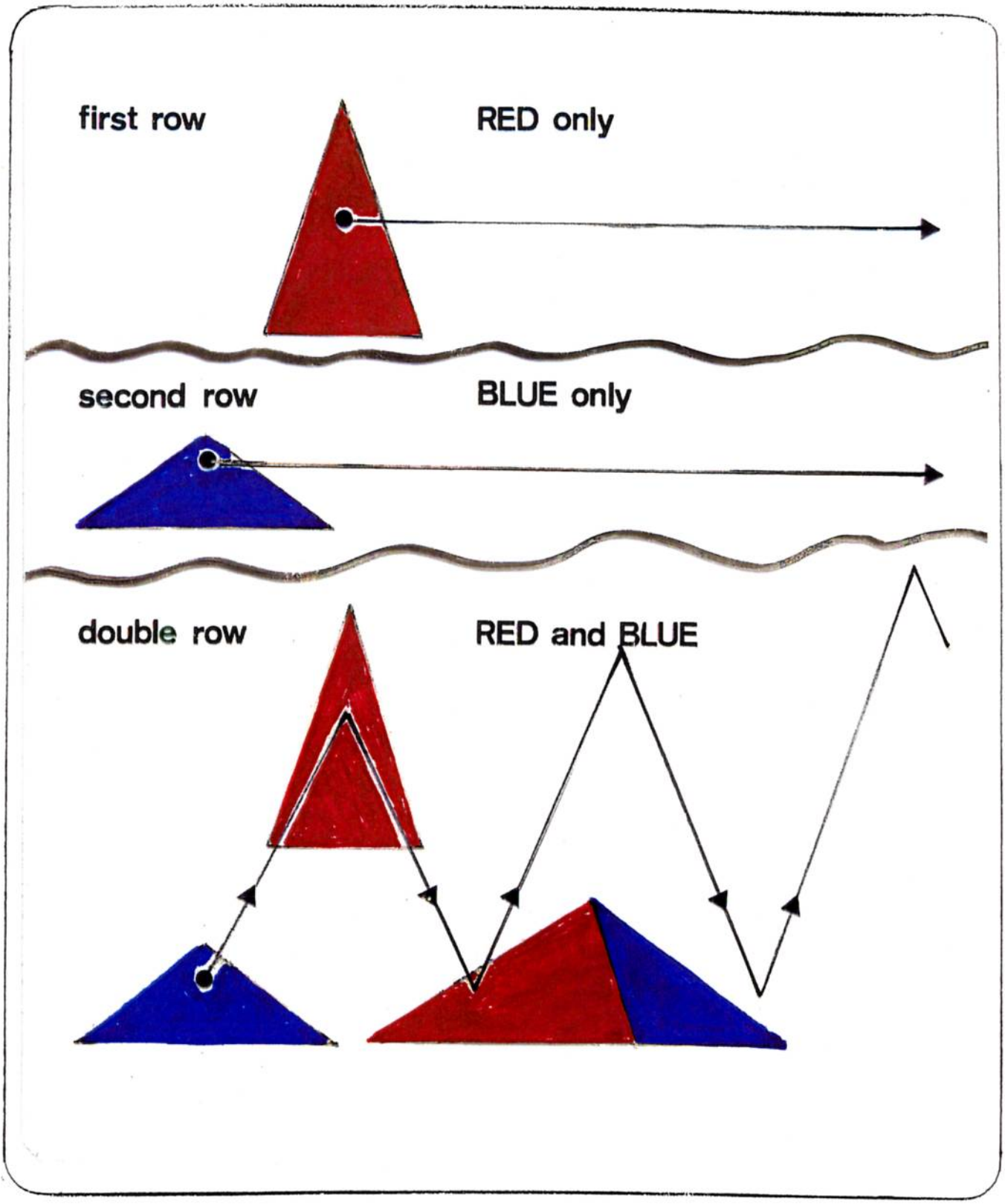
● Fill the double row of cutouts.
This time you'll need both red and blue triangles.

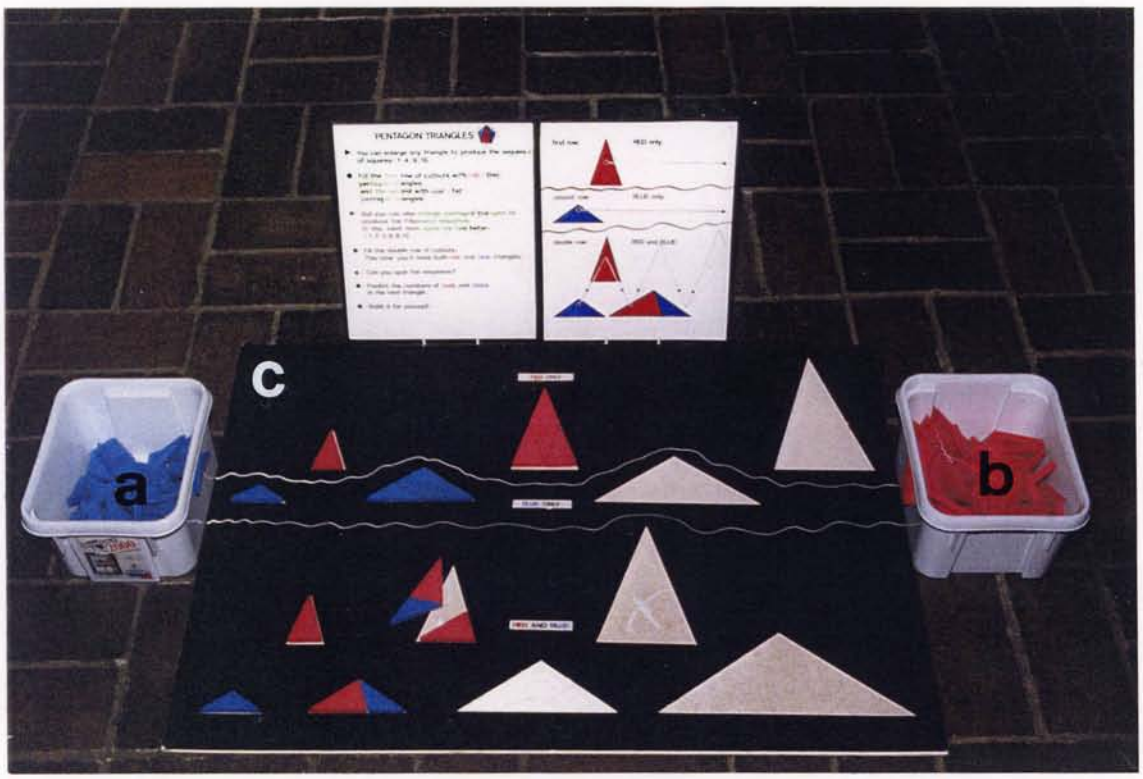
■ Can you spot the sequence?

● Predict the numbers of reds and blues in the next triangle.

● Build it for yourself.

	NUMBER	TITLE
GROUP		
STATION		(Above continued)
TOPIC		




PENTAGON TRIANGLES
a
b
c

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a,b	triangles defined by dissecting a regular pentagon along diagonals in expanded polyethylene:	Pentagon Triangles	Tarquin Publications (address above) on behalf of DIME
a	the 'fat' triangles in blue,		
b	the 'thin' triangles in red,		
	conveniently stored in an Addis Module 2000 unit 1 container		(see **THE STORAGE SYSTEM**)
c	2 sheets of mounting board, 750 mm x 500 mm, a black one with relevant triangles cut out as shown, stuck on a white, and upper surface varnished		local
	To make the cut-outs, assemble the triangles from the DIME pieces and draw round them. This ensures that the fit is not too tight.		
	The markings correspond to those on the 2nd caption board.		

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.4.5	PINE CONE SPIRALS
TOPIC		As 7.4.1

PINE CONE SPIRALS

- Look at the rounded end of the unpainted cone:

- Can you see clockwise spirals?
- Shut your eyes.
- Open them again.
- Can you see anticlockwise spirals?

▶ On the two painted cones these have been marked separately.

- Count them.


a


a


b


b

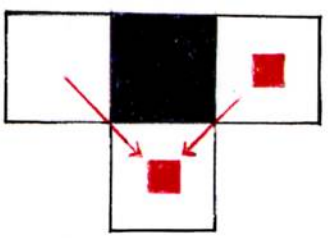
PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
	There are 3 pine cones, all varnished:		
	one unmarked,		
a	one painted to show clockwise spirals,		
b	one painted to show anticlockwise spirals.		
	These came from the forests of Brittany.		(U.K. specimens rather small)
	They may conveniently be displayed in plant pots as **7.4.1 c.**		

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.5.1	THE PASCAL SCANNER
TOPIC		Generating Pascal's Triangle

THE PASCAL SCANNER

▶ Here is a chessboard with 1 red cube in the middle square of the top row.

● Sweep the PASCAL SCANNER along this row. However many red cubes the robot's eyes see, put that total where his nose is:



● Now move the scanner down a square and sweep
the 2nd row,
then the 3rd,
the 4th,
the 5th.

▶ With the robot's help you have computed the first 6 rows of PASCAL'S TRIANGLE.

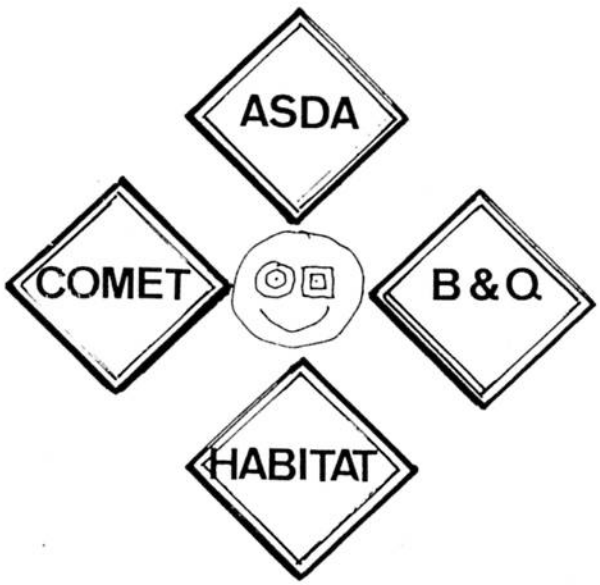
● Turn the black board over to check your adding.

