Topic: **Games**

Station: **3-D Os & Xs**

On the left are the 4 kinds of site and the number of lines-of-3 running through them:

The centre		13
A corner		7
A face centre		5
An edge centre		4

Consider only the two-player game.

We distinguish *attack*, creating winning lines, and *defence,* preventing your opponent doing so.

<u>Attack</u>
Choose a site with the greatest number of lines going through it.

<u>Defence</u>
Choose a site which will create a winning line such that, in blocking it, your opponent does not himself set up one.

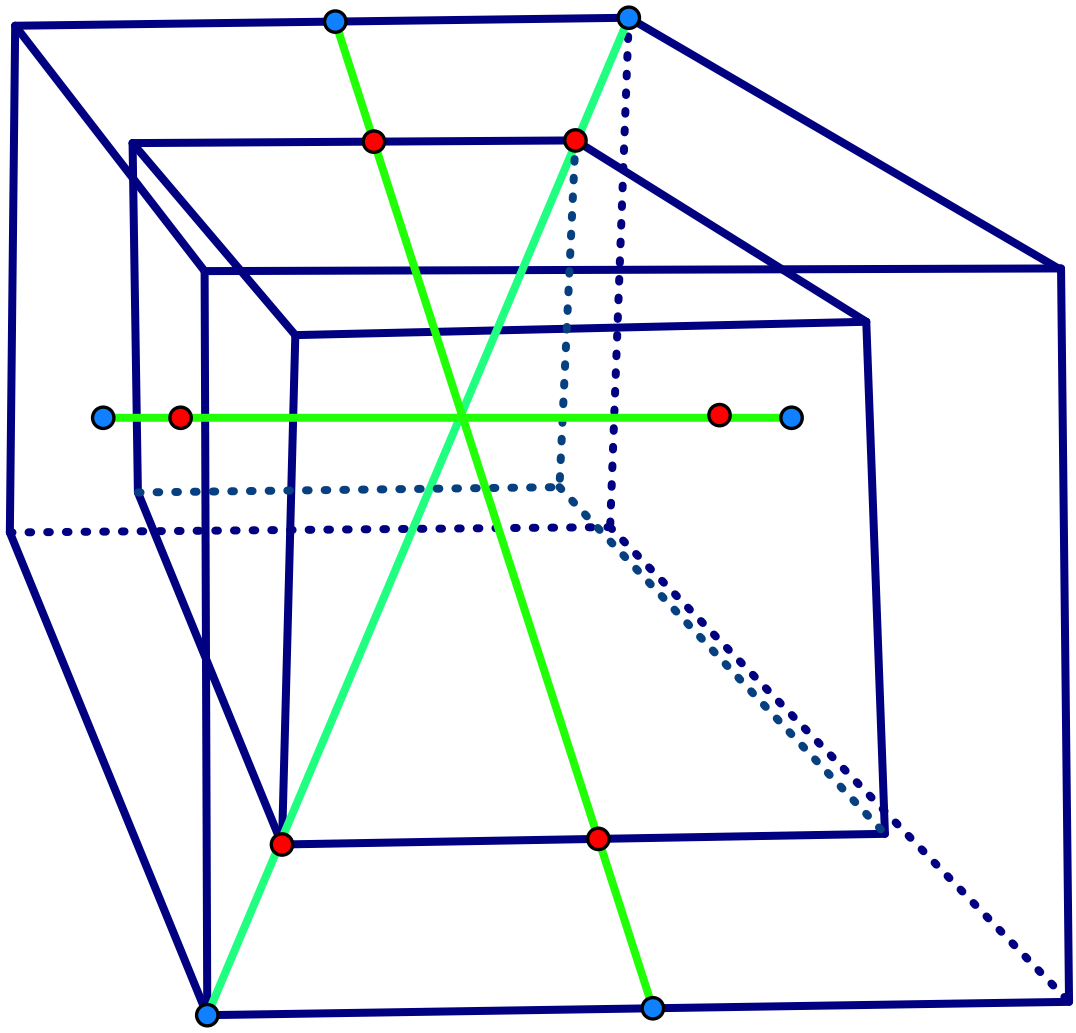
Your strategy is to set up *two* winning lines, since your opponent can only block one.

