Contents

The Magic Manual

Section 10

Heuristic

A guide for fabricators and users to stations from the Magic Mathworks Travelling Circus

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10. HEURISTIC

This word can be translated 'the art of discovery'. At these stations the investigators must fall back on first principles. They must discover for themselves the concepts involved. As throughout the Circus, the task is one of analysis, of dissecting the mathematical skeleton - the abstract structure - out of the physical corpse - the concrete embodiment. But, without a clue to the mathematics applicable, this is made one stage harder. Substations have therefore been introduced to offer help in this task:

10.1.1.2 serves **10.1.1.1**.
10.1.3.2 serves **10.1.3.1**.
10.1.6.1 serves **10.1.6.2**, which in turn serves **10.1.6.3**.
10.2.2 serves **10.2.1**.
10.4.1 and **10.4.3** serve **10.4.2**.
10.5.1 serves **10.5.2**, which in turn serves **10.5.3.1** and **10.5.3.2**.

10.1 This is a collection of games.

c **10.1.1 - 10.1.5** are all games of subtraction in the general sense that successive moves narrow the options. So would **10.1.6.2** be if the players placed single cubes alternately. In the version given here, however, it is a competition rather than a game, designed as preparation for **10.1.6.3**.

p In the investigative spirit of the topic, once the winning strategy is found, the players can change the game itself.

10.1.1

10.1.1.1 NIM

c The winning strategy uses the binary system in the way expounded in:

10.1.1.2 NIMROD

p If smaller boards are tried, as suggested by the hardboard mask, the advantage shifts as follows:

board	player favoured	winning move
III IIII IIIII	1st	2 matches from top row
II III IIII	1st	3 matches from bottom row
I II III	2nd	

Children are reluctant to reduce the board in this way. However, amongst others, these configurations occur as positions in the original game.

10.1.2 CHOMP

c The winning strategy is unknown in the general case. There is a proof that such exists for the first player. Sadly it is non-constructive.

p The special cases novices should study are a square board - 4 x 4 is possible here - and an n x 2 board - 6 x 2 is possible here: simple strategies exist for both.

10.1.3

10.1.3.1 THE PIE GAME

c The structure is revealed by an isomorphic game:

10.1.3.2 ALSO THE PIE GAME

In the first we have a model pie; in the second, a grid ruled in 3 directions. Corresponding to a number of slices in the former is a vector in the latter:

10.1.3.1	**10.1.3.2**
"Take n red slices."	"Move n units west."
"Take n blue slices."	"Move n units south."
"Take n red and n blue slices."	"Move n units west and n units south (i.e. southwest)."

With the given number of blue slices (8) and red slices (4), the first player can win by taking 1 blue slice.

p Exhibitors may choose to withhold **10.1.3.2** unless or until the time is ripe.

10.1.4 DOMINO BLOCK

c The second player can win by matching each move the first player makes with a point-symmetric one.

p Once the experimenters discover this strategy they can add interest by changing the number of players and the rules governing a move.

10.1.5 3-D Os & Xs

c On the 3 x 3 x 3 board, used here, the first player can win in 4 moves by seizing the central site. Note that 4 sites are distinguished by the number of lines they command:

centre (13), corner (7), centre-face (5), centre-edge (4).

p The game becomes interesting when the number of players is increased to 3 or 4 - which it can be quite simply by adding a new colour of marble.

10.1.6 TARTAN CHECKS

c 4 colours must meet in each vertex. Consequently, a cube of a given colour is bordered by 8 dissimilar ones. The rule generates 2 basic colour patterns:

abab...
cdcd...,
where cubes of a given colour define a square grid,

abcdabcd...
cdabcdab...,
where cubes of a given colour define a rhombic grid.

p The activity is structured as follows:

10.1.6.1 EDGES & CORNERS

The experimenters establish 'the facts of the case' by obeying the rules.

10.1.6.2 TARTAN BLOCK

A simple game forces them to consider the disabling effect of cubes in combination. (The optimal strategy is to build a complete double row (as in **10.1.6.1**) but then to displace the second row so that it forms the third - it is equally effective to adopt the same strategy with diagonals.)

10.1.6.3 THE CHECKS CHALLENGE

They apply what they have learnt.

A note on the title: the second word puns 'check' as a colour grid - of which a tartan is a special case, 'check' as a test that the stated rule has been followed and 'check' in the sense used in chess of 'putting a check on' your opponent's potential move(s).

10.2

10.2.1 SAFE QUEENS

c Beyond engendering an awareness of rows, columns and diagonals there would be little justification for this activity were it not for its inherent fascination.