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a,b	parts of 2 chessboards, combined as shown, on a basal sheet of hardboard Those shown have 40 mm squares, giving overall dimensions of 440 mm (width) x 240 mm (depth). The 'black board' referred to on the caption (but not illustrated) is a caption board as described. On the white face - initially concealed, therefore - the 1st 6 rows of Pascal's Triangle are reproduced.		local
c	1 cm cubes Though it is instructive to build towers with the cubes, as shown in the lower photograph, it is unnecessary.	Centicube: 08200	Economatics Ltd (address above)
d	the PASCAL SCANNER, marked as shown, a bridge in 3 mm perspex: top: 170 mm (wide) x 100 mm (deep); uprights, on shorter sides: 100 mm deep, therefore x 15 mm (high)		local

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.5.2	BACK TO THE CAR
TOPIC		As 7.5.1

BACK TO THE CAR

▶ If you're standing here ...

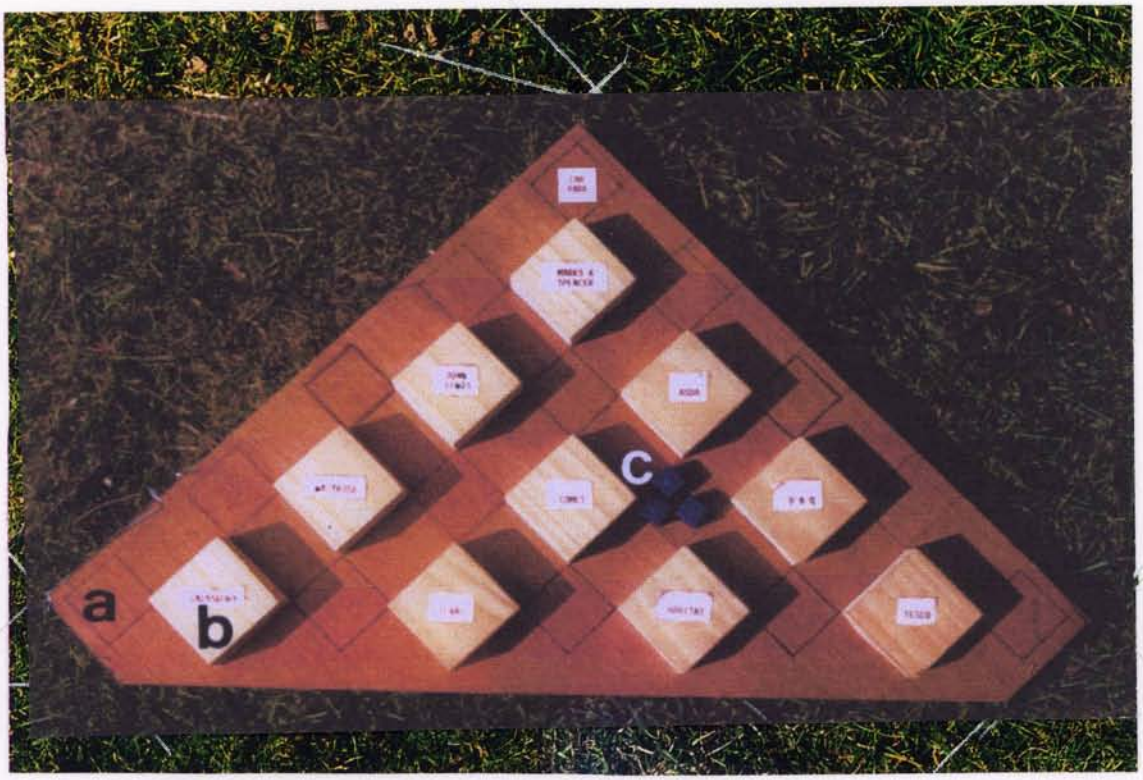


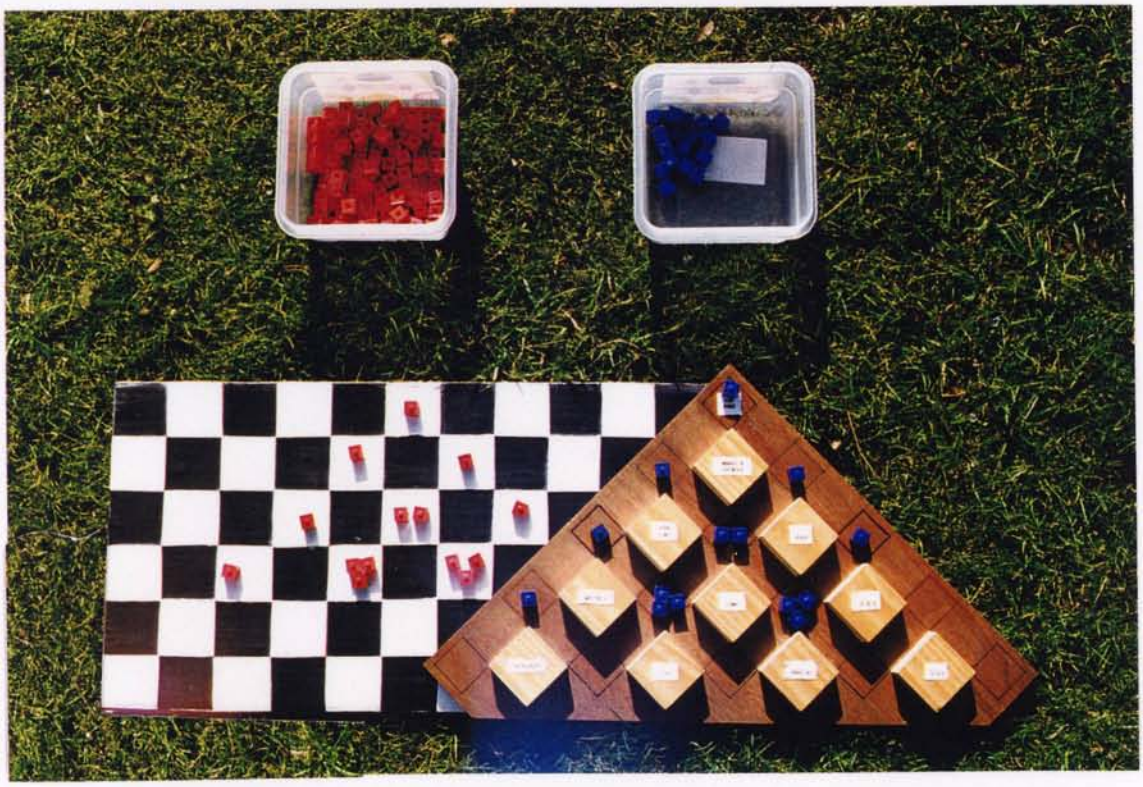
... there are 3 shortest ways to the car park.

- Walk your fingers along each.
- Put 3 blue cubes in that square.
- Do this for every square.

▶ (If you're already on the car park, there's 1 (!) so put 1 blue cube there.)

■ What have you made?


a
b
c



PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	base, in 9 mm plywood, cut to shape shown from a 400 mm square, 30 mm squares marked on it as shown, one labelled **CAR** **PARK**		local
b	'stores' in same material, 45 mm square, labelled as shown Appropriate local store names should be chosen and the caption modified accordingly.		
c	1 cm cubes The lower photograph shows corresponding stages in investigations **7.5.1** and **7.5.2**. Addis Module 2000 unit 1 containers may be used in place of those shown here.	Centicube: 08200	Economatics Ltd (address above) (see **THE STORAGE SYSTEM**)

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.5.3	CHOPS
TOPIC		As 7.5.1

CHOPS

▶ A 1-rod is chopped into:

1 piece

▶ A 2-rod is chopped into:

1 piece

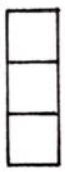
2 pieces

▶ A 3-rod is chopped into:

1 piece

2 pieces

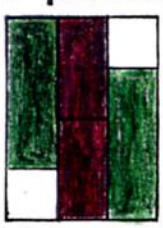
3 pieces



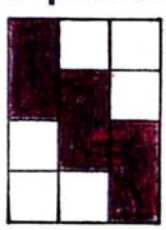
▶ A 4-rod is chopped into:

1 piece

2 pieces



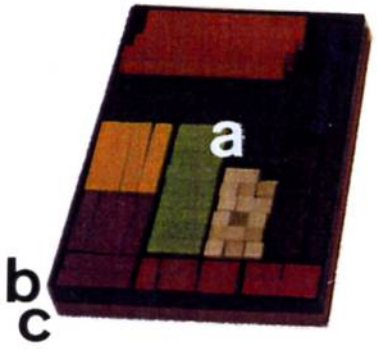
3 pieces

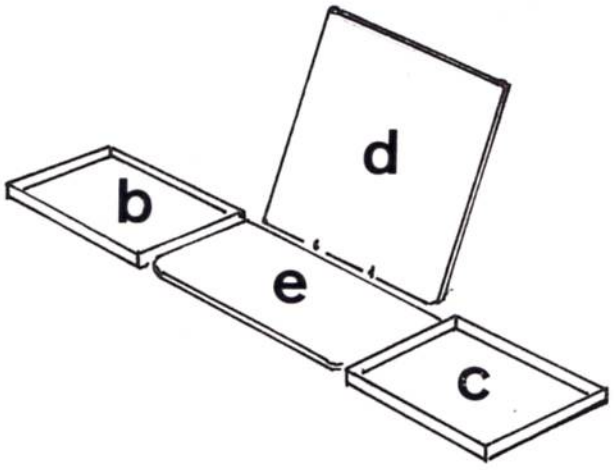


4 pieces

● Build the next row !

▼


a
b
c


d
b
e
c

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	Cuisenaire rods, in box 200 mm x 150 mm x 15 mm	NES Arnold catalogue: SY 358/4	NES Arnold Ltd (address above)
b - e	suggested arrangement:		
b	base of **a**, containing rods, set with length left-to-right		
c	lid of **a**, containing only those denominations of rod necessary to complete the task set but in rather greater quantity than necessary, similarly aligned		
d	caption board on plate stand		(see **THE CAPTION BOARDS**)
e	caption board as described, halved lengthways, set with length left-to-right and with white face uppermost		
	The function of **b**, which may be omitted, is to allow the ambitious student to extend the task to the 6-rod.		

