Contents

The Magic Manual

Section 5

Dissections

A guide for fabricators and users to stations from the Magic Mathworks Travelling Circus

5. DISSECTIONS

In contrast to the dissections of **3.6-7**, the examples here are ***a***symmetric.

5.1 are 2-dimensional; **5.2**, 3-dimensional.

The stations have been chosen for their variety. They are not intended to challenge *as puzzles* : they are almost all easily soluble if the experimenter uses the aids provided and works systematically. But that in itself demands mature skills.

5.1.1 and 5.1.2 are both based on **reptiles**, shapes which aggregate or split to produce a similar shape.

5.1.1 TANGRAM POLYGONS

c The tangram puzzle makes use of the √2 ratio and its powers produced by successively bisecting the right-angled isosceles triangle.

p The chosen format encourages people to solve the puzzles in 2 stages: to build sub-assemblies which in turn form the final polygon.

As a preliminary, the aid **TANGRAM SQUARES** may be used. Note that 2 of the 4 red pieces are congruent.

The dissections are not always unique. In such cases people who find one solution with ease should be encouraged to seek another.

5.1.2 THE RIDDLE OF THE SPHINX

c The 'sphinx' itself is the reptile: a ***hex* iamond**, (where a ***poly* iamond** is a polygon made from equilateral triangles sharing common edges). It is the particular hexiamond formed by joining 2 triamonds (regular trapezia) by non-corresponding edges.

p In the case of the sphinx scaled x 4, there is an aid but it is not explicit: the 'x 4' sphinx is to the 'x 2' sphinx as that itself is to the 'x 1'.

As with 5.1.1, the activity may be extended to finding all the solutions to the 'x 3' and 'x 4' sphinxes.

5.1.3 PENTOMINOES: THE 3 X 20 RECTANGLE

c By analogy with polyiamonds, **poly*ominoes*** are arrangements of n squares joined by edges. Their number rises rapidly with n. The set of ***pent* ominoes** has 12 members.

p Because 5 x 12 = 60, a **highly composite** number, one set of challenges is to make the 4 possible rectangles. Of these the most instructive is the 3 x 20. There are 2 solutions and one can be derived elegantly from the other.

Each piece has a letter code and the solutions are presented as words, giving the puzzlist a verbal handle on what is ostensibly a spatial task.

5.1.4 KURSCHAK'S COUNT

c The square can be dissected into 16 equilateral triangles and 32 isosceles triangles of 150°, containing a dodecagon with 12 equilateral triangles and 24 such isosceles triangles. Thus, if the square has area 4 square units, the dodecagon has area 3. The dodecagon is inscribed in a circle, which is in turn inscribed in the square. Since this circle therefore has unit radius, we can infer that the dodecagon inscribed in a unit circle has area 3 square units.

p The exercise is a piece of algebra made concrete.

5.2.1.1 THE SOMA CUBE

c The 3-D analogue of the polyomino is the **poly*cube***, an arrangement of n cubes joined by faces.

The Soma set comprises all polycubes, $n \leq 4$, which are not **cub*oids***. It happens that a cube of edge 3 units can be built from them (in 240 symmetrically distinct ways).

p There are few enough pieces that the puzzlists who exercise their ability to visualise them in chosen orientations, and can thus anticipate misfits, are favoured over those who proceed by trial and error. However, a solution consisting of 3 stacked plan views is provided. By proceeding systematically cell-by-cell and layer-by-layer, someone with a deficient 3-dimensional sense can therefore still solve the puzzle.

5.2.1.2 THE SOMA CUBE: MIRROR IMAGE

c Sections **2.1** and **3.7** have an equal claim on this station.

5 of the 7 Soma pieces have a plane of symmetry. The other 2 are therefore **chiral.** Not only that, but they are **enantiomorphic**. Consequently, any form made from the Soma pieces has a mirror image in which one of these pieces maps into the other.

p The experimenter builds one cube (or other Soma form) and inserts the pieces in corresponding positions in the image formed by the **MIRA** from section **2.1.1**.

5.2.2 $3^3 + 4^3 + 5^3 = 6^3$

c This dissection models the relationship.

(Advanced students can prove that this is a unique solution in consecutive integers to the equation

$$w^3 + x^3 + y^3 = z^3.)$$

p Though a photograph shows the (unique) solution, it requires careful study.

A calculator exercise reinforces the station's arithmetic significance.

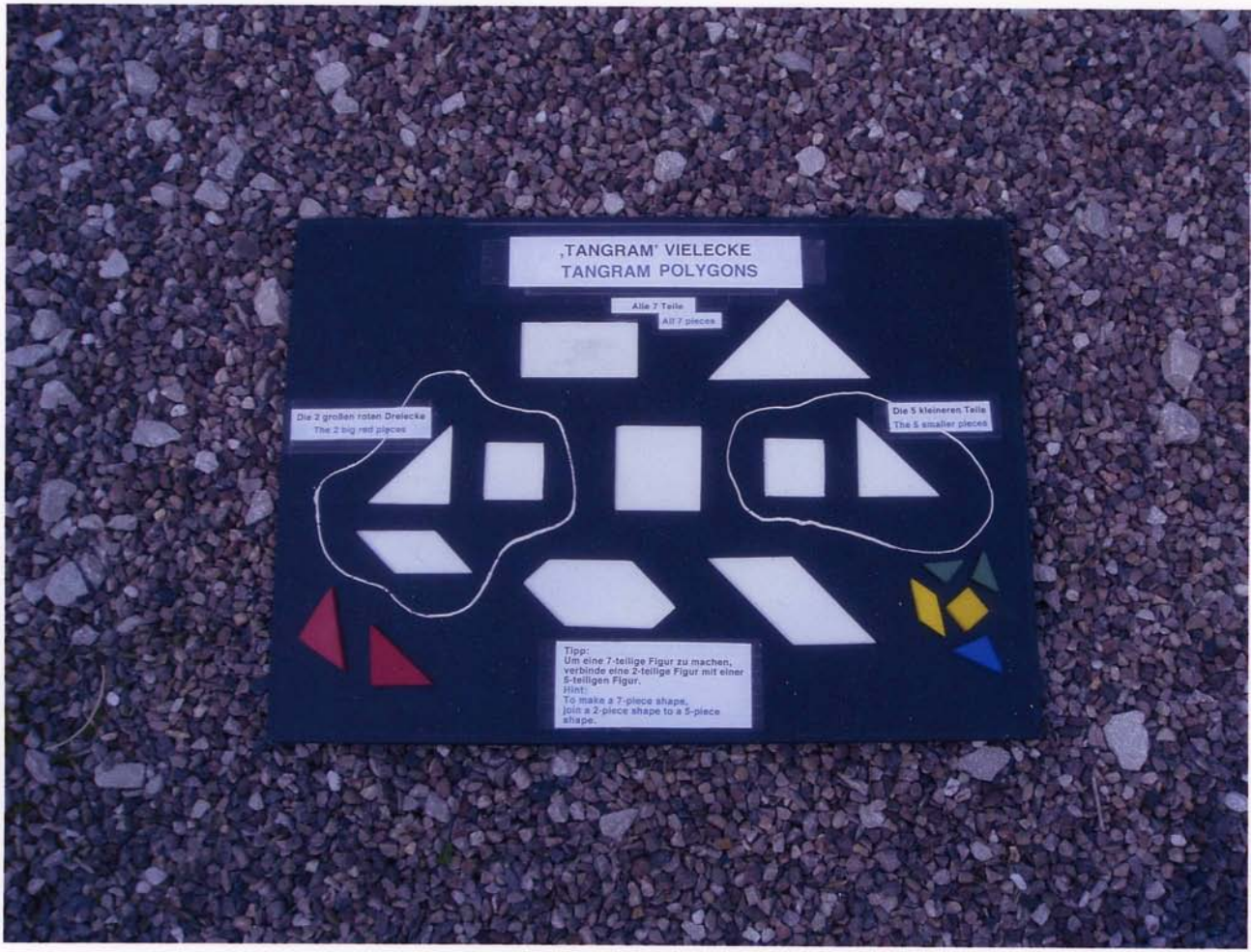
5.2.3 A 9-PIECE DIE

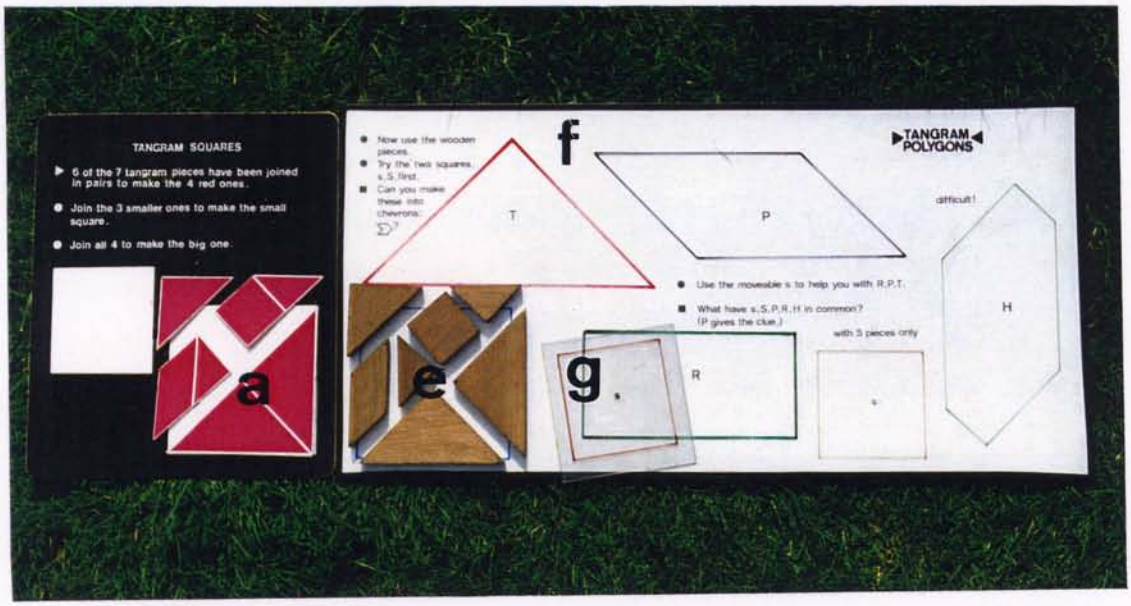
c Each face of the standard die has a different number of spots. The faces thus vary in symmetry and therefore in possible orientations according to the placing of the spots. It is this ambiguity which makes the puzzle difficult.