Allowing rotations about the centre and reflections in symmetry axes, there are 2 solutions in the 5 x 5 case, 6 in the 7 x 7. Students should be urged to seek all these rather than press on to the 8 x 8 case. Unfortunately, the challenge of the full board is hard to resist.

p Up to 7 x 7 one can profitably adopt a strategy (modified as necessary) - displacing the pieces by series of knights' moves or similar. As it turns out, one can only deal with the 8 x 8 case by changing from a strategic approach to an exhaustive one, systematically working one's way across the board modifying an initial alignment of 8 queens in a row. Children rarely make the switch and resort to an ad hoc approach, which rarely leads to a solution: the problem is too complex combinatorially.

The bigger the board considered, the more carefully one must check for alignments.

10.2.2 SAFE QUEENS ON A 5 X 5 BOARD

p If translations are allowed, there are only 2 7 x 7 solutions and 1 5 x 5. This second station allows people to confirm that fact in the second case. But its real purpose is to encourage them to extend the approach to the first.

10.3

p **10.3.1** and **10.3.2** have been paired because in each case the answer to the question posed is 'No'.

The two stations serve to distinguish physical and logical possibility.

Without a proof - however rough-and-ready - one's answers to the questions will not convince.

10.3.1 GRANDPA'S ARMCHAIR

p It's difficult to get a feel for what's going on here without recording the results of trial moves. This provides a good opportunity for students to devise their own ikons or symbols.

One can prove the (negative) result by considering, for example, what happens to the sum of the Cartesian coordinates of a chair corner when it executes a 1/4-turn.

The ancillary apparatus is intended for such investigations.

10.3.2 DOMINO TILINGS

p The caption prepares the investigators for failure - however reluctant they might be to accept it.

A further exercise distinguishes *necessary* and *sufficient* conditions.

10.4 WEAVES

p These are the functions of the ancillary pieces:

10.4.1 THE SPECIMEN WEAVE

... presents the general case.

10.4.2 THE CHUNKY WEAVE HOLE

... frames the question which it is the object of investigation **10.4** to answer: 'When is a check a possible weave?'

The dimensions of the hole are significant: it turns out that the condition necessary and sufficient for a check to be a possible weave is that in every 2 x 3 rectangle at least one colour must repeat in a row or column.

10.4.3 THE SEE-THROUGH WEAVE BOARD

... renders front and back of **THE CHUNKY WEAVE HOLE** visible simultaneously.

10.5

p This investigation has 3 stages and children can stop at any one:

10.5.1 LATIN SQUARES

The experimenters construct an order 3 Latin square, using coloured cups or coloured saucers.

10.5.2 GRAECO-LATIN SQUARES 1

They make an orthogonal pair of the same order, pairing the cups and saucers.

10.5.3 GRAECO-LATIN SQUARES 2

They attempt one of order 4:

10.5.3.1

... either by stacking pairs of coloured, interlocking cubes to make their cup-&-saucer pairs

10.5.3.2

... or by using the two attributes of playing cards: suit and value.

Most people find the second easier because the two component Latin squares are distinguished.

10.6 A DOMINO RECTANGLE

p A standard set of dominoes is packed in an 8 x 7 grid, the spot number for each cell recorded and the dominoes removed. The exercise is to replace the dominoes.

The activity is more or less interesting and challenging according to how the dominoes are arranged. The cunning arrangement here is due to David Fielker.

The station might have been included in **10.3** in that it is as much to do with where dominoes *can't* go as where they *can*.

SECTION		AGE RANGE					
HEURISTIC		Appropriate point of entry - not necessarily to the task set by the caption - and levels on which extension activities generated (some to be pursued off-site)					
STATION							
NUMBER	NAME	4	7	10	13	16	19+
10.1.1.1	NIM			*	*	*	*
10.1.1.2	NIMROD						*
10.1.2	CHOMP			*	*	*	*
10.1.3.1	THE PIE GAME			*	*	*	
10.1.3.2	ALSO THE PIE GAME					*	
10.1.4	DOMINO BLOCK			*	*	*	
10.1.5	3-D Os & Xs			*	*	*	
10.1.6.1	TARTAN CHECKS: EDGES & CORNERS		*	*	*		
10.1.6.2	TARTAN CHECKS: TARTAN BLOCK			*	*		
10.1.6.3	TARTAN CHECKS: THE CHECKS CHALLENGE			*	*	*	
10.2.1	SAFE QUEENS			*	*	*	*
10.2.2	SAFE QUEENS ON A 5x5 BOARD				*	*	
10.3.1	GRANDPA'S ARMCHAIR				*	*	
10.3.2.1	DOMINO TILINGS 1				*	*	
10.3.2.2	DOMINO TILINGS 2				*	*	*
10.4.1	WEAVES: THE SPECIMEN WEAVE					*	*
10.4.2	WEAVES: THE CHUNKY WEAVE HOLE					*	*
10.4.3	WEAVES: THE SEE-THROUGH WEAVE BOARD					*	*
10.5.1	LATIN SQUARES		*	*	*		
10.5.2	GRAECO-LATIN SQUARES 1			*	*	*	
10.5.3.1	GRAECO-LATIN SQUARES 2				*	*	*
10.5.3.2	A PLAYING CARD ALTERNATIVE			*	*	*	*
10.6	A DOMINO RECTANGLE			*	*	*	