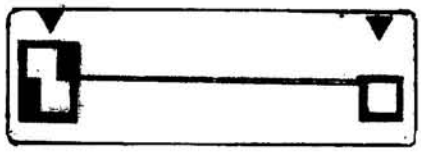
	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.5.4	NUMBER-BUILDING 4
TOPIC		7.3.2 read from Pascal's Triangle

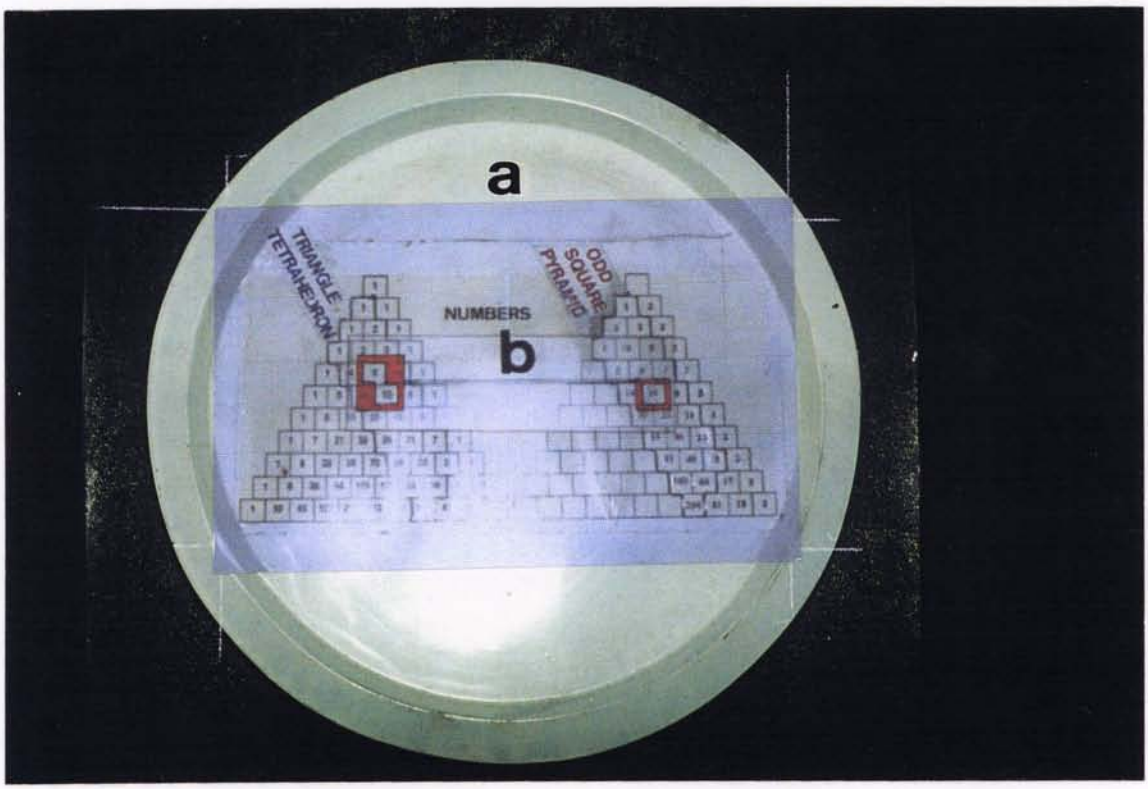
NUMBER-BUILDING 4

- **Move the 'key' around.**

- **What is the secret it unlocks, the link between**

... these numbers **and** **... this number?**




a
TRIANGLE
TETRAHEDRON
NUMBERS
b
ODD
SQUARE
PYRAMID

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	tea tray, 300 mm diameter,* bearing 2 parallel triangular arrays: on the left, **7.1.5 d** down to row 10, on the right, a triangle derived from it by adding the numbers in diagonally adjacent cells, headed, and with certain diagonals indicated, as shown - the colours correspond to those of **4.4 b** and **e** respectively		local
b	Glodex rectangle, 135 mm (wide) x 35 mm (deep), squares framed as shown *history as **7.1.5 c**; this arrangement eye-catching but without any other advantage		

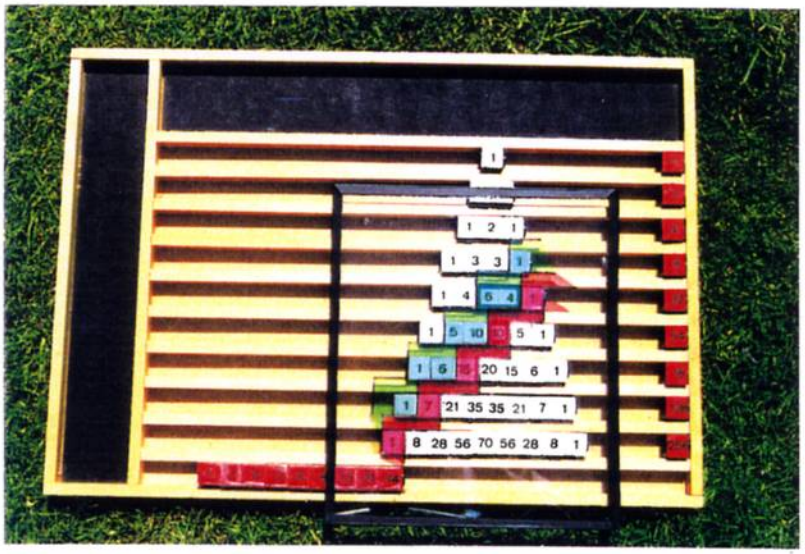
	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.5.5	FIBONACCI AND PASCAL
TOPIC		The Fibonacci sequence read from Pascal's Triangle

FIBONACCI AND PASCAL

- These 2 pictures show how you add green to make red on Pascal's Triangle and so produce the Fibonacci sequence.

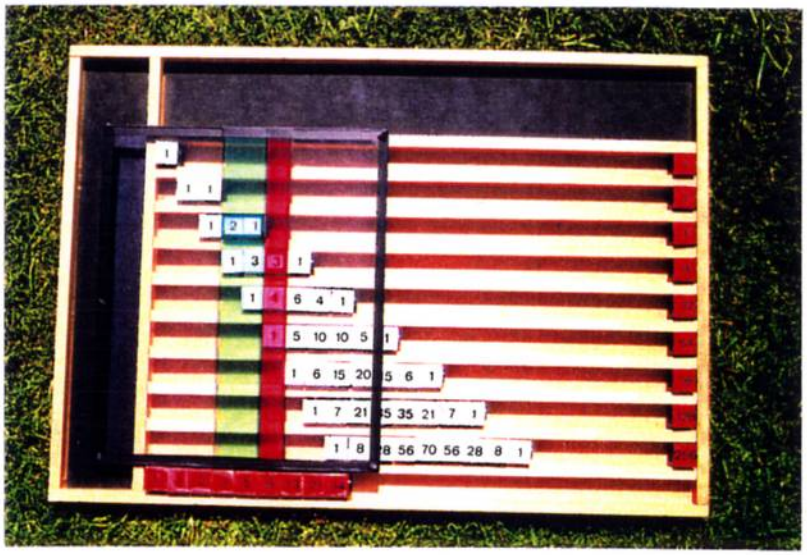
- Slide the rows to make this:

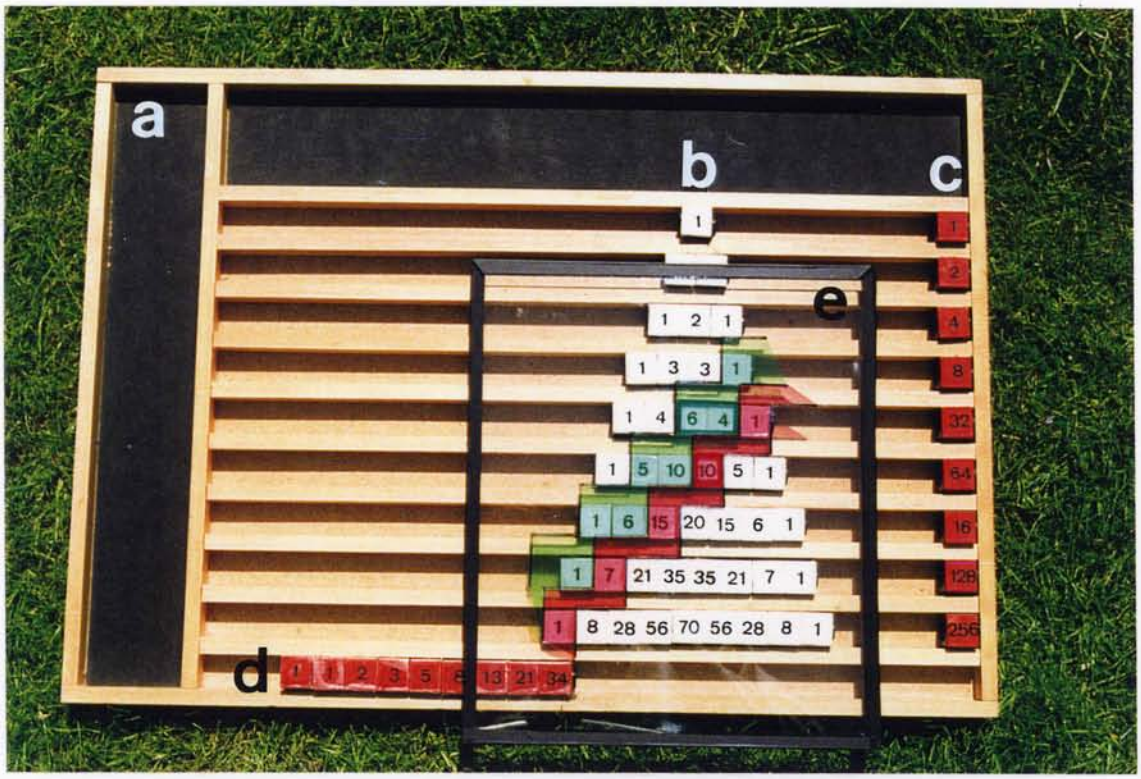
- Slide the clear sheet around and check the adding.