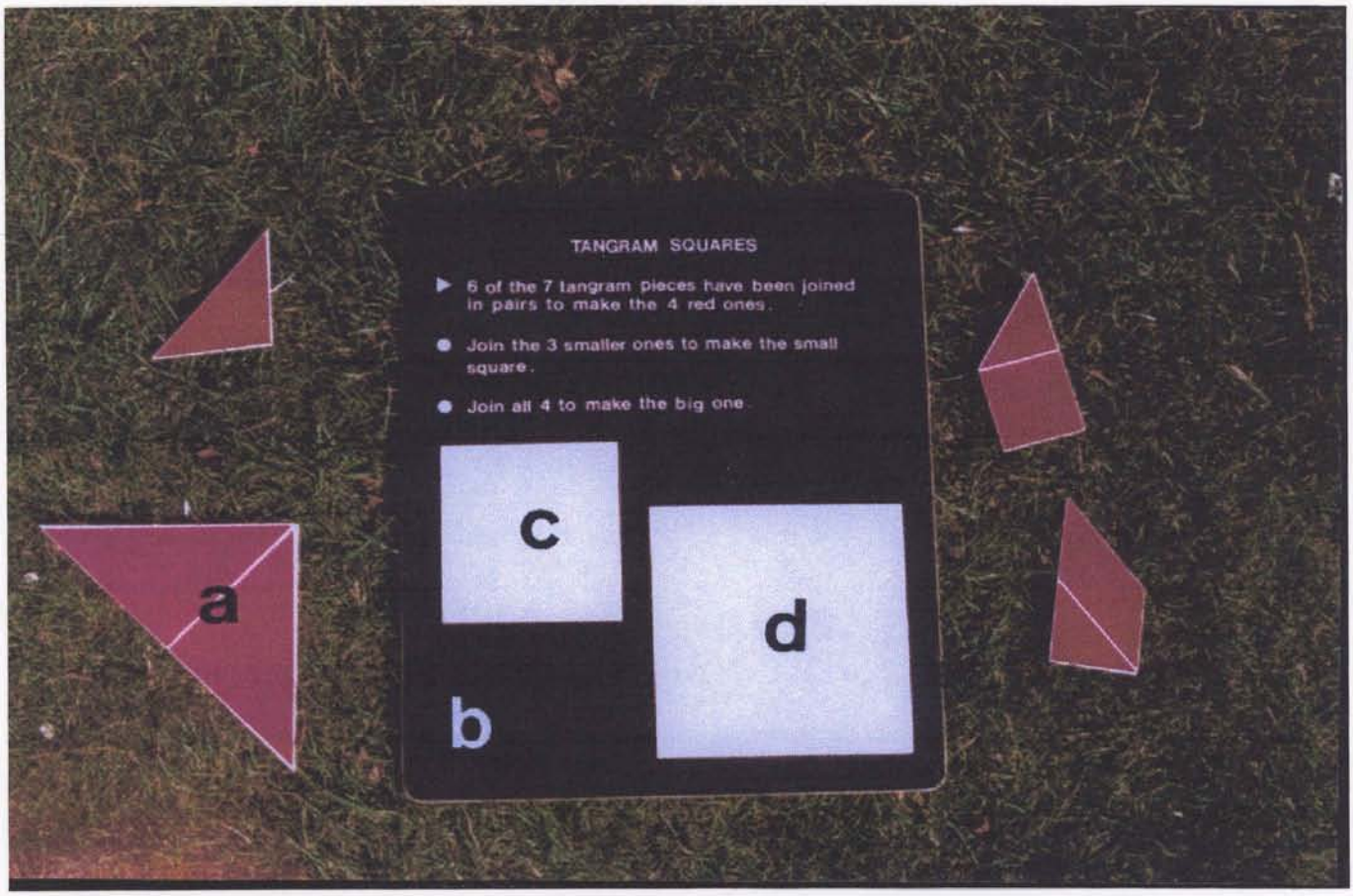
p The comparison die bears an important clue to solving the puzzle. Even so, it is important to sort the pieces carefully. 4 contain the 8 die corners, 4 only edges, 1 neither. Only 2 of the pieces are congruent and therefore interchangeable.

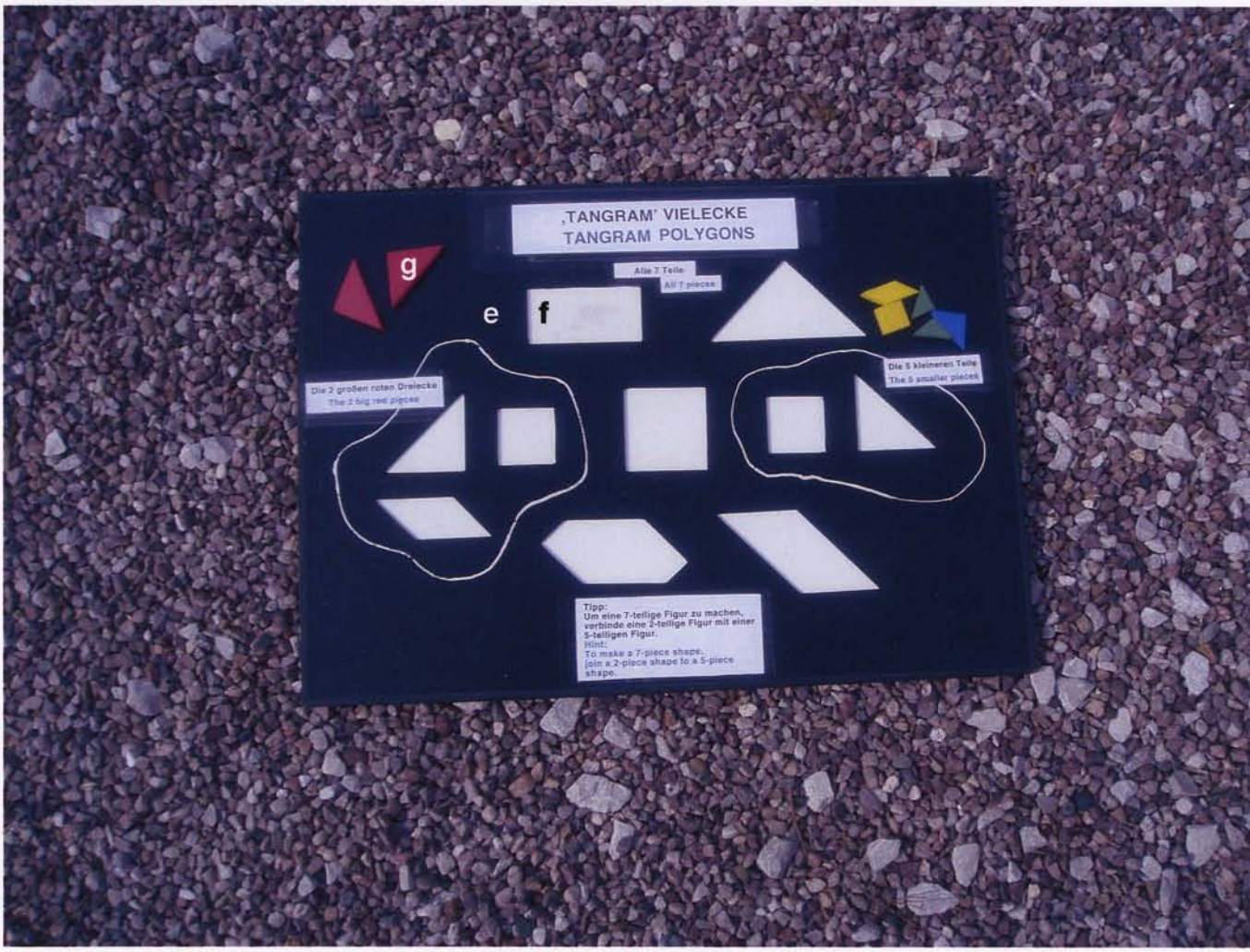
SECTION		AGE RANGE					
DISSECTIONS		Appropriate point of entry - not necessarily to the task set by the caption - and levels on which extension activities generated (some to be pursued off-site)					
STATION							
NUMBER	**NAME**	**4**	**7**	**10**	**13**	**16**	**19+**
5.1.1	TANGRAM POLYGONS		*	*	*	*	
5.1.2	THE RIDDLE OF THE SPHINX			*	*	*	
5.1.3	PENTOMINOES: THE 3 X 20 RECTANGLE		*	*	*	*	
5.1.4	KURSCHAK'S COUNT				*	*	
5.2.1.1	THE SOMA CUBE		*	*	*	*	
5.2.1.2	THE SOMA CUBE: MIRROR IMAGES				*	*	
5.2.2	$3^3 + 4^3 + 5^3 = 6^3$			*	*	*	
5.2.3	A NINE-PIECE DIE			*	*	*	

	INSTRUCTION NEEDED				SUPERVISION NEEDED			SERVICING NEEDED		
→	Needs no explan-ation or caption	Caption enough for most people	Needs aural prompt	Visitors must be talked through stages	None	Benefits from a helper's input	Session must be directed	None	A little	Much
5.1.1	*					*		*		
5.1.2	*				*			*		
5.1.3	*				*			*		
5.1.4			*			*		*		
5.2.1.1	*				*			*		
5.2.1.2		*			*			*		
5.2.2		*				*		*		
5.2.3		*				*		*		


‚TANGRAM' VIELECKE
TANGRAM POLYGONS
Alle 7 Teile
All 7 pieces
Die 2 großen roten Dreiecke
The 2 big red pieces
Die 5 kleineren Teile
The 5 smaller pieces
Tipp:
Um eine 7-teilige Figur zu machen, verbinde eine 2-teilige Figur mit einer 5-teiligen Figur.
Hint:
To make a 7-piece shape, join a 2-piece shape to a 5-piece shape.