	INSTRUCTION NEEDED				SUPERVISION NEEDED			SERVICING NEEDED		
→	Needs no explanation or caption	Caption enough for most people	Needs aural prompt	Visitors must be talked through stages	None	Benefits from a helper's input	Session must be directed	None	A little	Much
10.1.1.1		*			*			*		
10.1.1.2				*	*			*		
10.1.2		*			*			*		
10.1.3.1		*			*			*		
10.1.3.2			*		*			*		
10.1.4		*			*			*		
10.1.5		*			*			*		
10.1.6.1		*			*			*		
10.1.6.2		*				*		*		
10.1.6.3		*				*		*		
10.2.1			*			*		*		
10.2.2			*			*		*		
10.3.1		*				*			*	
10.3.2.1		*				*		*		
10.3.2.2		*				*			*	
10.4.1			*			*		*		
10.4.2			*			*		*		
10.4.3			*			*		*		
10.5.1		*				*		*		
10.5.2		*				*		*		
10.5.3.1		*				*		*		
10.5.3.2		*				*		*		
10.6		*				*		*		

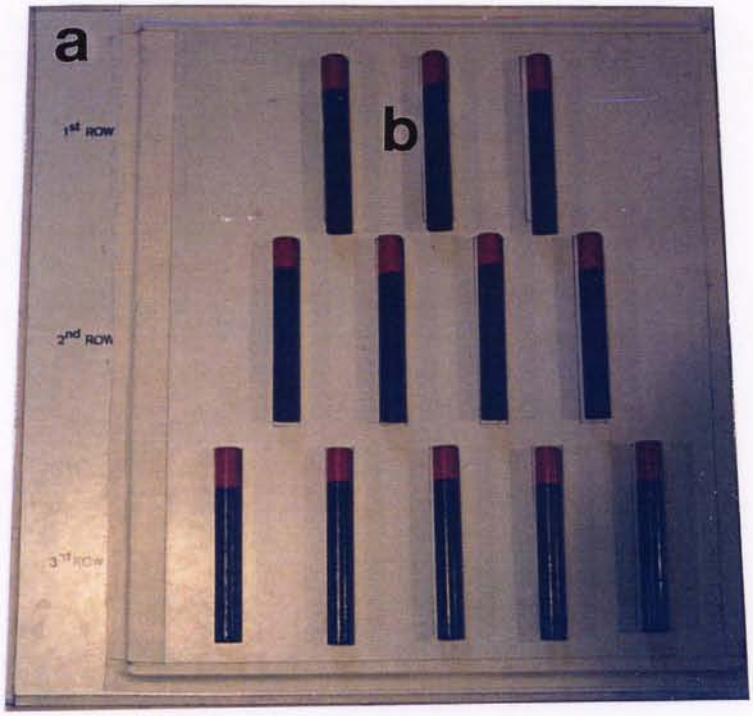
	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.1.1	NIM
TOPIC	A game with strategy defined by 10.1.2	