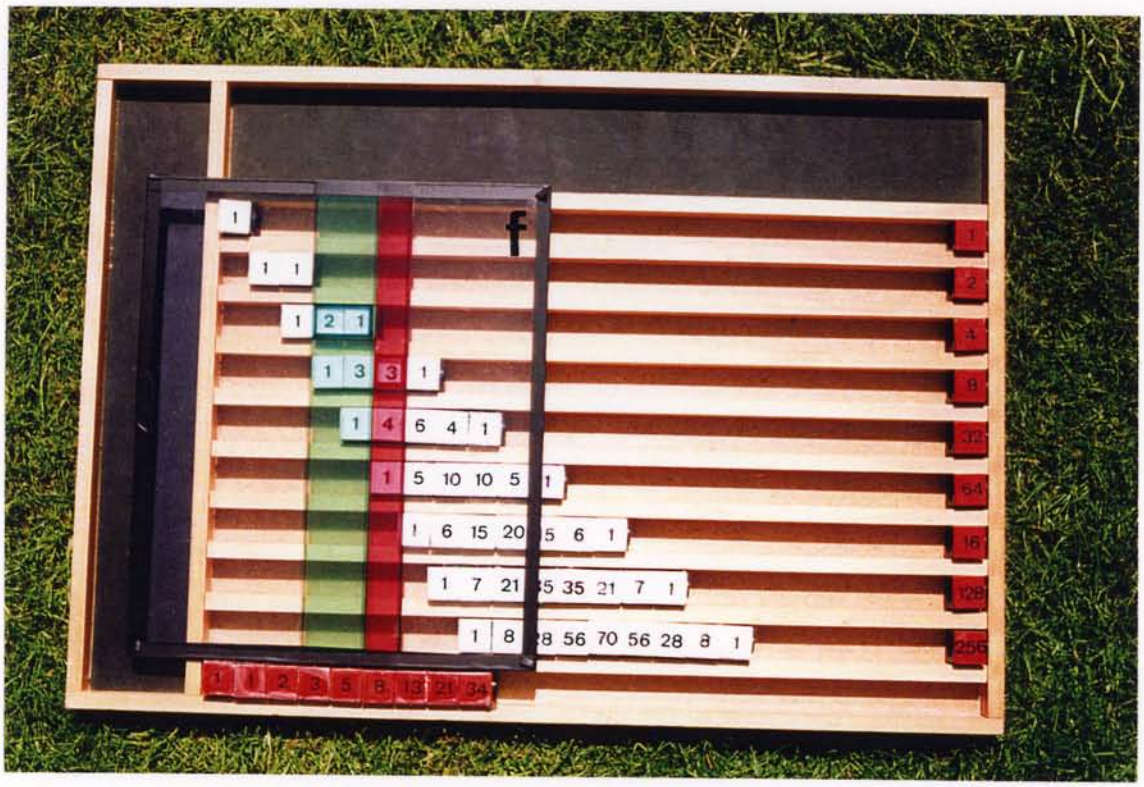


- Slide the rows to make this:

- Slide the clear sheet around and check the adding.




a
b
c
e
d
1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 1 2 3 5 8 13 21 34
1
2
4
8
32
64
16
128
256


f
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 1 2 3 5 8 13 21 34
1
2
4
8
32
64
16
128
256

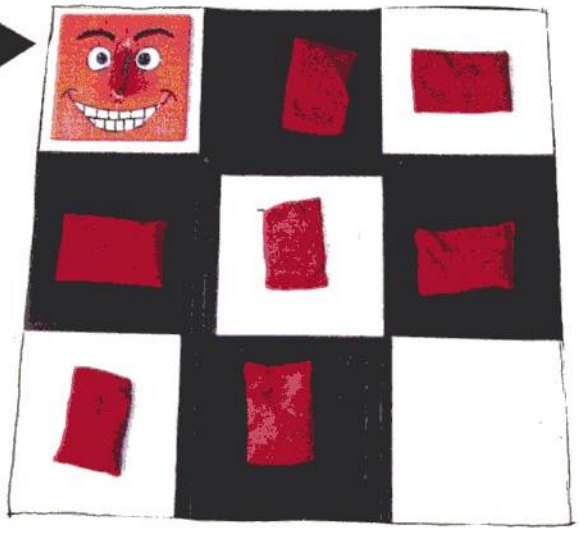
PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	frame, 570 mm (wide) x 400 mm (deep) x 20 mm (thick), sold as:	Histogram board: SY 055/8	NES Arnold Ltd (address above)
b,c,d	(black) Multilink cubes	Multilink: SY 007/9	v.s.
	with 20 mm square tiles	red and white square tiles	Tarquin Publications (address above)
	bearing the numbers shown, stuck on:		
b	cubes with white tiles, those constituting each Pascal's Triangle row stuck together		
c	cubes with red tiles bearing powers of 2		
d	cubes with red tiles bearing the Fibonacci sequence stuck together		
e,f	Glodex sheets, 250 mm (wide) x 300 mm (deep), each carrying pieces cut from green and red acetate, groups of cells outlined upon them in felt-tip of the same colour		
	Those numbers in cells framed in green sum to those in cells framed in red.		
	Accordingly, numbers covered by green acetate and those covered by red acetate sum to the same total.		
	The 'red' total is a term in the Fibonacci sequence; the 2 preceding are 'green' subtotals.		

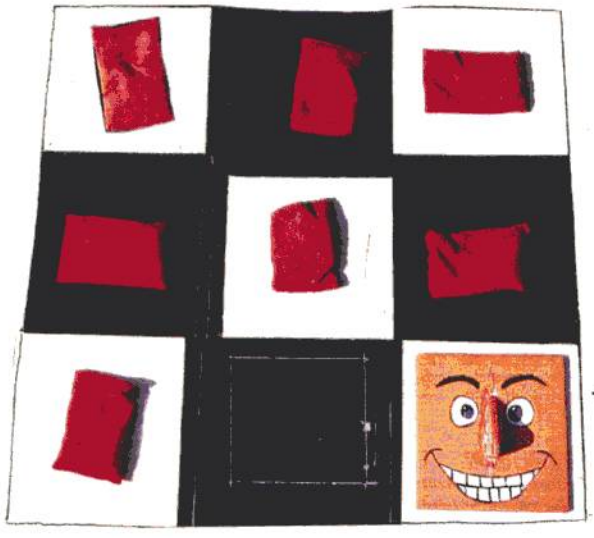
	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.6.1	SLIDING SAM
TOPIC	The sequence with n^{th} term $8n + 5$, n = square edge - 2	

SLIDING SAM

▶ Object: to get Sam ...

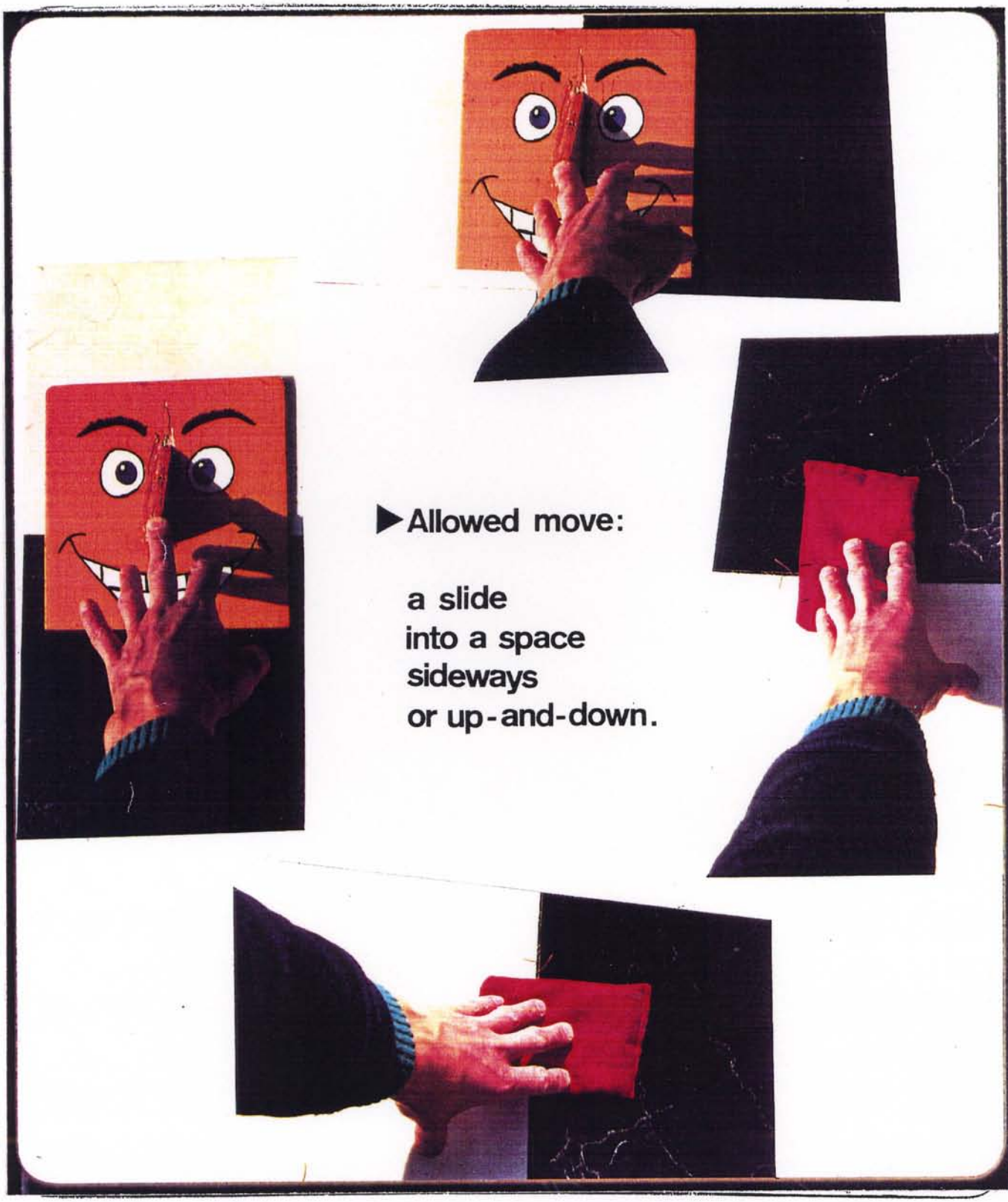
from here ▶ (the top left square)





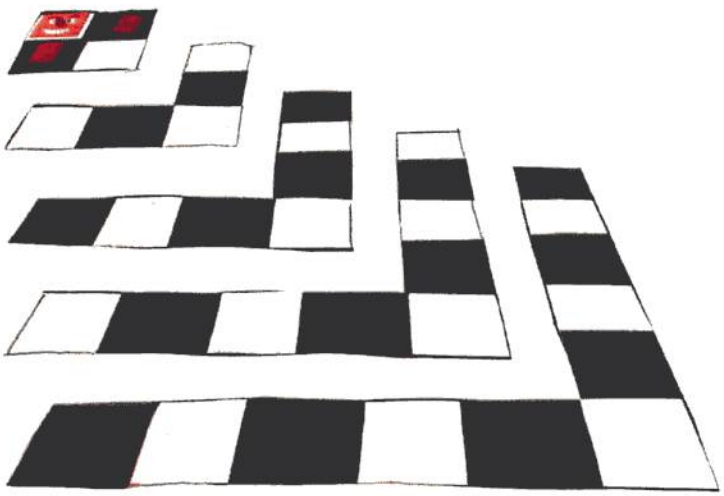
◀ ... to here (the bottom right square)

	NUMBER	TITLE
GROUP		
STATION		(Above continued)
TOPIC		



	NUMBER	TITLE
GROUP		
STATION		(Above continued)
TOPIC		

▶ You can make the board bigger by adding 'L's:



■ How many moves does it take for
a 2x2 board?
a 3x3 board?
a 4x4 board?