TANGRAM SQUARES
6 of the 7 tangram pieces have been joined in pairs to make the 4 red ones.
Join the 3 smaller ones to make the small square.
Join all 4 to make the big one.
a
e
f
Now use the wooden pieces.
Try the two squares, s,S first.
Can you make these into chevrons:
T
P
TANGRAM POLYGONS
difficult!
H
Use the moveable s to help you with R,P,T.
What have s,S,P,R,H in common? (P gives the clue.)
with S pieces only
R
g


TANGRAM SQUARES
6 of the 7 tangram pieces have been joined in pairs to make the 4 red ones.
Join the 3 smaller ones to make the small square.
Join all 4 to make the big one.
a
b
c
d

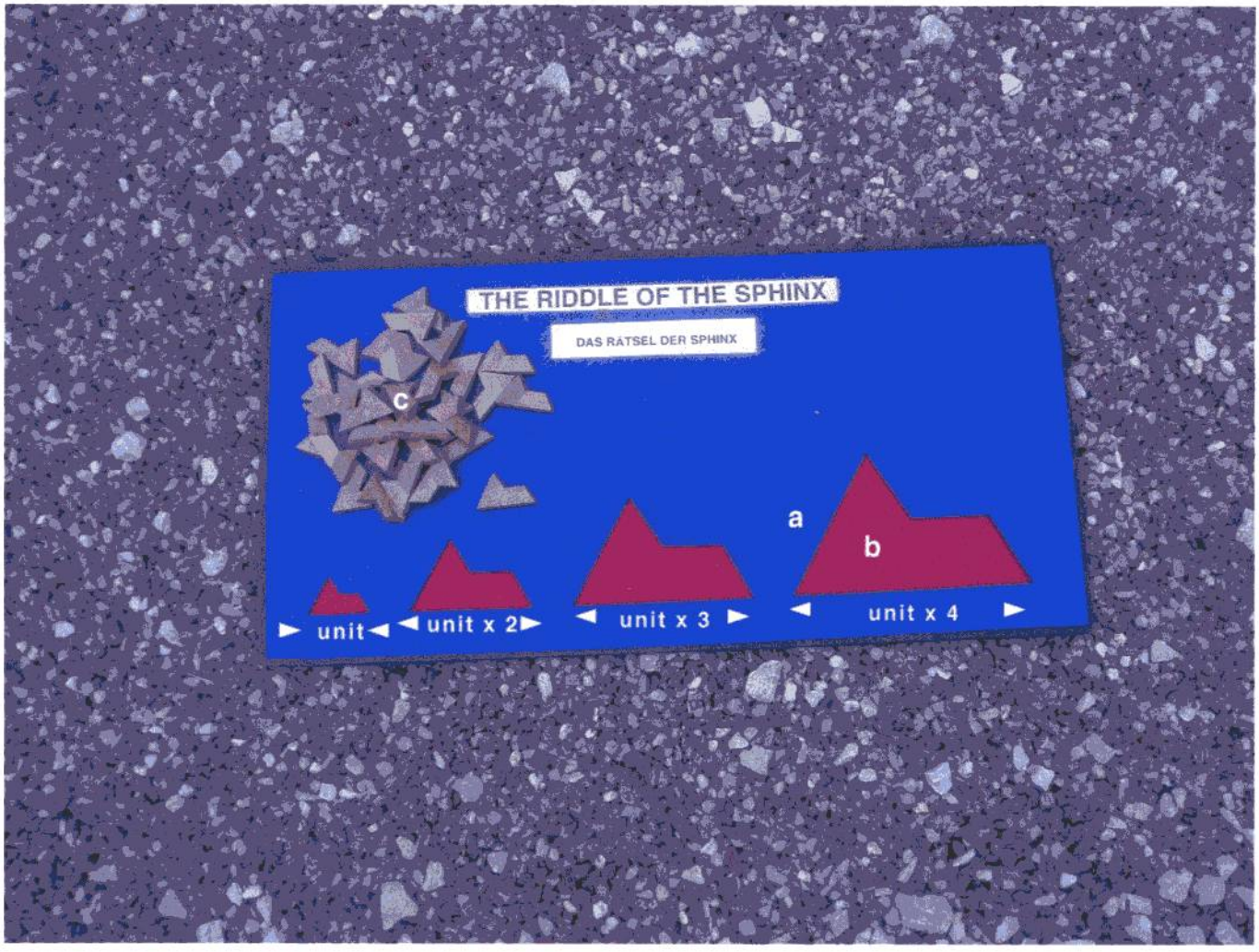

‚TANGRAM' VIELECKE
TANGRAM POLYGONS
g
e
f
Alle 7 Teile
All 7 pieces
Die 2 großen roten Dreiecke
The 2 big red pieces
Die 5 kleineren Teile
The 5 smaller pieces
Tipp:
Um eine 7-teilige Figur zu machen, verbinde eine 2-teilige Figur mit einer 5-teiligen Figur.
Hint:
To make a 7-piece shape, join a 2-piece shape to a 5-piece shape.

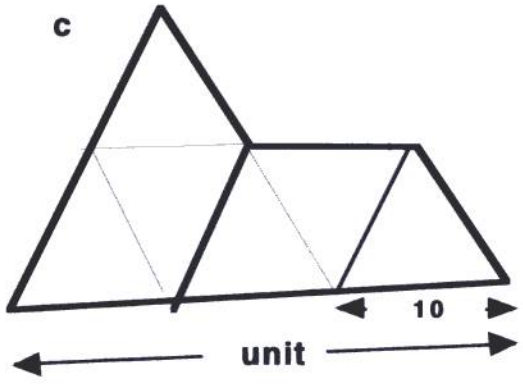
PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	card tangram supplied with:	*Tangrams* by Jon Millington	Tarquin Publications (address above)
	stuck on 1 mm (white) Plasticard and cut into segments as shown		local
b	caption board as described, with		
c, d	white Plasticard squares of the respective sizes stuck on		local
e	3 mm matt black PVC sheet, 500 mm x 700 mm, with cutouts as shown, taped to	Amarifoam	Amari Plastics (address above)
f	similar white sheet	Amarifoam	Amari Plastics (address above)
g	tangram pieces in Plastazote (expanded polyethylene)	DIME 7-piece Tangram	Tarquin Publications (address above)

	NUMBER	TITLE
GROUP	5	DISSECTIONS
STATION	5.1.2	THE RIDDLE OF THE SPHINX
TOPIC		Dissections based on aggregates of equilateral triangles