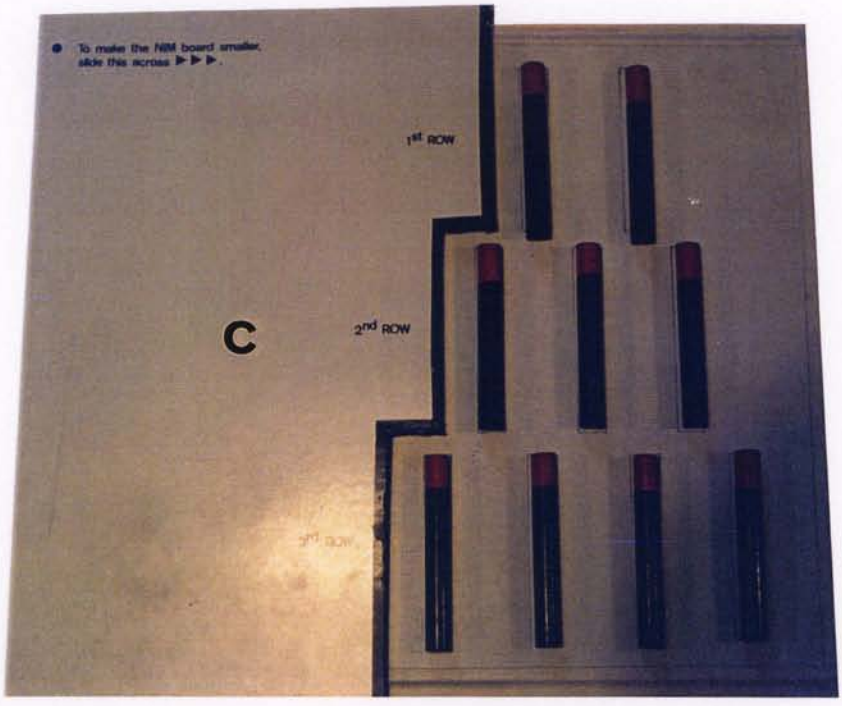
NIM

A GAME FOR 2

- On your move
  take any number of matches
  but from only one row.

- Force your opponent
  to take the last match
  and you win!


a
1st ROW
b
2nd ROW
3rd ROW


To make the NIM board smaller, slide this across ▶▶▶.
1st ROW
c
2nd ROW
3rd ROW

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	baseboard: 375 mm square, 2 sheets of white-faced hardboard stuck back-to-back, 'match' positions marked with rectangles 37.5 mm apart horizontally, 15 mm vertically, rows labelled: **1st row** **2nd row** **3rd row** There is no mathematical significance in locating the 'matches' but the arrangement makes them easier to count.		local
b	'matches': 100 mm lengths of 12.5 mm diameter wood dowel, one flat face sanded, on which they can rest, painted as shown		local
c	- optional mask, with which baseboard can be modified to decrease length of each row in steps of 1 'match': white-faced hardboard, cut as shown from rectangle 250 mm wide x 375 mm deep, rows labelled as above, bearing caption: **To make the NIM board smaller, slide this across.**		local

	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.1.2	NIMROD
TOPIC		A game strategy based on preserving parity and the binary system

NIMROD

- ▶ The blue cubes show the number of matches in each row as a sum of 4s, 2s and 1s.
- ▶ The pink cubes show the 4, 2 and 1 column totals as a sum of 2s and 1s.
- ● Right up to the endgame, ensure there is no pink cube in a "1" box.
- ▶ The endgame begins when only one row has more than one match.
- ● Now ensure there is a pink cube in the last "1" box.

NIMROD		MATCH NUMBERS		
		4	2	1
ROW	TOP			
	MIDDLE			
	BOTTOM			
COLUMN TOTALS	2			
	1			

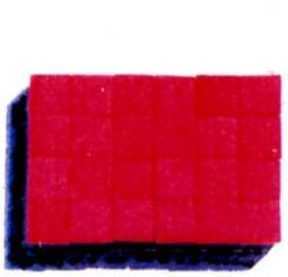
PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
	chart shown marked on caption board as described, faced with Glodex, squares drawn to match size and colour of appropriate:	DIME cube	Tarquin Publications (address above)
	(see **1.10 a** for specification)		
	Multilink would serve as well.	Multilink: SY 007/9	NES Arnold Ltd (address above)
	Provide 6 blue, 4 red.		

	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.1.3	CHOMP
TOPIC		Another subtraction game. The winning strategy is known only for 2 x n and n x n boards.

CHOMP

A GAME FOR 2

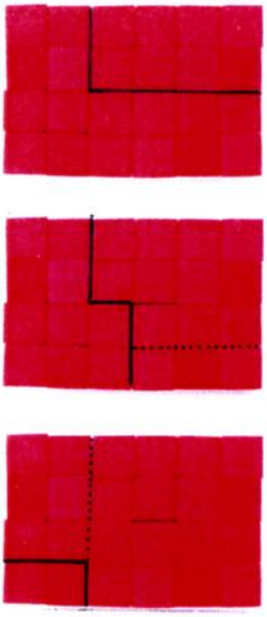
- From this box of Turkish Delight take turns chomping square or oblong bites from the top right:



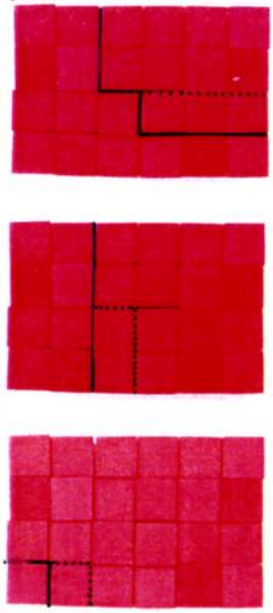
The last piece is poisoned: take the last chomp and you lose!