If you go first, you have a win on your 4th move by commanding the centre site.

There are 3 situations, depending on the kind of site your opponent chooses in response.

In these games we start at the point where you have taken the centre site and your opponent (Yellow) has made his first move. We stop at the point where you (Red) have created two winning lines.

As a measure of how good a position is, we can give each player a 'potential'. This is the number of lines remaining which he could in principle complete. Both players start with a potential of 49, the total number of lines running through all sites. One way to determine this number is to imagine the 3-cube encased in a 5-cube. Each line in the 3-cube lies on one starting from a point on the 5-cube and ending on a point diametrically opposite. The number of lines is therefore $\frac{5^3-3^3}{2} = 49$.

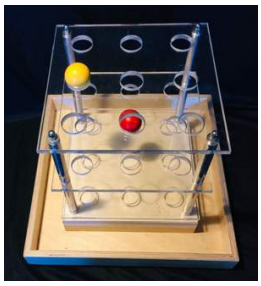
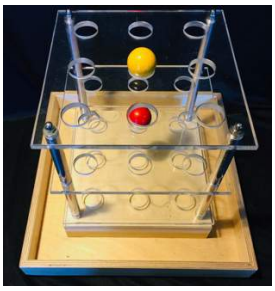
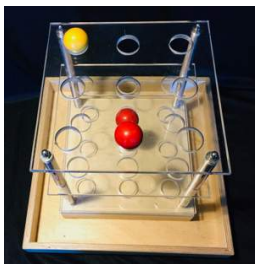
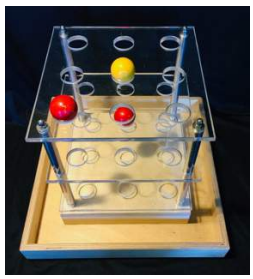
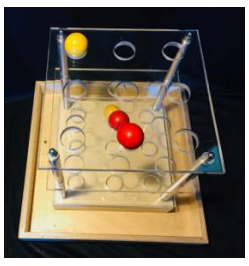
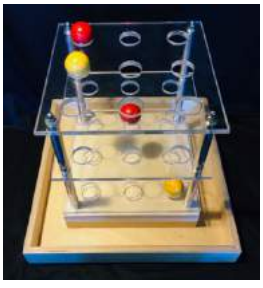
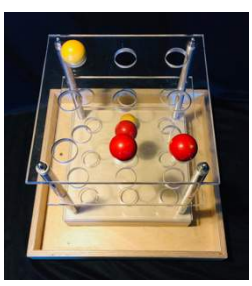
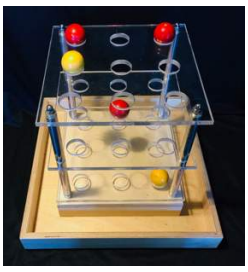
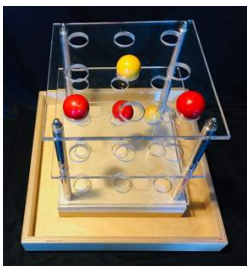


After the first move, you retain a potential of 49; your opponent's has dropped to 49 -13 = 36.

Following the 'thumbnail' summary of the three possible game types, we shall analyse them.