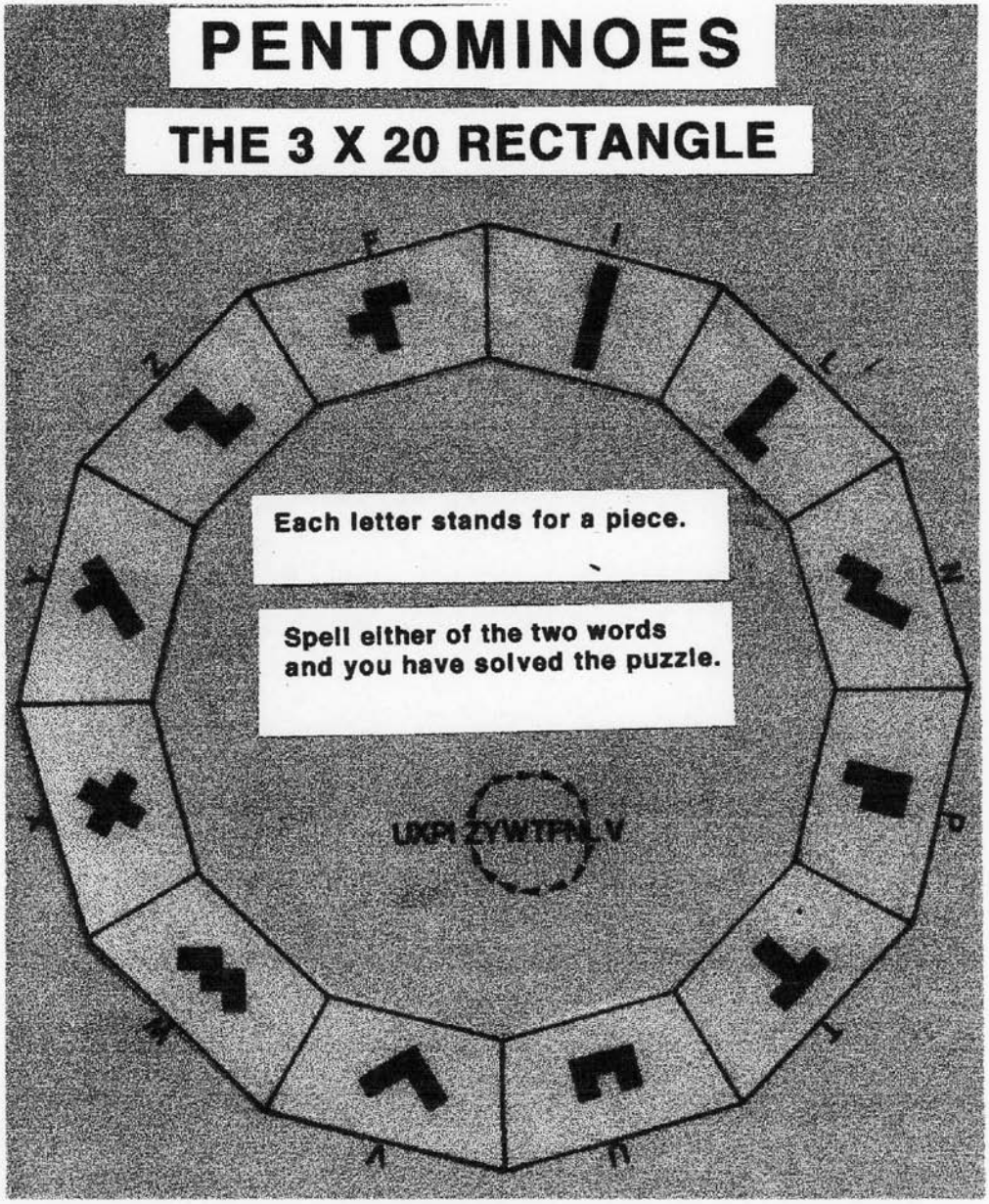
SPHINX SOLUTIONS

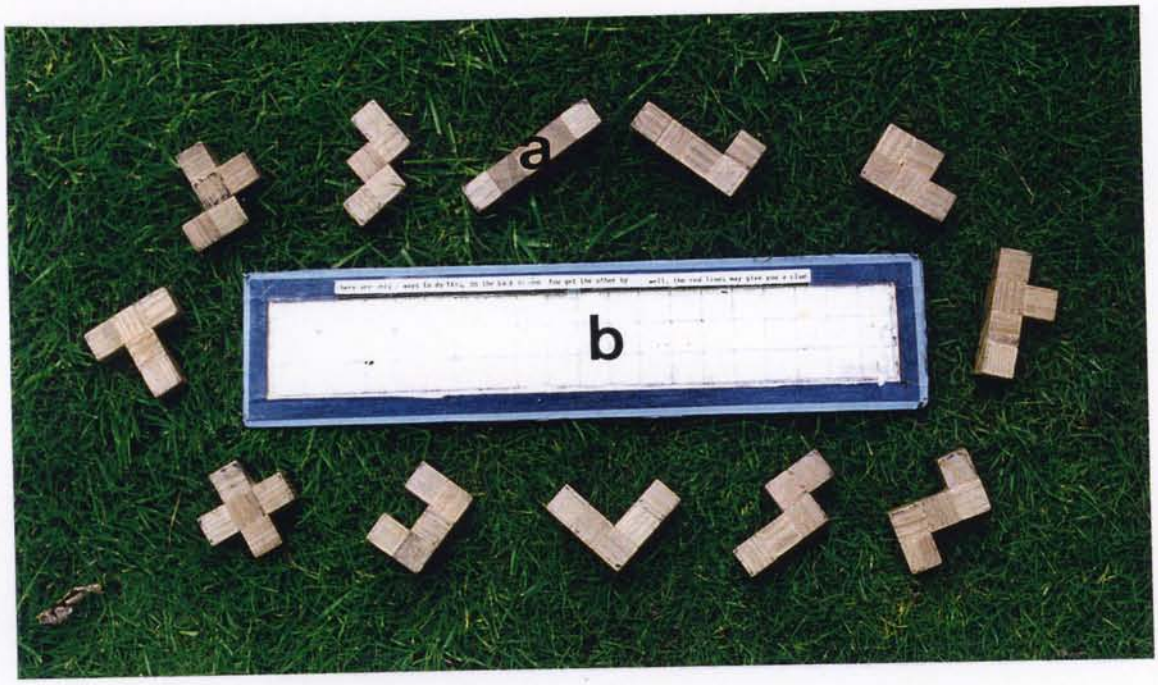
- **Turn over to see solutions other students have found.**
- **Are there any more?**

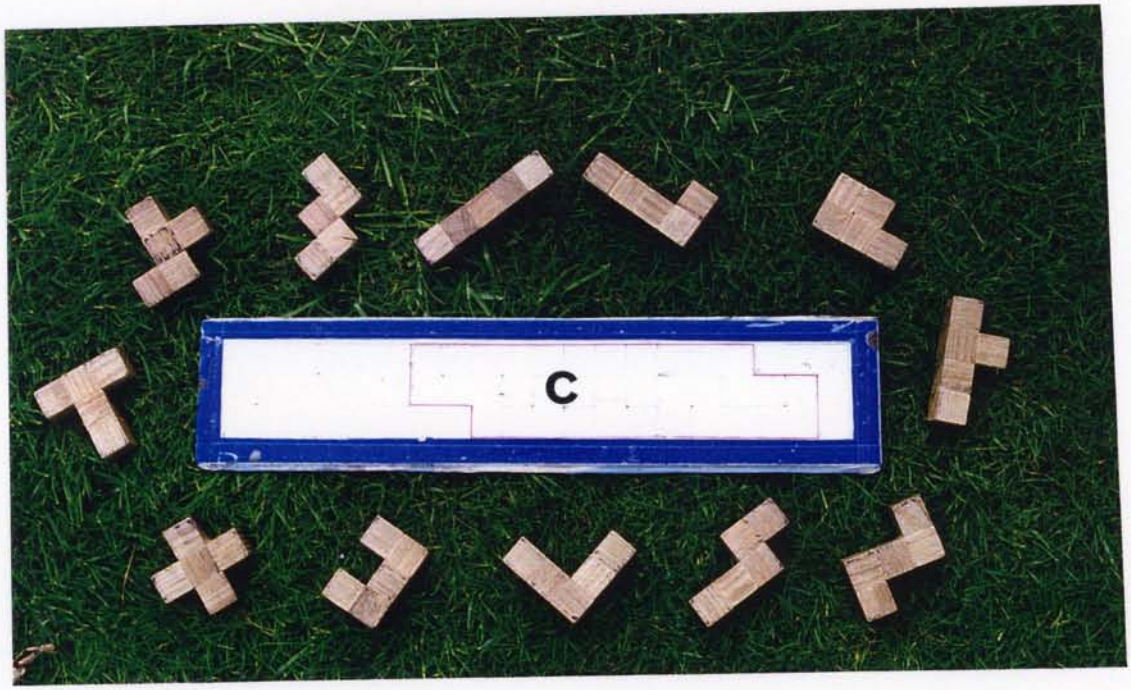

THE RIDDLE OF THE SPHINX
DAS RÄTSEL DER SPHINX
c
a
b
unit
unit x 2
unit x 3
unit x 4


c
10
unit

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	3 mm sheet of blue PVC, 410 mm x 820 mm, with cutouts as shown, taped to	Amarifoam	Amari Plastics (address above)
b	a similar sheet in red	Amarifoam	Amari Plastics (address above)
c	10 mm wood 'sphinxes' as shown*, $1^2 + 2^2 + 3^2 + 4^2 = 30$ needed * The figure shows how the sphinx can be made by cutting a regular hexagonal prism into 2 trapezia and joining them by non-corresponding faces.		local


PENTOMINOES
THE 3 X 20 RECTANGLE
Each letter stands for a piece.
Spell either of the two words
and you have solved the puzzle.
UXPIZYWTRLV


a
b


c

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	pentominoes, from 20 mm wood cubes	Pentacubes	Tarquin Publications (address above)
b, c	**c** = **b** verso		
	2 sheets of Glodex, 420 mm x 80 mm, sandwich a sheet on both sides of which is a 3 x 20 grid of 20 mm squares. The '**c**' side is marked as shown. A half-turn rotation of this section of the puzzle yields the second solution.		local
	On each face is a raised, 10 mm border in 3 mm PVC.	Amarifoam, 3 mm	Amari Plastics (address above)

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	pentominoes, from 20 mm wood cubes	Pentacubes	Tarquin Publications (address above)
b, c	**c** = **b** verso		
	2 sheets of Glodex, 420 mm x 80 mm, sandwich a sheet on both sides of which is a 3 x 20 grid of 20 mm squares. The '**c**' side is marked as shown. A half-turn rotation of this section of the puzzle yields the second solution. On each face is a raised, 10 mm border in 3 mm PVC.	Amarifoam, 3 mm	local Amari Plastics (address above)

	NUMBER	TITLE
GROUP	5	DISSECTIONS
STATION	5.1.4	KURSCHAK'S COUNT
TOPIC	Congruent shapes as units of area in an algebraic argument	

KURSCHAK'S COUNT

▶ **Here you see a black regular 12-sided polygon ('dodecagon'). Its corners ('vertices') lie on a circle of radius 1 unit. This fits in the red square. The square therefore has area 2 x 2 units = 4 units. The square contains: 4 triangles, 8 half-rhombuses and the dodecagon.**