Example:

A



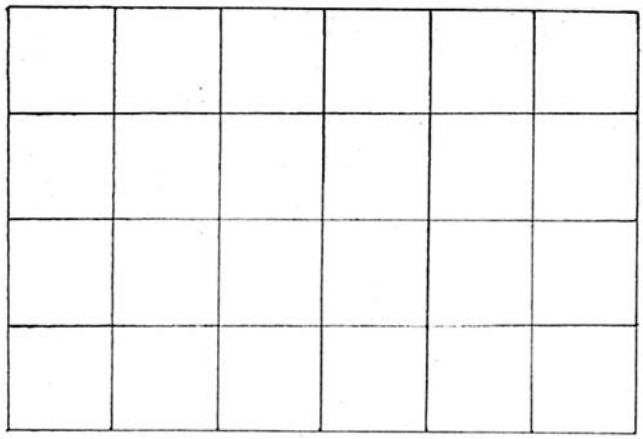
B



wins!

CHOMP

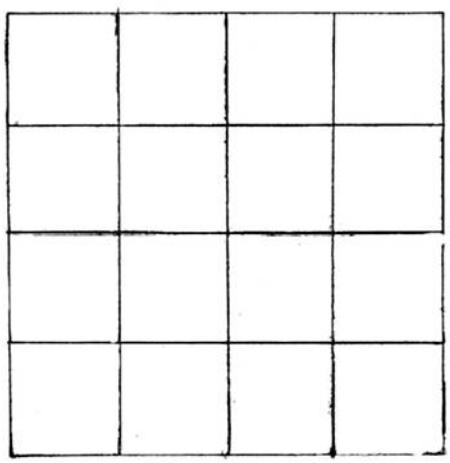
- Take turns chomping out rectangular - oblong or square - bites **in this direction:**

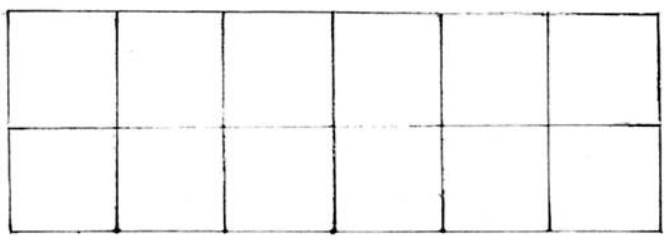


▶ Take the last chomp and you lose.

- Try these boards.

- Can you find winning strategies?





PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
	2 caption boards as described, marked as shown (**reproduced here at 50%**) Supply at least 40 of: i.e. enough to enable 2 boards to be used simultaneously	DIME cube	Tarquin Publications (address above)

	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.1.3.1	THE PIE GAME
TOPIC	A game with strategy explored through 10.1.3.2	

THE PIE GAME

A GAME FOR 2

- **Here are:**
  8 slices of blueberry pie,
  4 slices of cherry.

To win all 12, play the 'pie' game:

Take turns.
On your go, turn over ...