● Fill in the table.
Predict the numbers for
a 5x5 board?
a 6x6 board?

● Try to achieve them!

	NUMBER	TITLE
GROUP		
STATION		(Above continued)
TOPIC		

TALLY COUNTER

Press to count.

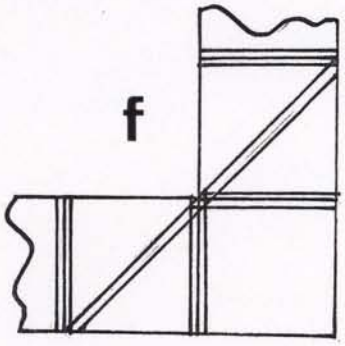
Wind back
to 0000.

	NUMBER	TITLE
GROUP		
STATION		(Above continued)
TOPIC		

BOARD	MOVES	DIFFER - ENCE	DIFFER - ENCE
2 x 2			
3 x 3			
4 x 4			
5 x 5			
6 x 6			

▶ If you've taken the fewest moves, you should get 0s in the last column.


a
b
c
d
e


f

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	square vinyl tiles, black and white, 250 mm, taped as set of **L**s which nest as shown		local
f	= **a** verso Tape tiles front and back for strength. Reinforce corners as shown.		
b	'Sam' as 200 mm square of 6.5 mm plywood, with 'nose' of same held by screw countersunk into back and glue, painted as locally appropriate		
c	bean bags, red,	NES Arnold catalogue: L 3062/14 (**3 packs needed**)	NES Arnold Ltd (address above)
	conveniently stored in an Addis Module 2000 unit 4 container		(see **THE STORAGE SYSTEM**)
d	tally counter, on caption board preceding (order through supplier of sports equipment)		local
e	recording board preceding, faced with Glodex for use of dri-wipe pen		(see **THE CAPTION BOARDS**)

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.6.2	THE FERRY PROBLEM
TOPIC	As 7.6.1, n = number of men - 1	

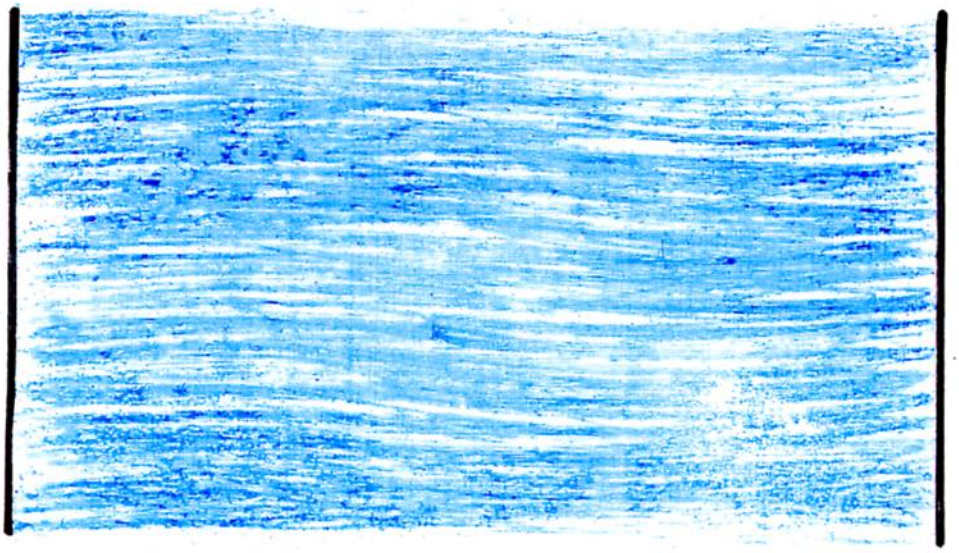
THE FERRY PROBLEM

▶ 1 man and 2 boys have to cross a river.

The boat holds 1 man or 2 boys.
All can row.

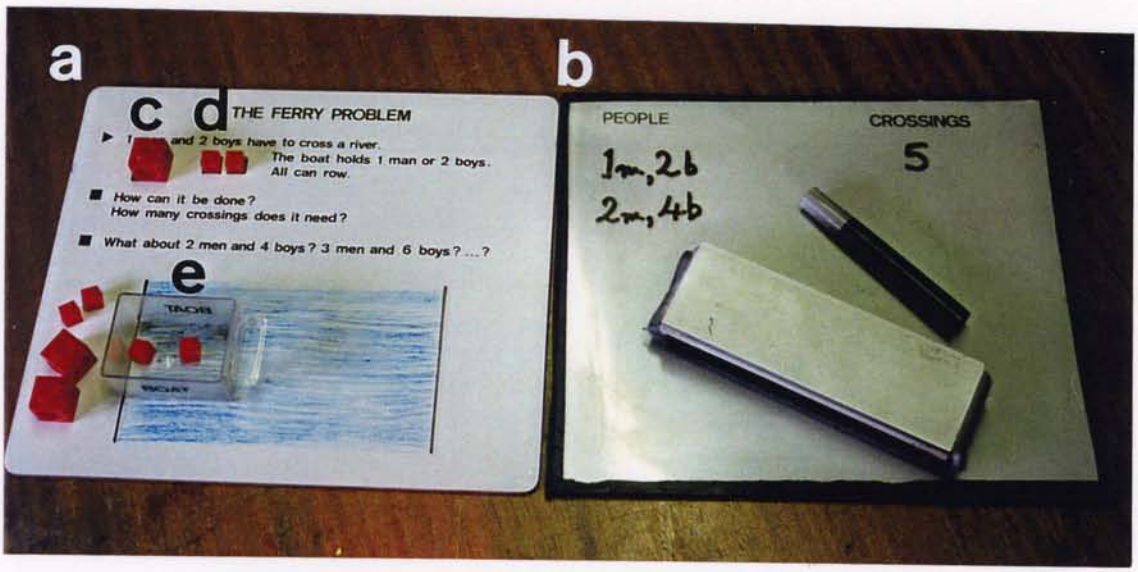
■ How can it be done?
How many crossings does it need?

■ What about 2 men and 4 boys? 3 men and 6 boys? ...?



PEOPLE

CROSSINGS


a
b
c
d
e
THE FERRY PROBLEM
and 2 boys have to cross a river.
The boat holds 1 man or 2 boys.
All can row.
How can it be done?
How many crossings does it need?
What about 2 men and 4 boys? 3 men and 6 boys? ...?
PEOPLE
CROSSINGS
5
1m, 2b
2m, 4b

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	caption board		
b	recording board, shown in use Though no writing is intended on **a**, it is advisable to face this board with Glodex too.		(see **THE CAPTION BOARDS**)
c	2 cm cubes	Multilink: SY 007/9	NES Arnold Ltd (address above)
d	1 cm cubes	Centicube: 08200	Economatics Ltd (address above)
e	'boat': storage drawer from chest of same intended to hold small items, 65 mm (long) x 45 mm (broad) x 35 mm (high), labelled **BOAT** on port and starboard sides A model boat could be substituted for this.		local

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.7	LEAPFROG
TOPIC	The sequence with n^{th} term $n(n + 2)$, n = number of (red) frogs	

LEAPFROG

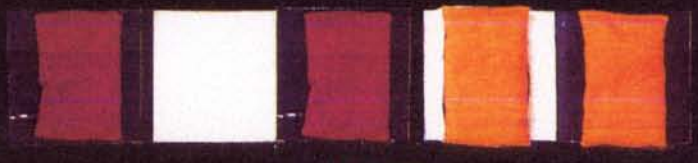
- Here is a row of lily pads on a pond with frogs.
- The red and yellow frogs want to change places:

START

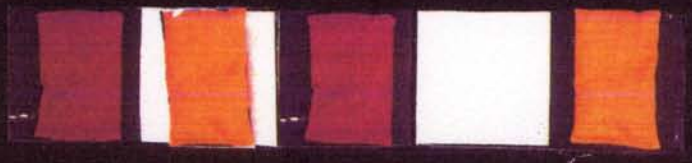
FINISH

- Allowed moves:

a SLIDE



a JUMP



- How many moves does it take?
- Try 1, 2, 3, 4 frogs each side. Fill in the table. Predict the number for 5. Try to achieve it!