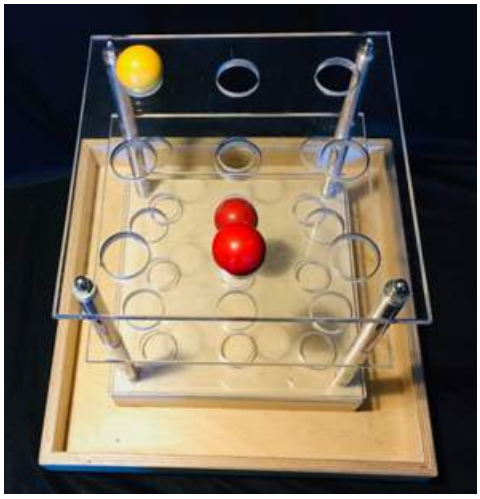
In each move after his first, Red is one move off a win through the centre. Yellow can only make lines using the 26 'surface' sites.

Although the potential is a number, it is only of qualitative significance: it does no more than indicate who is in the better position.

Game 1	Game 2	Game 3
		
		
		
		
		

Game 1

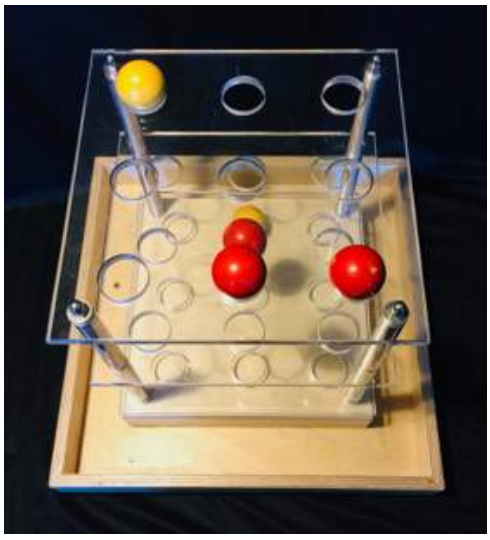
Red's new potential: 49 - 7 = 42.



Yellow's new potential: 36 - 4 = 32.

Yellow must block Red's line on the next move but it will not help him form a line of his own. Red will therefore be free to set up a new line without having to block a Yellow line.

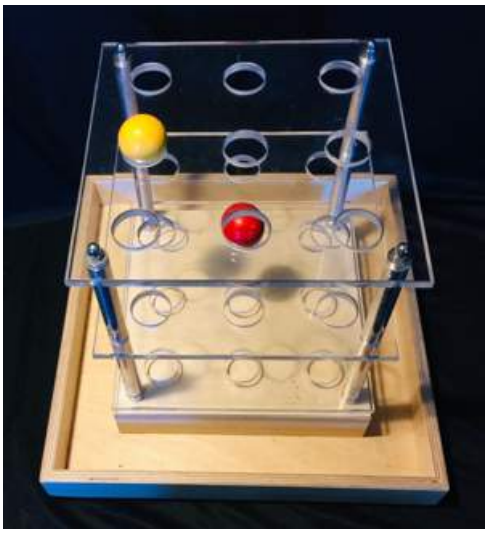
Red's new potential: 42 - 4 = 38.



Yellow's new potential: 32 - 7 = 25.

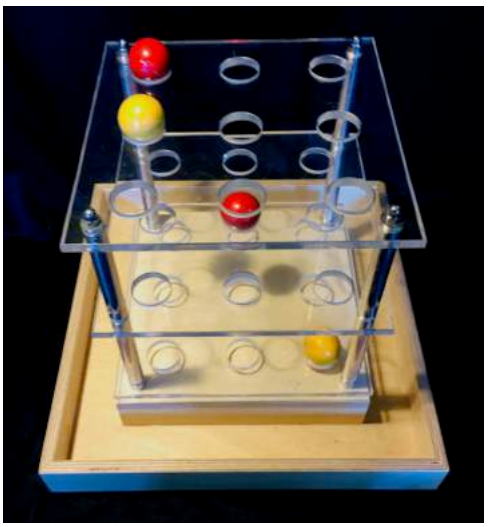
The new line Red has set up would help Yellow but it's too late: Red has set up two lines and yellow will only be able to block one.