A) any number of blueberry slices

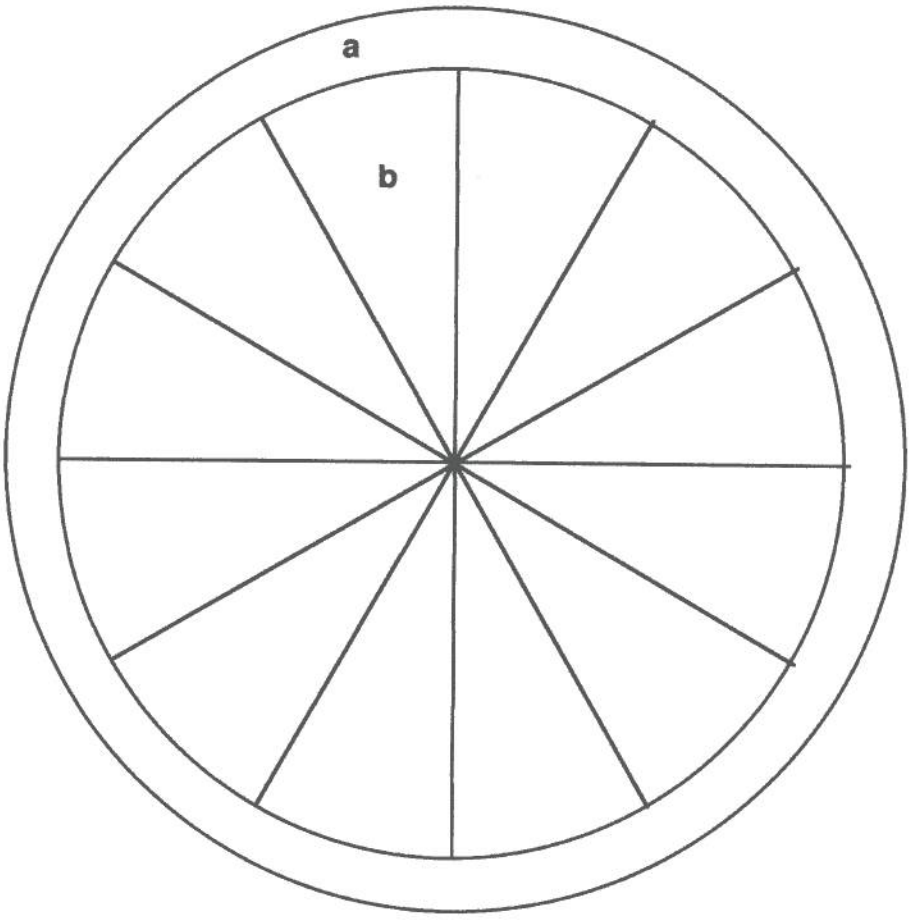
or

B) any number of cherry slices

or

C) the same number of each .

Who turns over the last slice, wins.


a
b

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	disk, 6 mm MDF, painted white, 200 mm diameter		local
b	sectors, 6 mm MDF, cut from disk 175 mm diameter, 4 painted red, 8 blue		local

	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.1.3.2	ALSO THE PIE GAME
TOPIC		A transformation of 10.1.3.1

ALSO THE PIE GAME

- **The counter starts in the north-east corner and ends in the south-west corner.**
- **The grid forces you to move either ...**