	NUMBER	TITLE
GROUP		
STATION		(Above continued)
TOPIC		

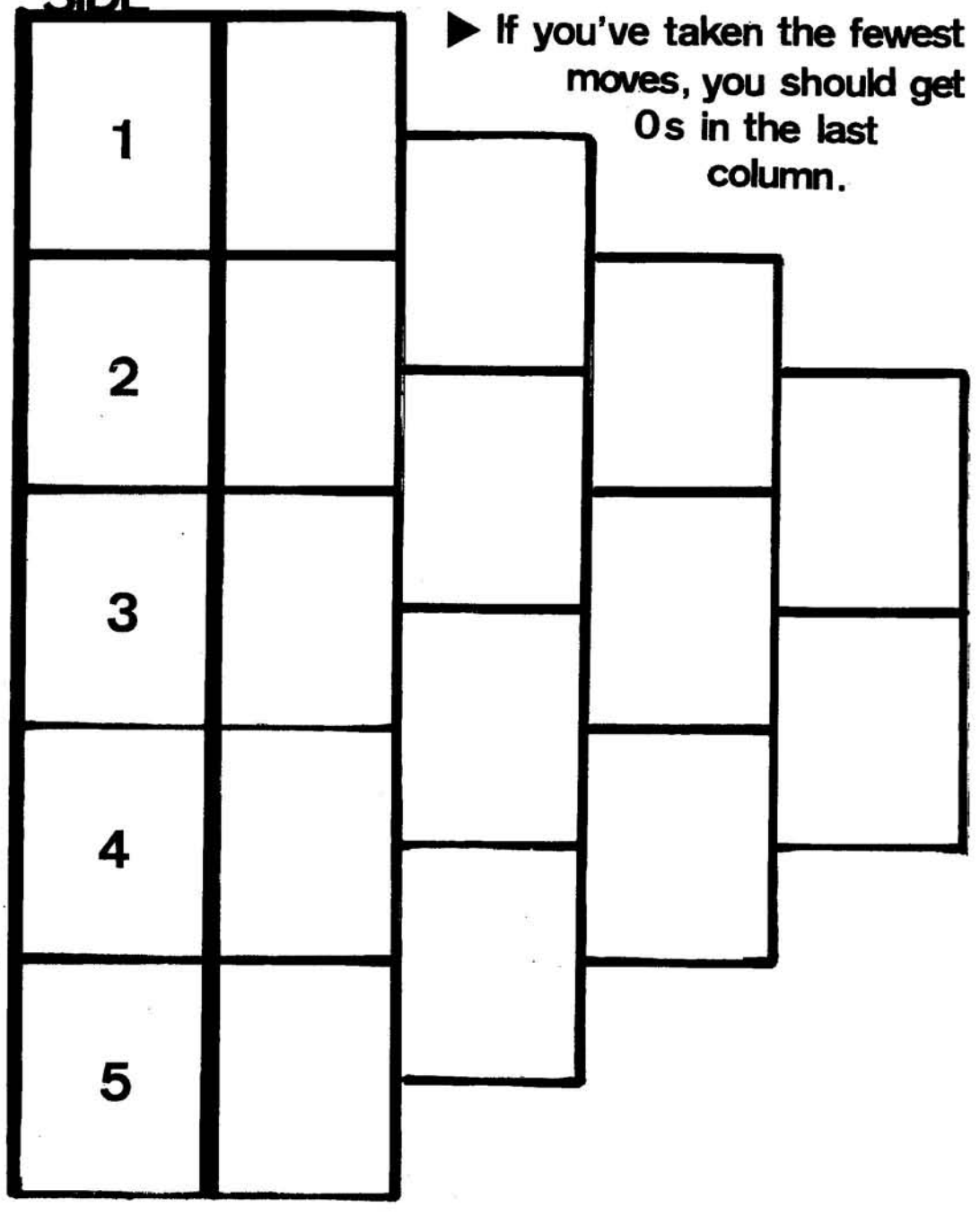
TALLY COUNTER

Press to count.

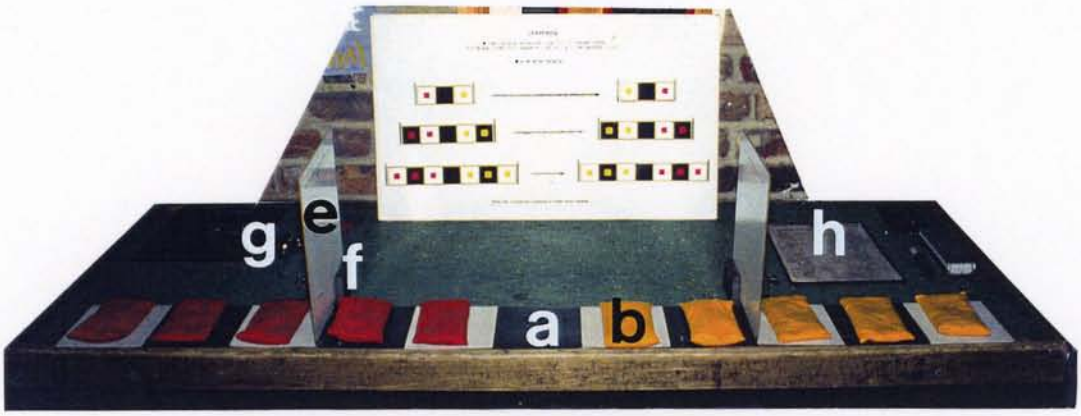
Wind back
to 0000.

	NUMBER	TITLE
GROUP		
STATION		(Above continued)
TOPIC		

FROGS MOVES DIFFER DIFFER DIFFER
EACH -ENCE -ENCE -ENCE
SIDE



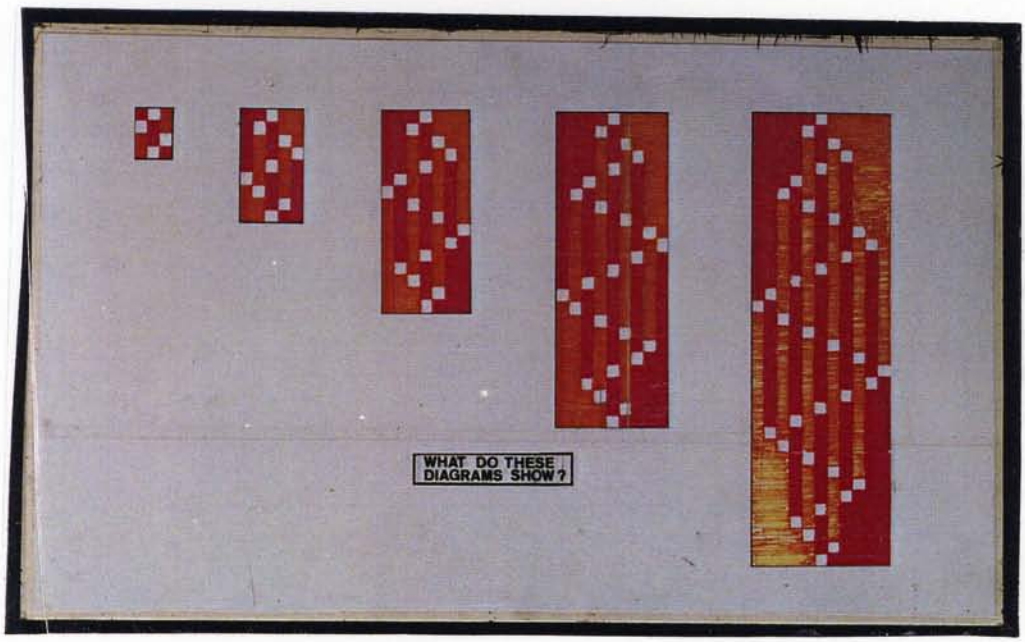
▶ If you've taken the fewest moves, you should get 0s in the last column.