● **Fill the dodecagon with triangles and rhombuses. Count them.**

■ **How many units in area is the dodecagon?**

A

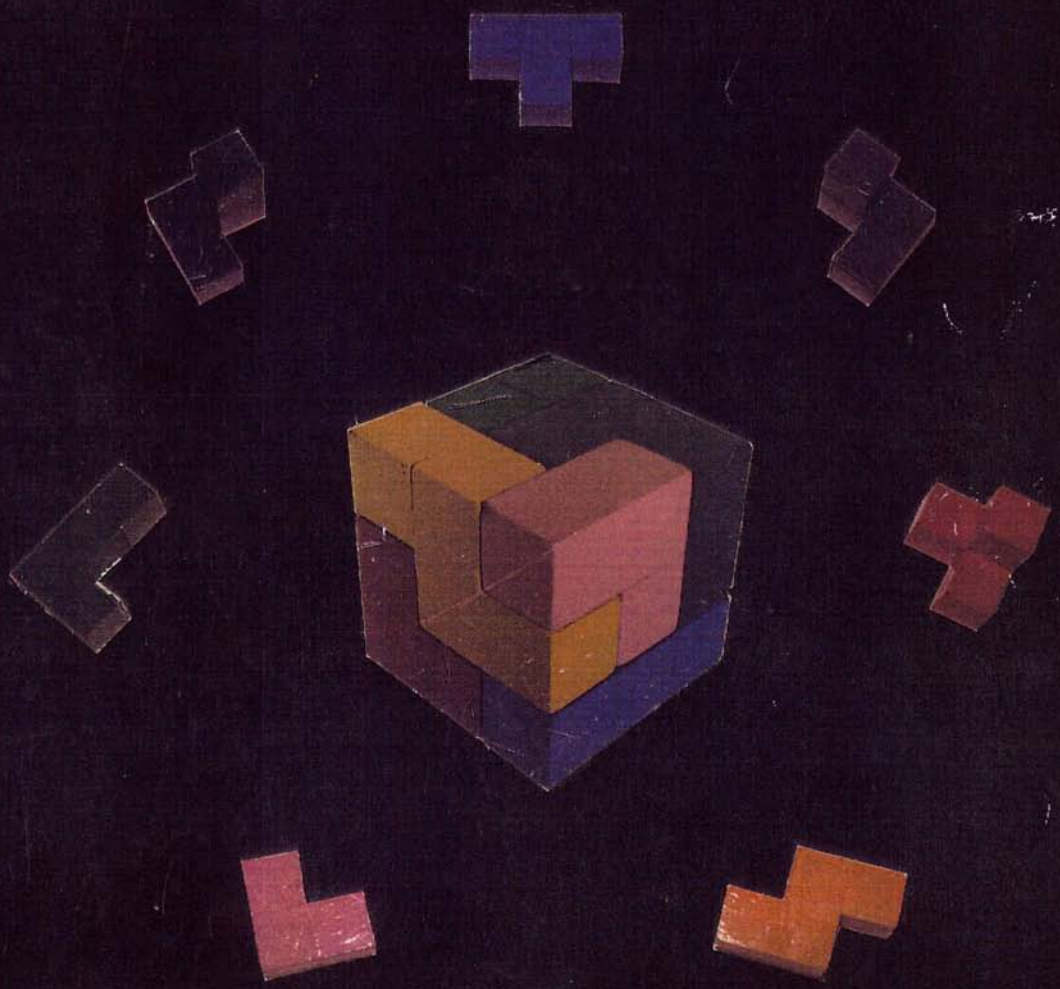

a
KURSCHAK'S COUNT
Here you see a black regular 12-sided polygon ('dodecagon'). Its corners ('vertices') lie on a circle of radius 1 unit. This fits in the red square. The square therefore has area 2 x 2 units = 4 units. The square contains: 4 triangles, 8 half-rhombuses and the dodecagon.
Fill the dodecagon with triangles and rhombuses. Count them.
How many units in area is the dodecagon?
b
c
d

B


KURSCHAK'S COUNT
Here you see a black regular 12-sided polygon ('dodecagon'). Its corners ('vertices') lie on a circle of radius 1 unit. This fits in the red square. The square therefore has area 2 x 2 units = 4 units. The square contains: 4 triangles, 8 half-rhombuses and the dodecagon.
Fill the dodecagon with triangles and rhombuses. Count them.
How many units in area is the dodecagon?

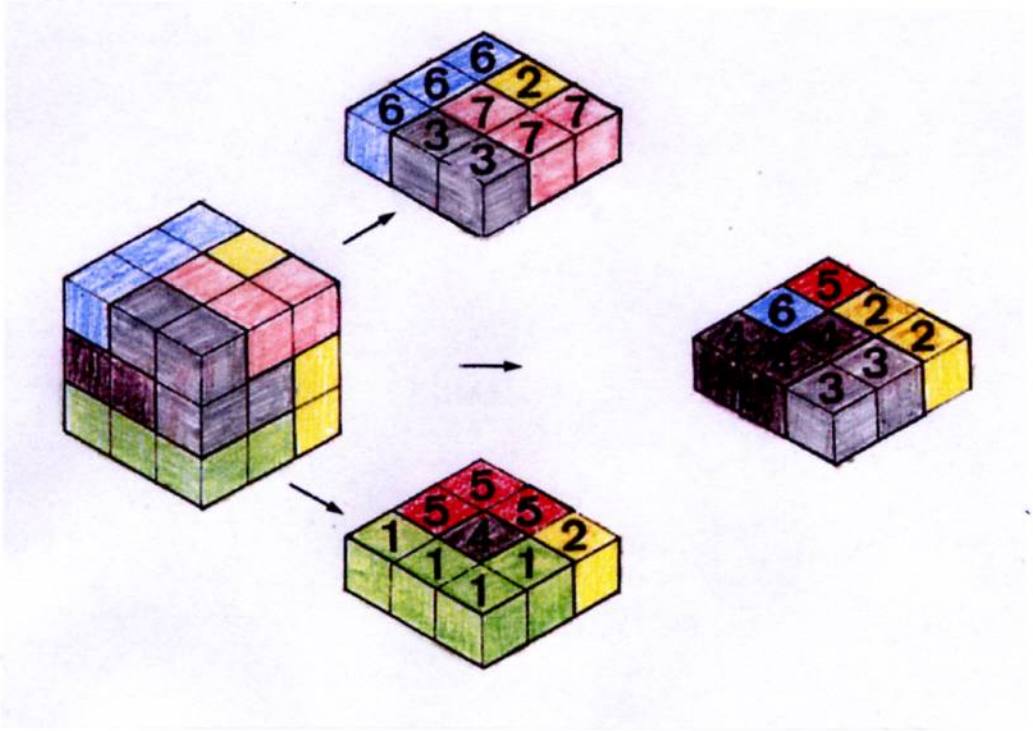
PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
A	the puzzle as presented		
B	the puzzle completed		
a	actual caption board		
b, c	prisms of ff. shapes:	Pattern Blocks	NES Arnold Ltd (address above)
b	equilateral triangles, 12		
c	30° rhombuses, 12		
d	The regions between the dodecagon and the square are ready-filled. The puzzlist is therefore required only to complete the dodecagon itself on the base provided by a piece of 3 mm black PVC. Note that the outer 150° isosceles triangles are 30° rhombuses bisected by their longer diagonals.	Amarifoam	Amari Plastics (address above)

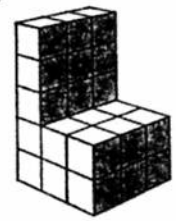
THE SOMA CUBE



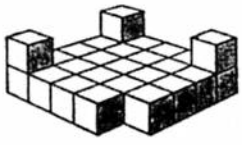
Turn over for help.