(A) south

or

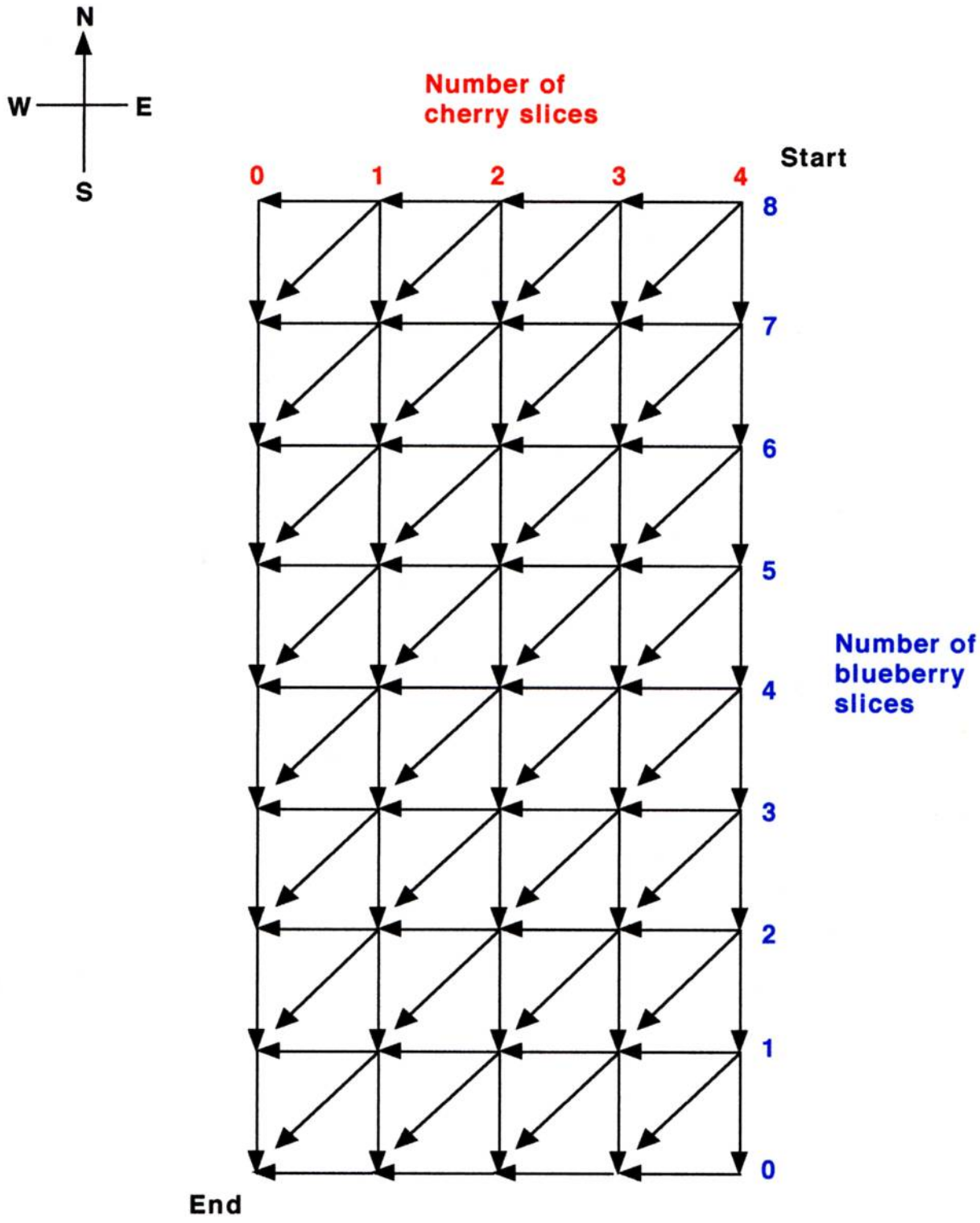
(B) west

or

(C) south-west.

- **On your go,**
  move the counter a chosen number of spaces in a straight line.

  Who makes the last move, wins.


N
W
E
S
Number of
cherry slices
0
1
2
3
4
Start
8
7
6
5
4
3
2
1
0
Number of
blueberry
slices
End

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
	The illustration shows the game board to scale. The graphic is mounted on a caption board as described, faced with Glodex. A 1-cm counter (yellow) is provided, also a dri-wipe pen if requested.		

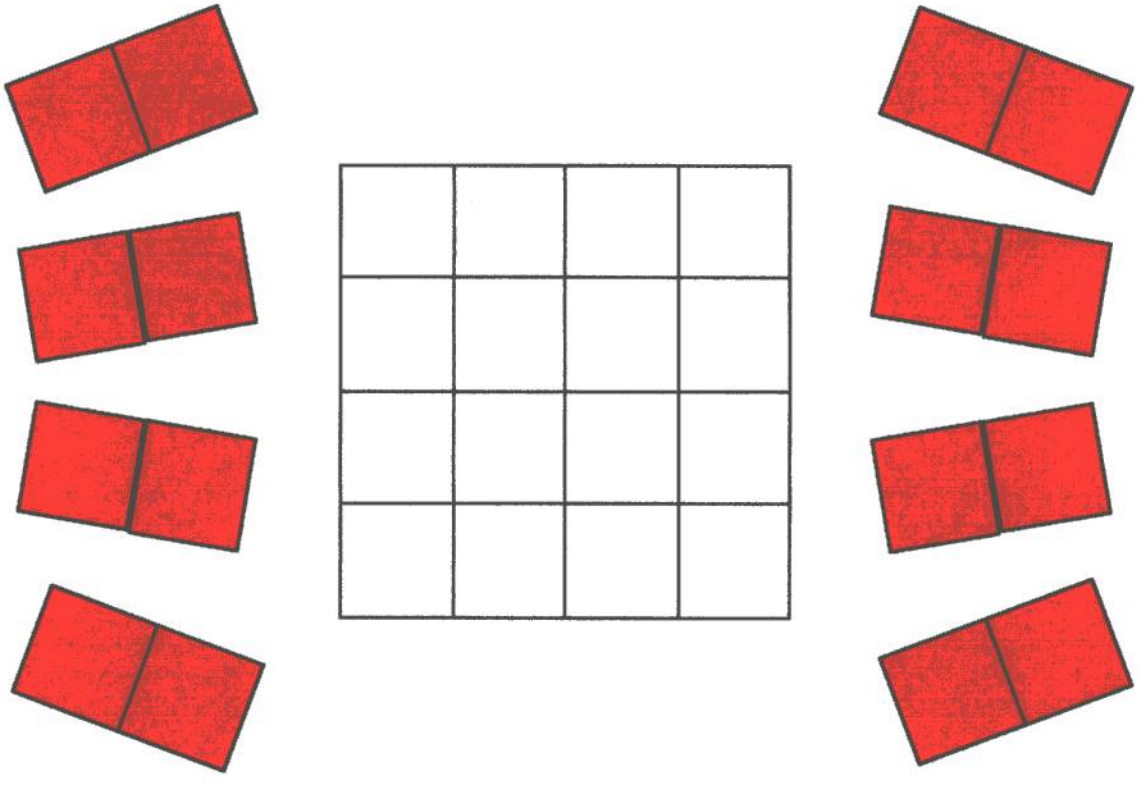
	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.1.4	DOMINO BLOCK
TOPIC	A game with a simple strategy based on point symmetry	

DOMINO BLOCK
A Game for 2

- **You each have 4 dominoes.**

  Take turns.
  Lay them on this grid.