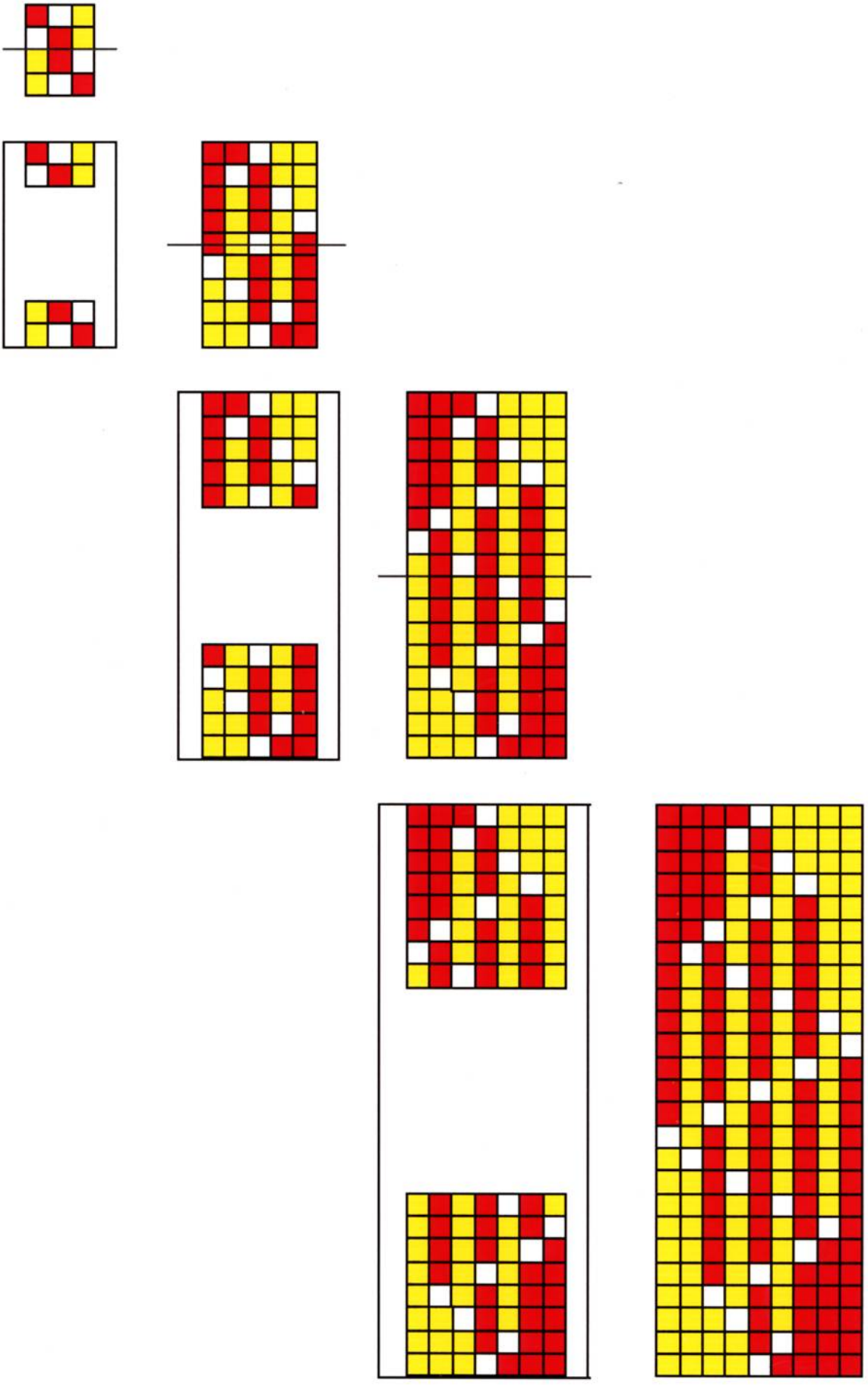

g
e
f
a
b
h

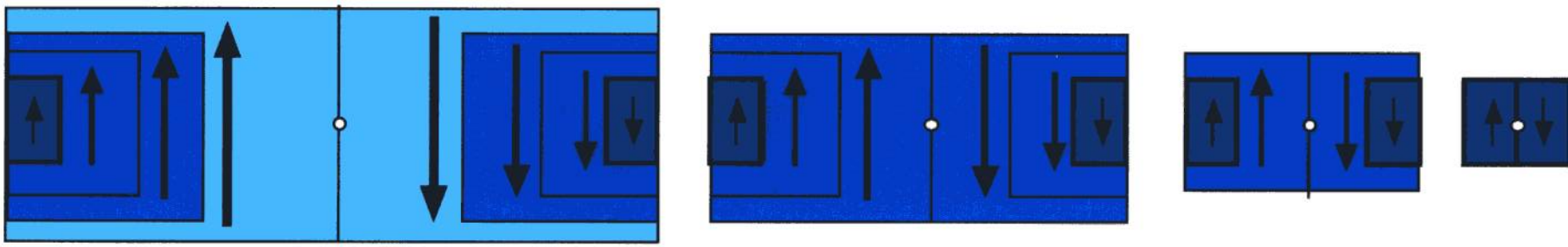

e
f
c
d
g
e
h
3 5 2
8 7
15
f

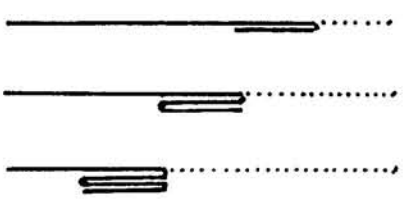
	NUMBER	TITLE
GROUP		
STATION		(Above continued)
TOPIC		

i







PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a,b,e,f	table-scale version of the apparatus		
c,d,e,f	floor-scale version of the apparatus		
e,f	These units are moved to define the length of the 'row of lily pads'. Note that the same can be achieved by folding 'lily pads' under:  but visitors are loath to return the extended 'row' to its shortest form ready for the next group by doing this.		
e	caption board as described, bearing the instruction **Move 'bookend' outwards to make more squares.**		
f	slotted base	STA-395-W	Griffin Education (address above)
a,e	'row of lily pads'		
a	square vinyl tiles, 150 mm, joined by tape front and back		local
c	square vinyl tiles, 300 mm, joined likewise		local
b,d	'frogs'		
b	bean bags: red yellow	NES Arnold catalogue: L 3062/14 L 3062/27	NES Arnold Ltd (address above)
d	6 mm plywood squares, 200 mm, with drawer handles, held by glue and screws countersunk into the back, painted respective colours		local
g	= **7.6.1 d**		
h	recording board, shown in use		(see **THE CAPTION BOARDS**)
i	caption preceding (This may be omitted: it is a distraction to all but the ablest students.) 2 sheets of white-faced hardboard stuck back-to-back, bearing the designs shown and the question **WHAT DO THESE DIAGRAMS SHOW?**		

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.8	THE HANOI PAGODA
TOPIC		The sequence with n^{th} term $2^n - 1$, n = number of cups

THE HANOI PAGODA

1 ▶
CUP

- **Moving 1 cup at a time and never putting a large on a small, move the tower from SAND to ROCK.**

- **How many moves do you need for a tower**

  2 ▶
  CUPS

  - **1 cup high?**
  - **2 cups high?**
  - **3 cups high?**
  - **4 cups high?**
  - **5 cups high?**
  - **6 cups high?**
  - **7 cups high?**

  - **n cups high?**

  3 ▶
  CUPS

	NUMBER	TITLE
GROUP		
STATION		(Above continued)
TOPIC		

	NUMBER	TITLE
GROUP		
STATION		(Above continued)
TOPIC		

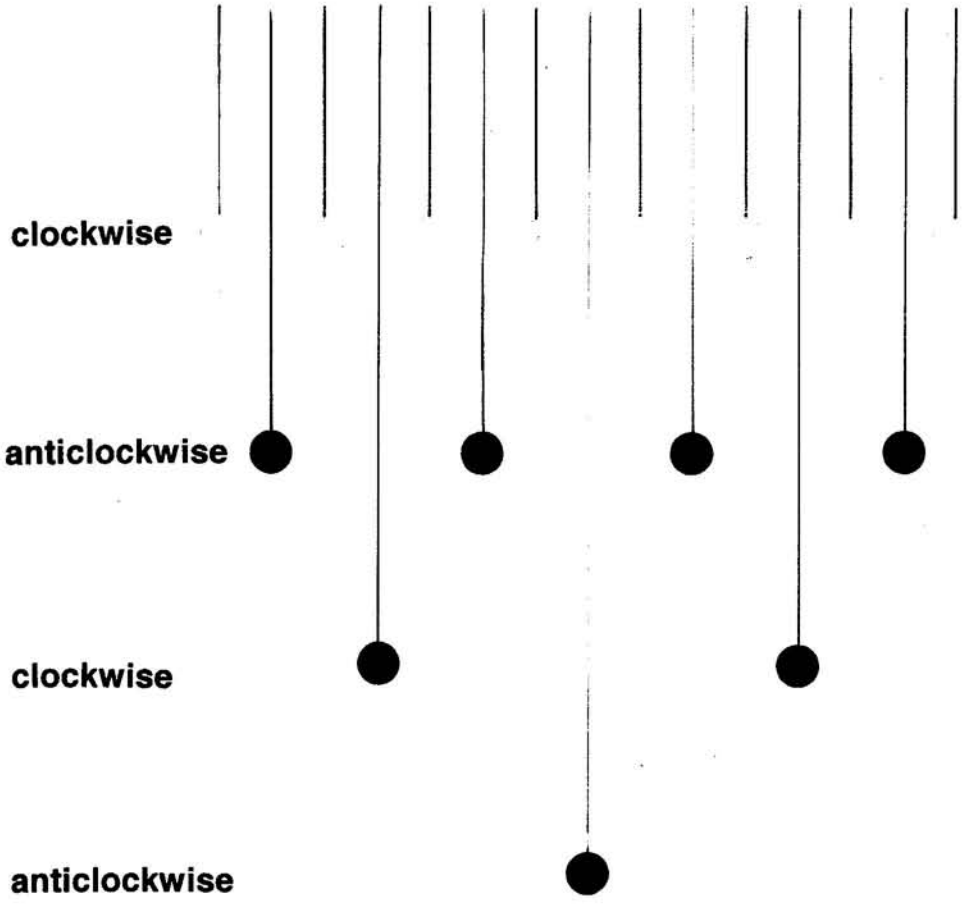
CUPS	MOVES
1	
2	
3	

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a,b	full caption The photograph shows how the 2 parts are joined.		
a	(standard)		
b	150 mm x 100 mm photographs, sandwiched between 2 Glodex sheets, 1200 mm (wide) x 300 mm (high/deep)		
c	toy, sold under the name:	Building Beakers	local
d	(alternative to the prototype shown here:) 2 caption boards as described: one black face upwards, with 3 circles of diameter slightly greater than that of the largest cup cut out, stuck on a second, white face upwards, labelled **SAND GRAVEL ROCK** in positions corresponding to cut-outs in upper board		
e	recording board, with text preceding		(see **THE CAPTION BOARDS**)

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.8.1	(Above continued)
TOPIC		

USE YOUR RULER TO SOLVE THE HANOI PUZZLE.

- For towers up to 4 cups high, move the cups in this order: ------->



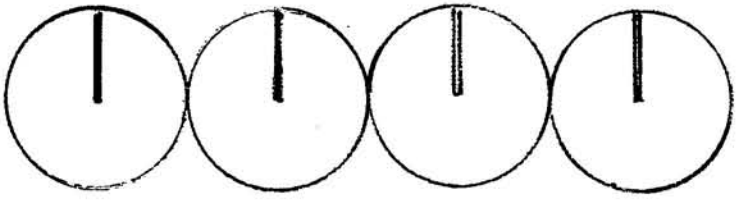
- Study the pattern.
- Extend it for towers 5 cups, 6 cups, 7 cups, ... n cups high.

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.8.2	THE HANOI COMPUTER
TOPIC		As 7.8.1

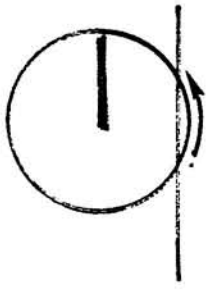
THE HANOI COMPUTER

- ▶ The white letters show destinations: S=SAND, G=GRAVEL, R=ROCK.
- ▶ Each wheel shows how the corresponding cup moves.
- ▶ When its white line passes under a black arrow, that cup moves to the destination marked:

- ● Zero the lines:

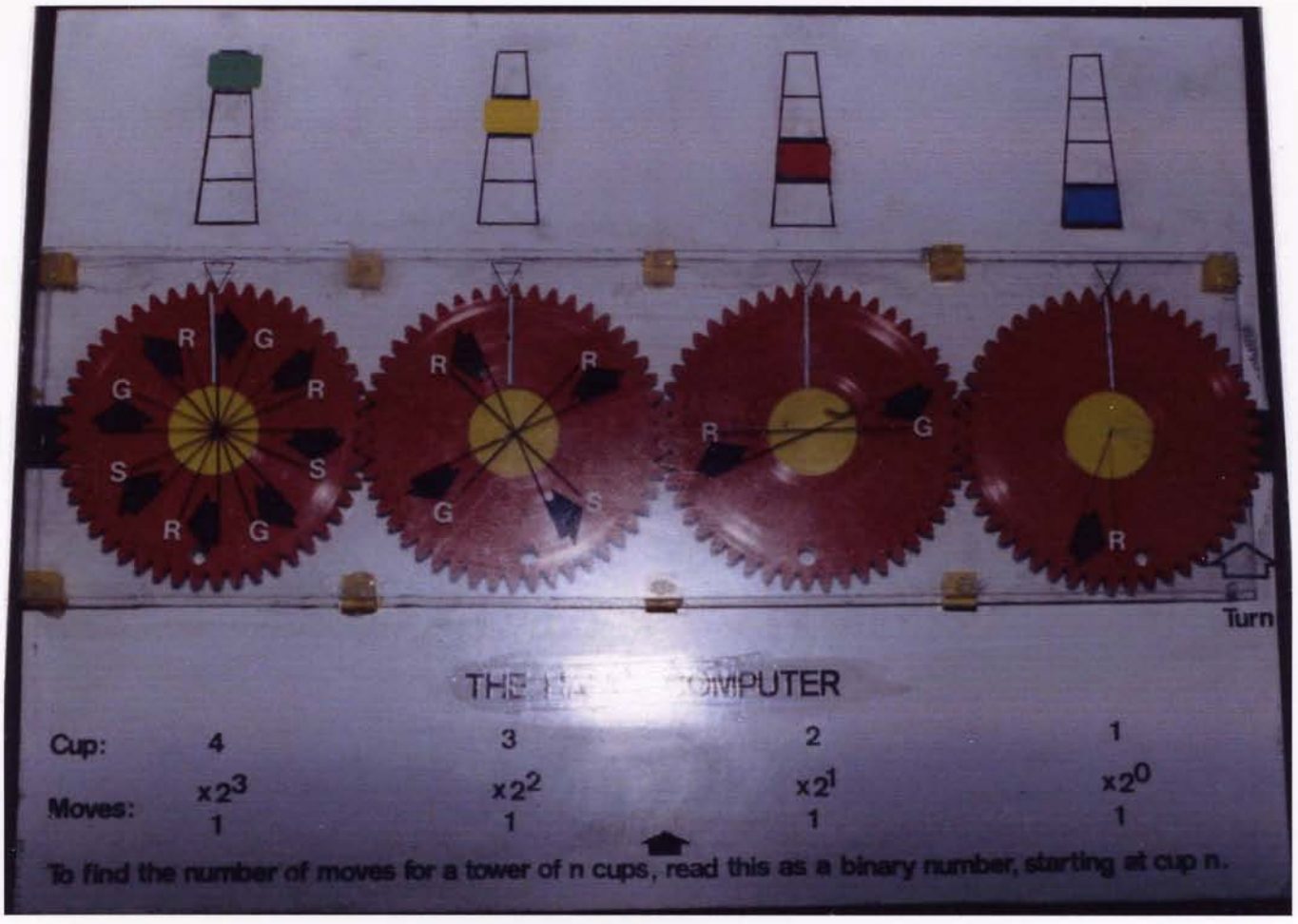


- ● Turn the righthand wheel slowly anticlockwise:

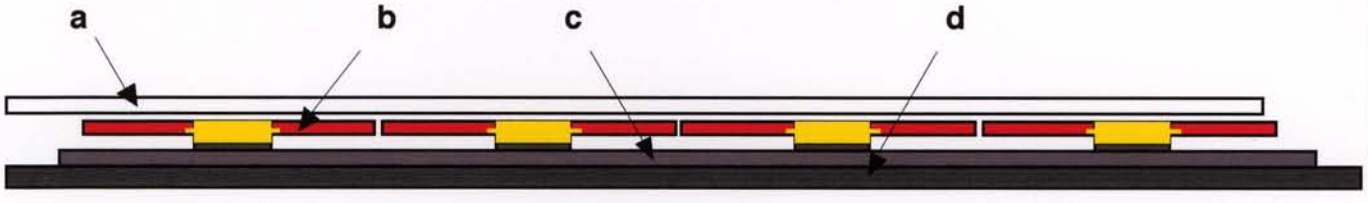


- ● Observe.

1


Turn
THE [illegible] COMPUTER
Cup: 4 3 2 1
Moves: x2^3 x2^2 x2^1 x2^0
1 1 1 1
To find the number of moves for a tower of n cups, read this as a binary number, starting at cup n.

2


a
b
c
d

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
1	VIEW IN PLAN		
2	VIEW IN SECTION		
a	sheet of 3 mm Glodex, 400 mm x 120 mm, marked as shown, spaced 12 mm from:		local
d	baseboard, consisting of 2 sheets of white-faced hardboard, 420 mm x 300 mm, stuck back-to-back		local
b	48-tooth gear, 100 mm diameter, marked as shown: 4 needed		(see **1.6 f**)
c	strip of 'loop' velcro, 400 mm x 20 mm, stuck to **d**		local

	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.9.1	BASE 2 WEIGHINGS
TOPIC		The r.h. pan sequence as 1 to 15 in binary notation

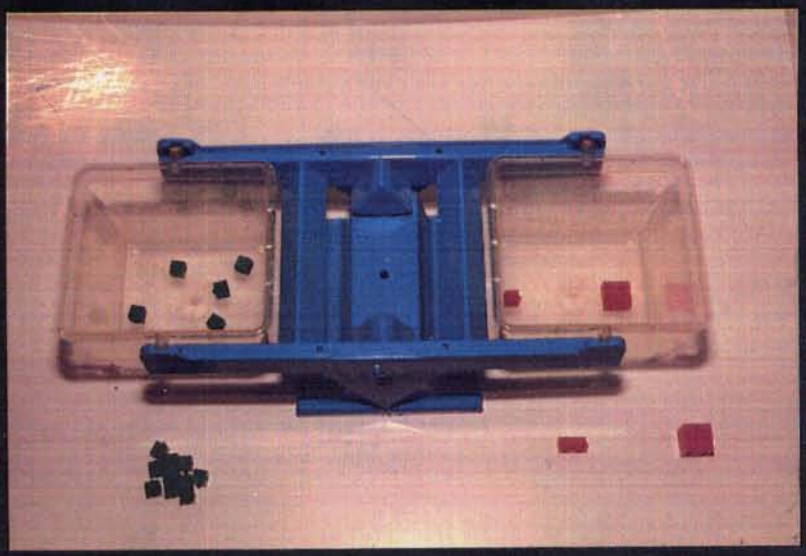
BASE 2 WEIGHINGS

- Balance every number of loose green cubes  from 1 to 15 in the LEFT bucket ...

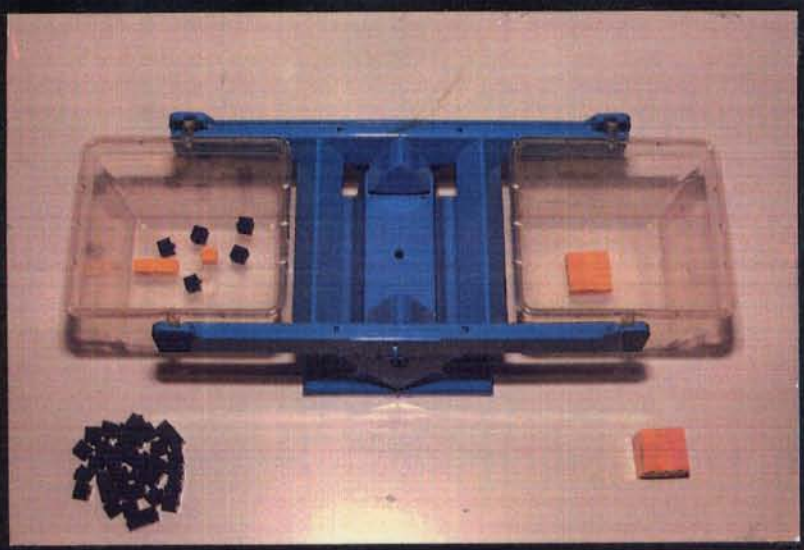
... by putting 1 or more of these in the RIGHT bucket:

	NUMBER	TITLE
GROUP		
STATION		(Above continued)
TOPIC		(Above and below)

How to weigh 5 green cubes



How to weigh 5 blue cubes

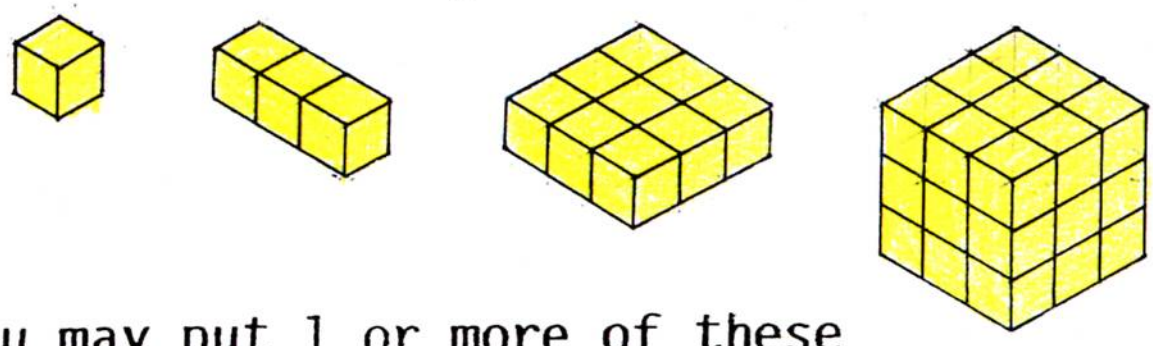


	NUMBER	TITLE
GROUP	7	SEQUENCES
STATION	7.9.2	BASE 3 WEIGHINGS
TOPIC	The sequence of pan differences as 1 to 40 in ternary notation	

BASE 3 WEIGHINGS

- **Balance every number of loose blue cubes from 1 to 40 in the LEFT bucket ...**

... with these 4 yellow masses:



You may put 1 or more of these in the LEFT bucket, the RIGHT bucket or BOTH.

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
	The 3 captions are arranged in order. The central caption serves the outer two: the upper photograph shows a weighing performed in base 2, the subject of the 1st board; the lower, the same weighing in base 3, the subject of the 3rd board.		
	2 2-pan balances	Super Beamer Balance, transparent buckets: 08413	Economatics Ltd (address above)
	The masses are 1 cm cubes	Centicube: 08200	v.s.
	The green and blue sets comprise individual cubes; the red and yellow sets, cubes joined as shown in the respective captions. For these masses to be used to the accuracy required, the amount of cement should be minimal.		