	NUMBER	TITLE
GROUP		
STATION		(Above continued)
TOPIC		

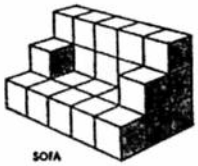




CHAIR

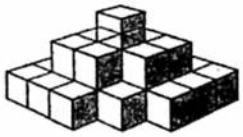


CASTLE

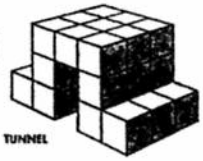


SOFA

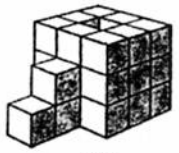
BATHTUB



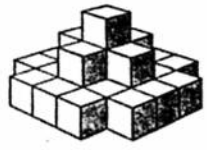
STEAMER



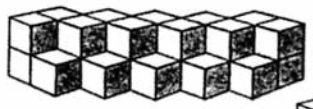
TUNNEL



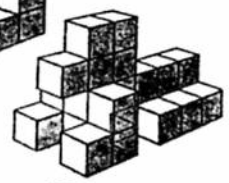
WELL



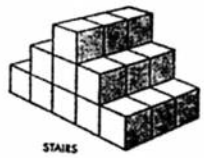
PYRAMID



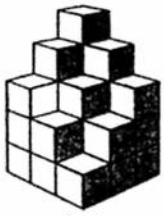
WALL



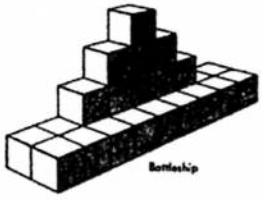
DOG



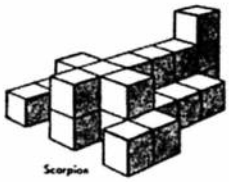
STAIRS



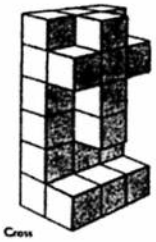
Crystal



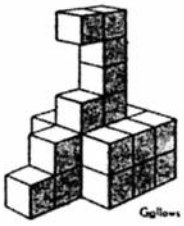
Battleship



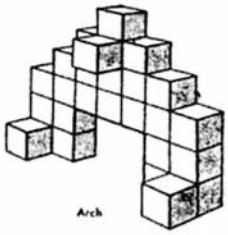
Scorpion



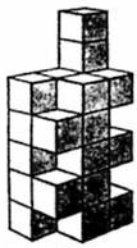
Cross



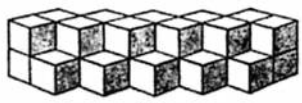
Gallows



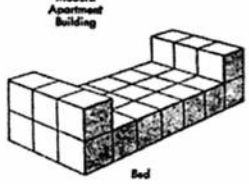
Arch



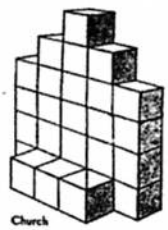
Modern Apartment Building



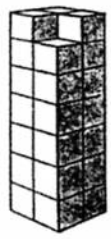
Wall



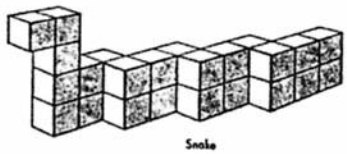
Bed



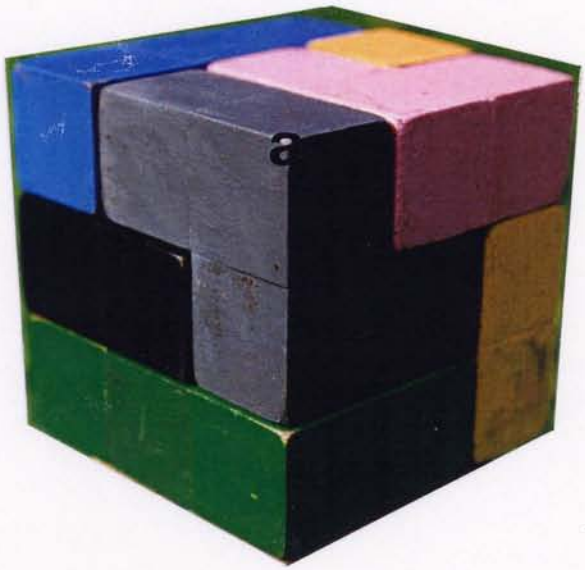
Church

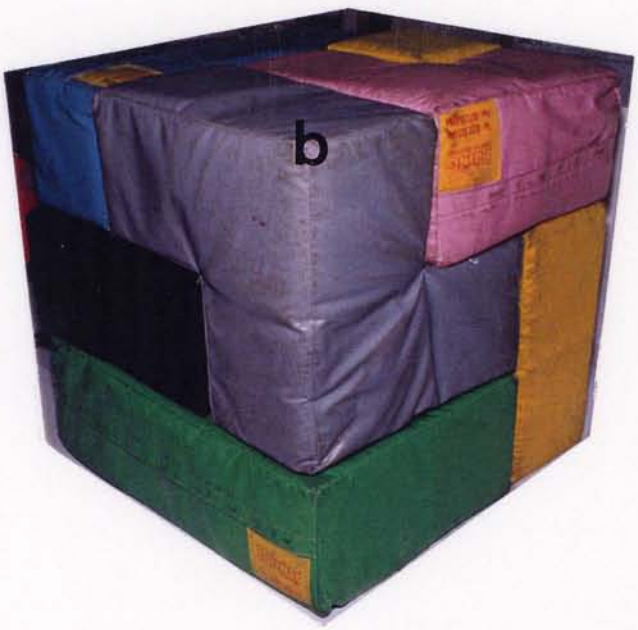


Tower



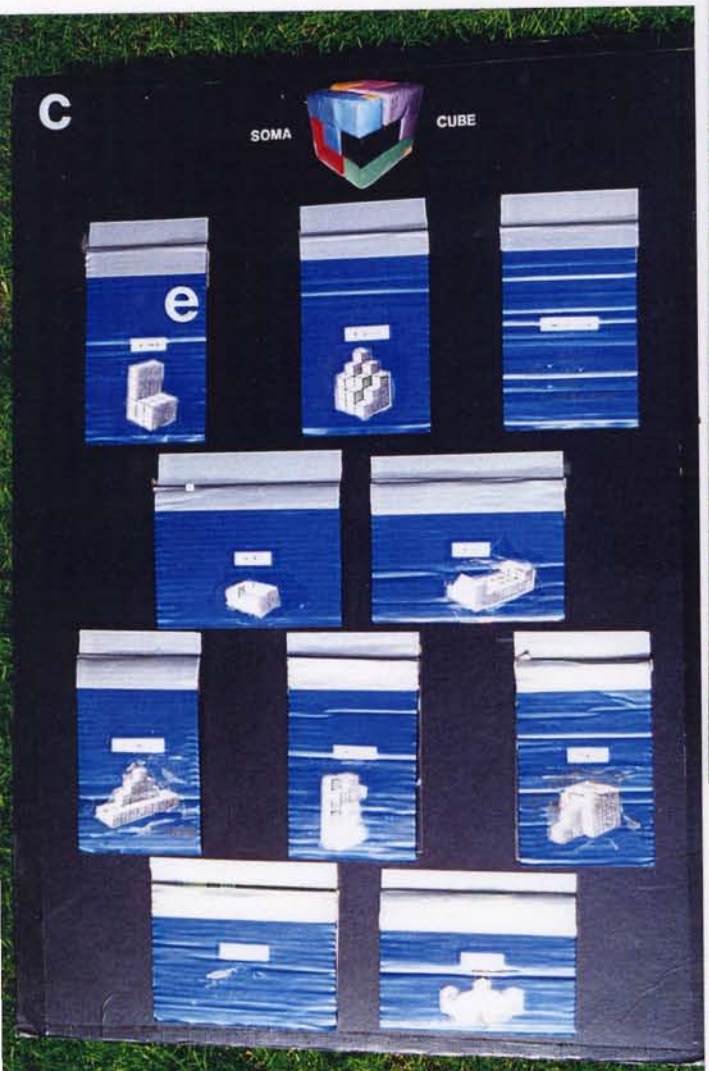
Snake

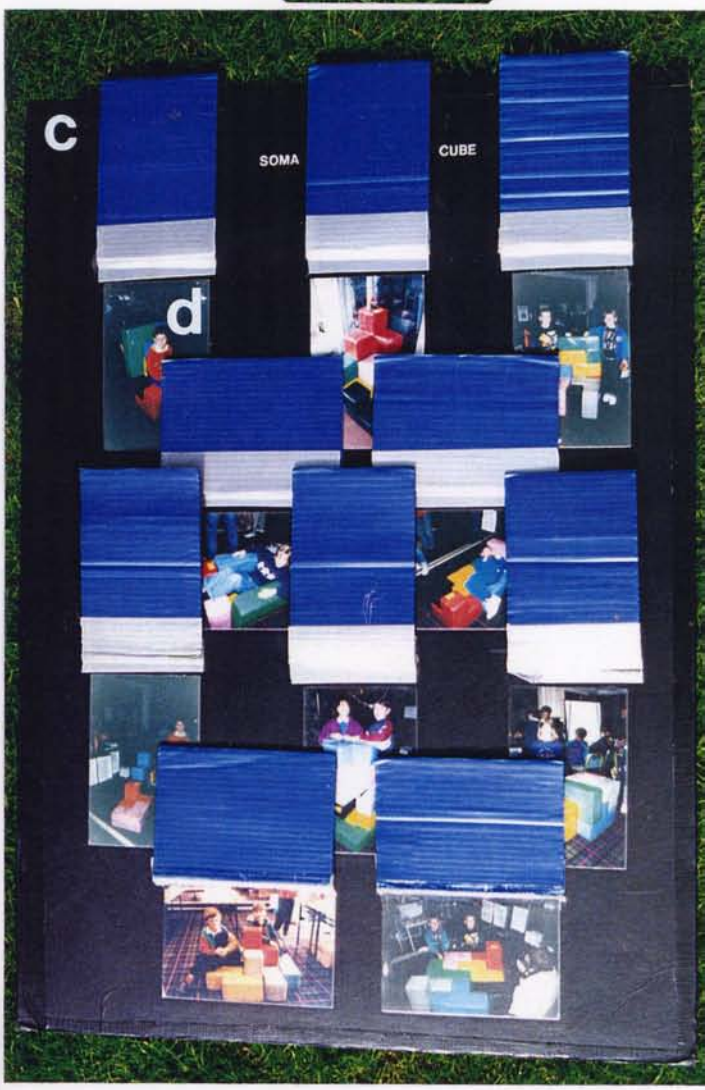

a


b


a
b


a


c
SOMA
CUBE
e


c
SOMA
CUBE
d

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	a wood Soma cube from 20 mm cubes, supplied plain, each of the 7 polycubes painted a different colour, the colours chosen to match those in which Multilink cubes are available, so that children can copy the pieces using that kit	Soma Cube Multilink	Tarquin Publications (address above) NES Arnold Ltd (address above)
b	**a** scaled x 10 but in vinyl-covered foam	Soma Cube	Boing Creative Solutions The Old Malthouse 6 Pennywell Road Bristol BS5 0TJ T +44 1272 555544 F +44 1272 550786
c	mounting board, 800 mm x 600 mm, laminated/covered with Transpaseal; should be laid flat or at a shallow angle so that the flaps **e** stay open when folded back		local
d	photograph of an assembled shape, e.g. a chair		
e	Corriflute flap with taped hinge, bearing a drawing of the object but without a clue to how the component pieces might be assembled to make it: (ideally) one therefore attempts the puzzle using **e** only, then folds back the flap to obtain help from **d**; (in practice, visitors opt immediately for **d**, finding that a sufficient challenge)		local
d, **e**	For suitable objects* see, e.g.: *Note that some objects can be transformed into others by moving only 1 piece, e.g.: cube & 'crystal', cube & 'chair'.	*More Mathematical Puzzles and Diversions*, Martin Gardner	a Penguin distributor

	NUMBER	TITLE
GROUP	5	DISSECTIONS
STATION	5.2.1.2	THE SOMA CUBE: MIRROR IMAGE
TOPIC	The significance of chirality in the solution of 3-D dissection puzzles	

THE SOMA CUBE:
How to Build a Mirror Image

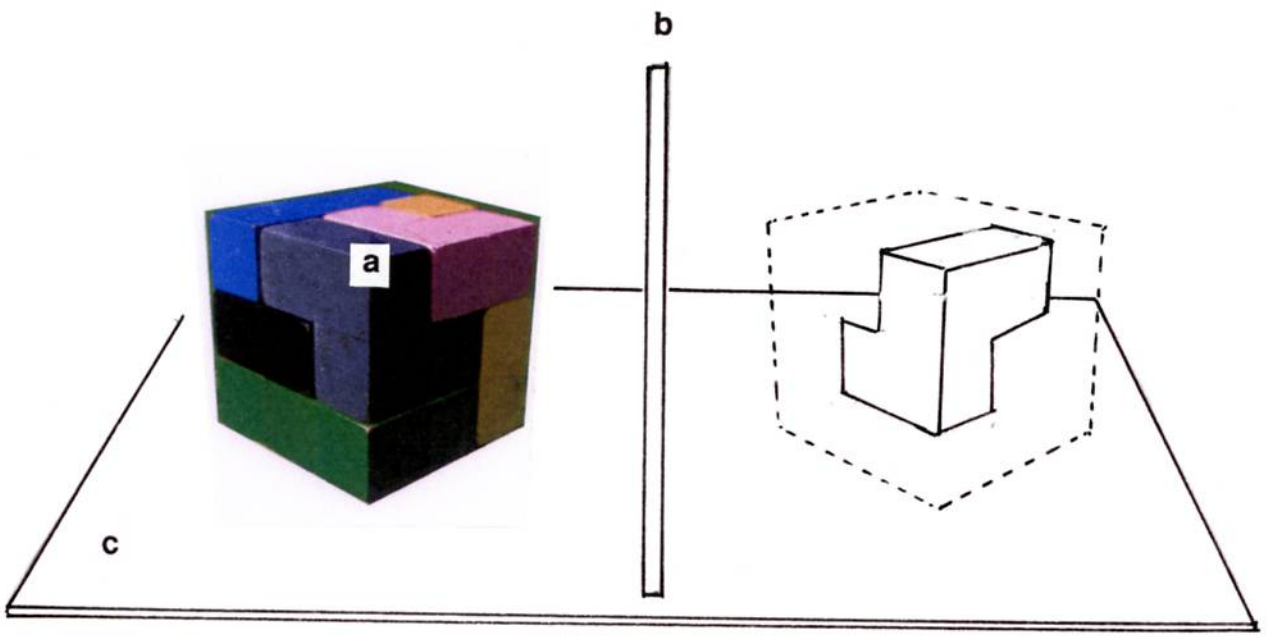
- **When you have succeeded in building a Soma Cube, take two sets of pieces to the mirror-window for comparing left- and right-handed objects.**