Game 2

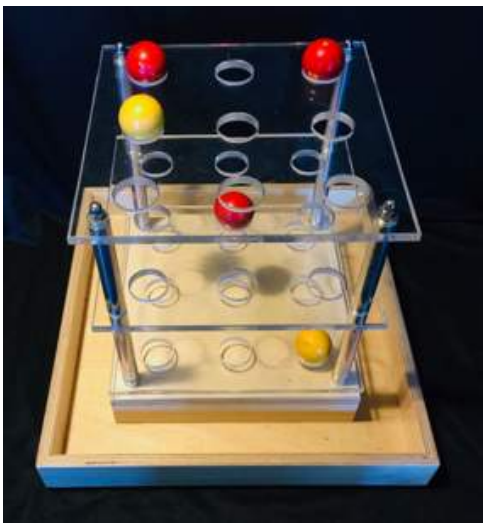


Red's new potential: 49 - 4 = 45.

Yellow's new potential: 36 - 7 = 29.



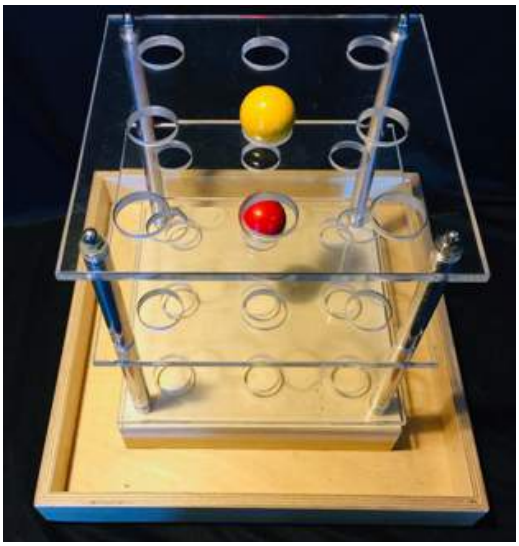
Red's new potential: 45 - 7 = 38.



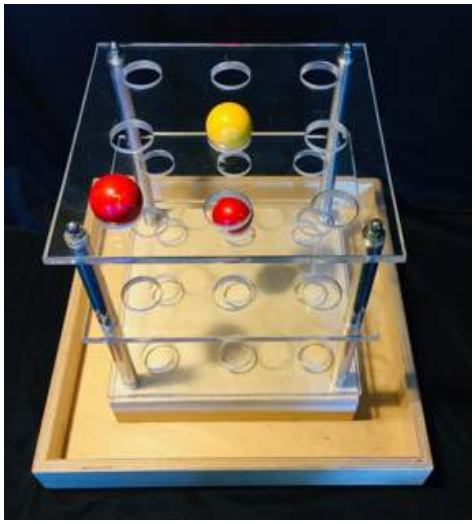
Yellow's new potential: 29 - 7 = 22.

At no point has Red allowed Yellow to construct a line.

Game 3

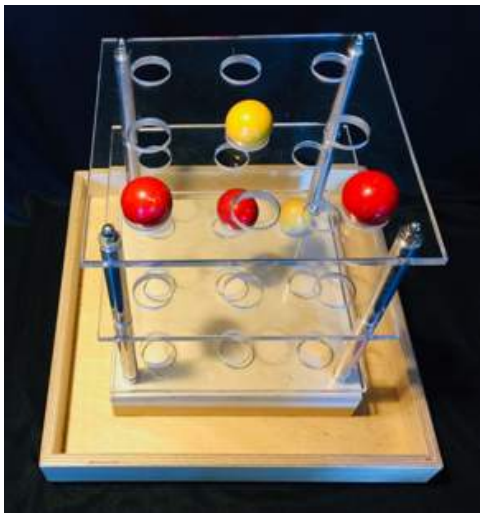


Red's new potential: 49 - 5 = 44.



Yellow's new potential: 36 - 7 = 29.

Red's new potential: 44 - 7 = 37.



Yellow's new potential: 29 - 7 = 22.

Again, Yellow has had no chance to begin a line.