- **Take the grey piece and the black piece from one set. Use the mirror-window to compare them.**

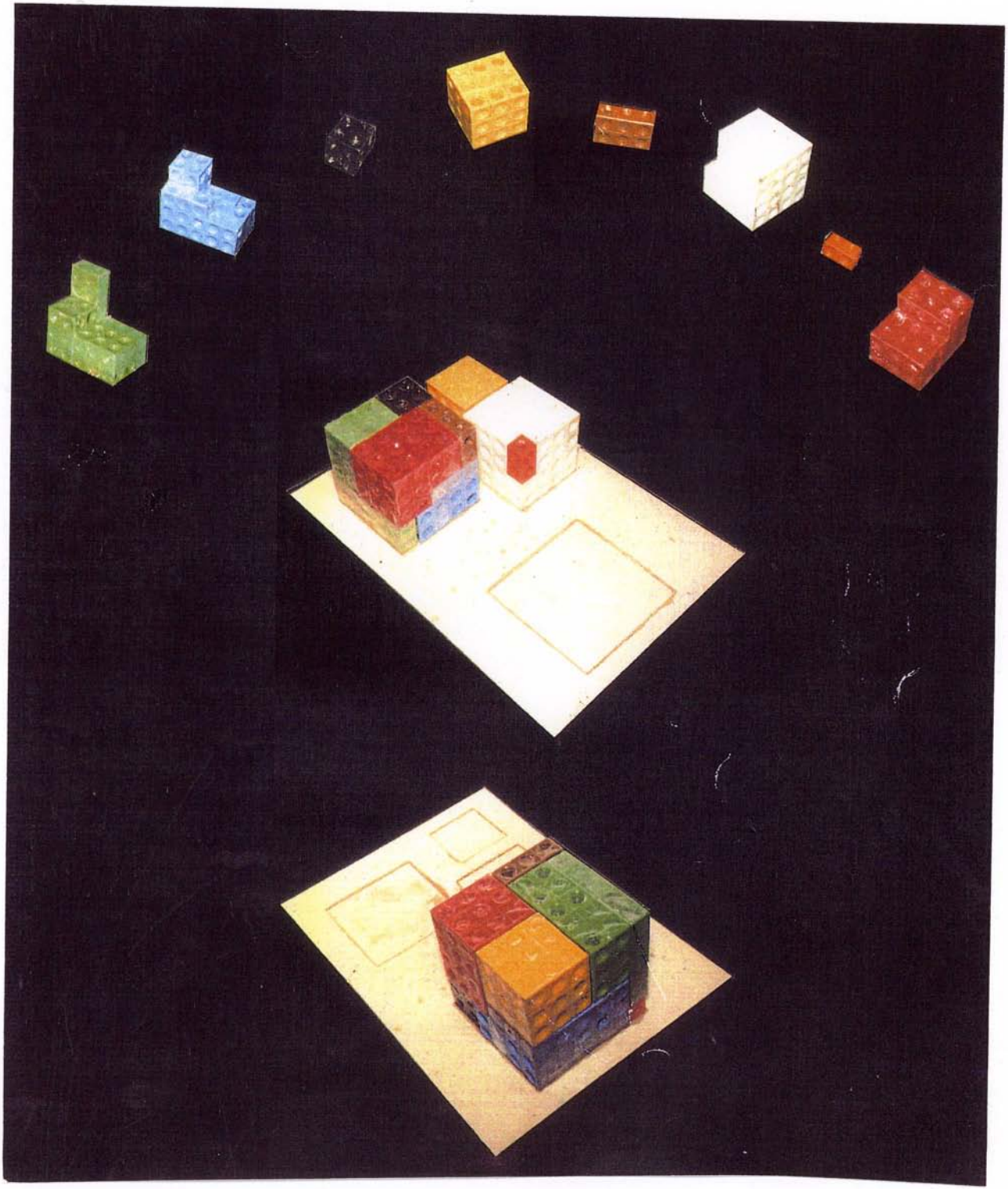
- **Use *one* set to build a cube. Sit it to the *left* of the mirror-window.**

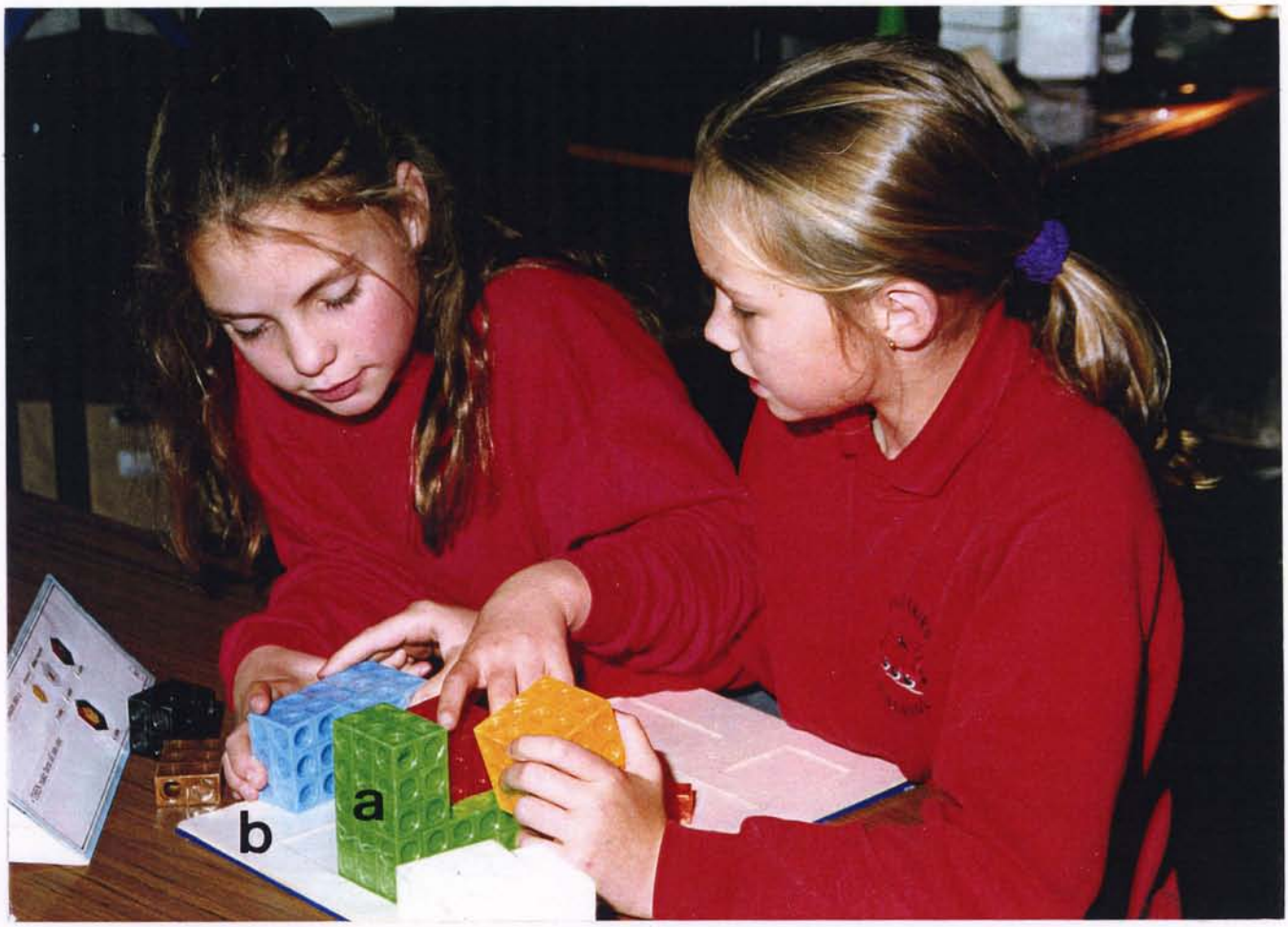
- **Use *the other* set to the *right* of the mirror-window to build the mirror image. *From what you have discovered in the earlier experiment, you must set the grey piece so that the reflection is black and the black piece so that the reflection is grey.***

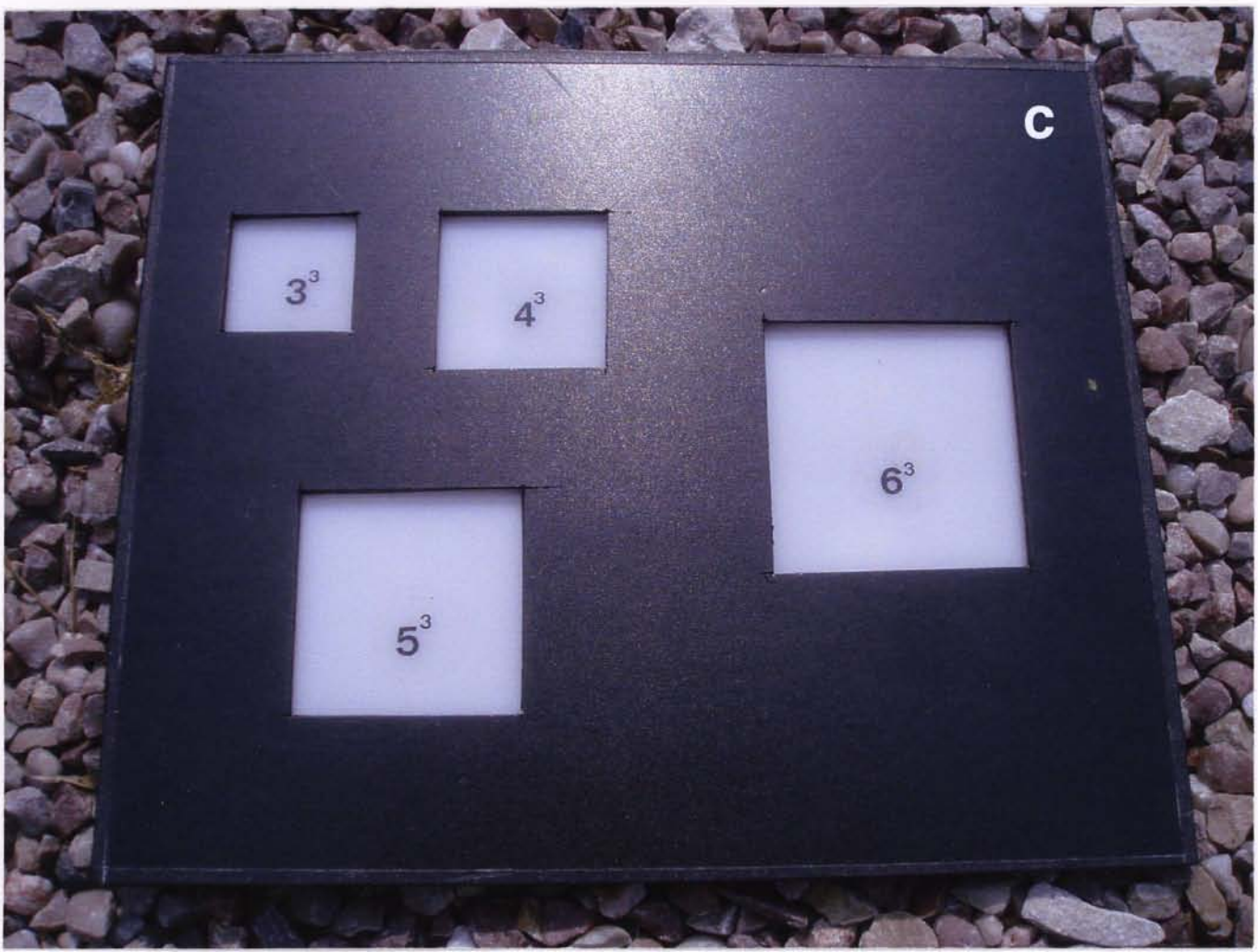
- **Apply the same principle to building a left-handed ...**
  bath
  bed
  chair
  well
  .
  .


b
a
c

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	as **5.2.1.1 a**, 2 required	(see same)	(see same)
b, c	The apparatus of **2.1.1** can be used or a second, smaller version built specially for this station. The essential components are:		
b	a matt black surface		local
c	a clear sheet, e.g. of 3mm Glodex		local
	In the figure, the right-hand piece corresponding to the *grey* on the left has been picked out on the completed mirror-cube. The drawing shows that this is in fact the *black* piece.		




a
b


c
3^3
4^3
6^3
5^3

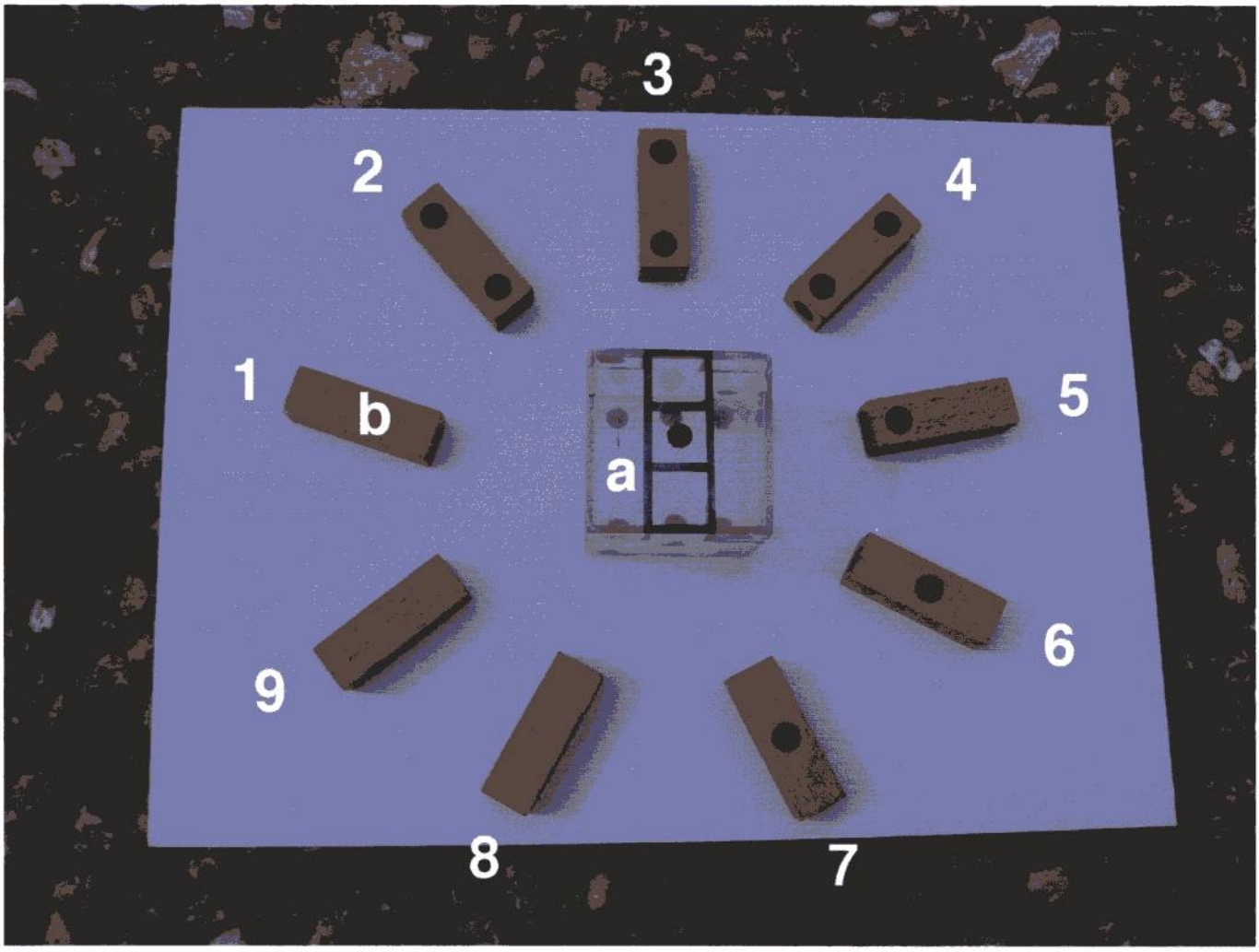
PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	pieces shown on caption here made from Multilink and covered with Transpaseal	Multilink	NES Arnold Ltd (address above)
b	baseboard for Multilink version, 360 mm x 270 mm:		
	2 layers of Plasticard stuck together, the upper with cutouts to receive the assembled cubes, i.e. squares respectively of edge: 62 mm, 82 mm, 102 mm, 122 mm, to allow 2 mm clearance		local
	The puzzle is also available in wood:	Three-Four-Five-Six	Tarquin Publications (address above)
	as with **5.2.1**, the pieces can be painted to match the Multilink colours.		
c	baseboard for wooden version to illustrate the visual effect of using contrasting colours for upper and lower layers, (here Amarifoam)		

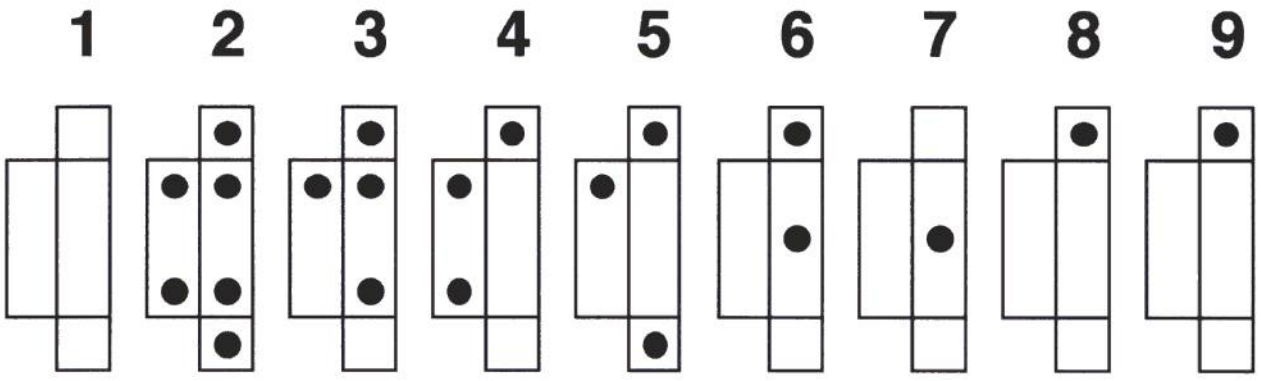
	NUMBER	TITLE
GROUP	5	DISSECTIONS
STATION	5.2.3	A 9-PIECE DIE
TOPIC		Sorting and matching

A 9-PIECE DIE

- **Use the 9 wooden pieces to build the perspex die.**

- **The black lines on the '1' face will help you to succeed.**


3
2
4
1
b
a
5
9
6
8
7


1
2
3
4
5
6
7
8
9

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	perspex cube, 45 mm edge, marked in two ways: 1) as standard die, viz. spots on opposite faces total 7. Note that this still allows different spot arrangements. Therefore another die cannot necessarily be substituted. 2) to show that the central 3 puzzle pieces lie at right angles to the outer 6.		local
b	9 square prisms in wood, 15 mm square x 45 mm, marked as shown on the corresponding nets		local