  The last to go, wins.



a

b

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a, **b**	based on a 40 mm square		
a	red Perspex, 3 mm, x 8		local
b	grid as shown drawn on caption board as described, faced with Glodex		local
	If the grid is also drawn on a 300 micron acetate sheet - or, better, on 3 mm Glodex - and an extra set of **a** made, the game can also be played on the O.H.P.		

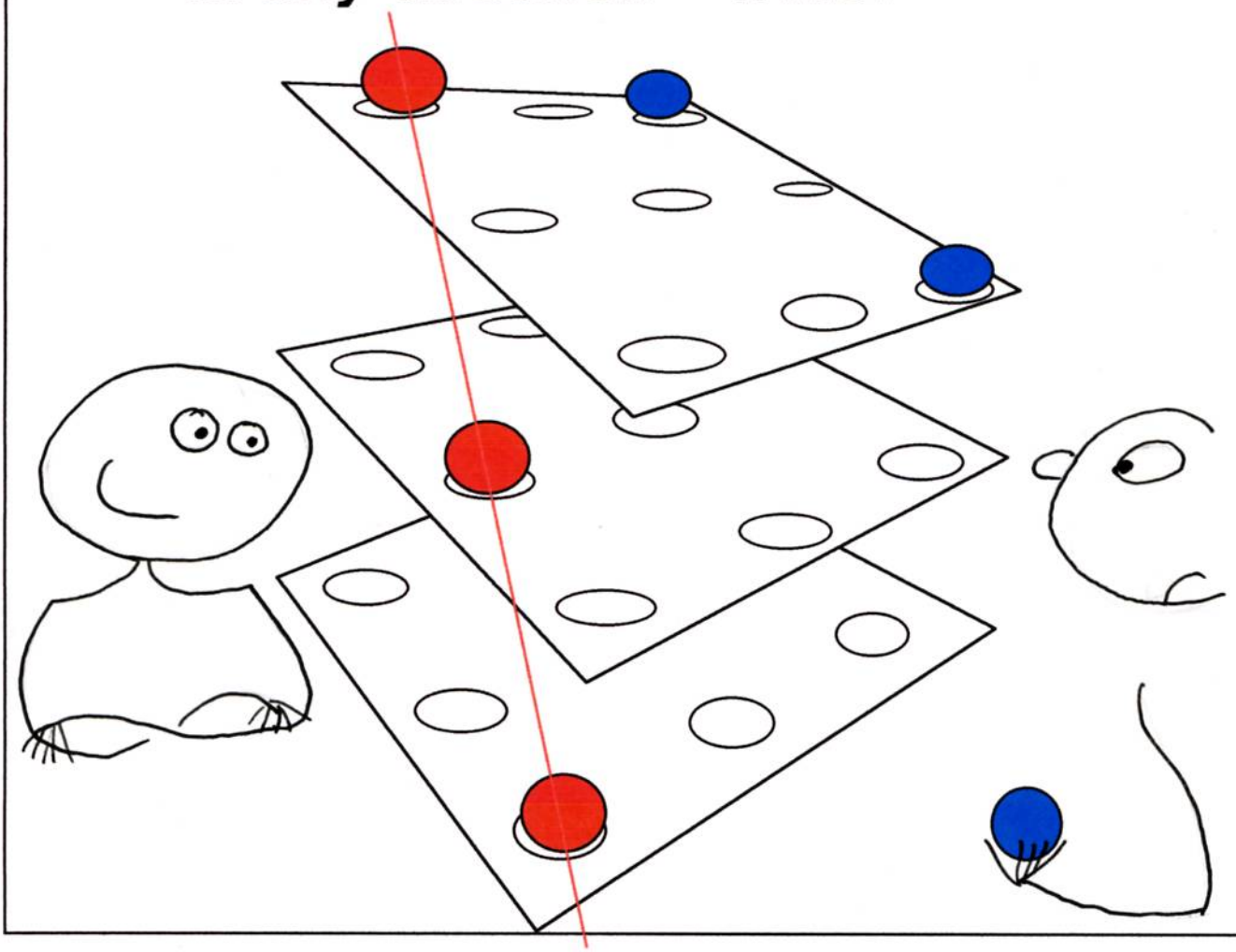
	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.1.5	3-D Os & Xs
TOPIC		A positional game with a strategy based on creating alternative moves

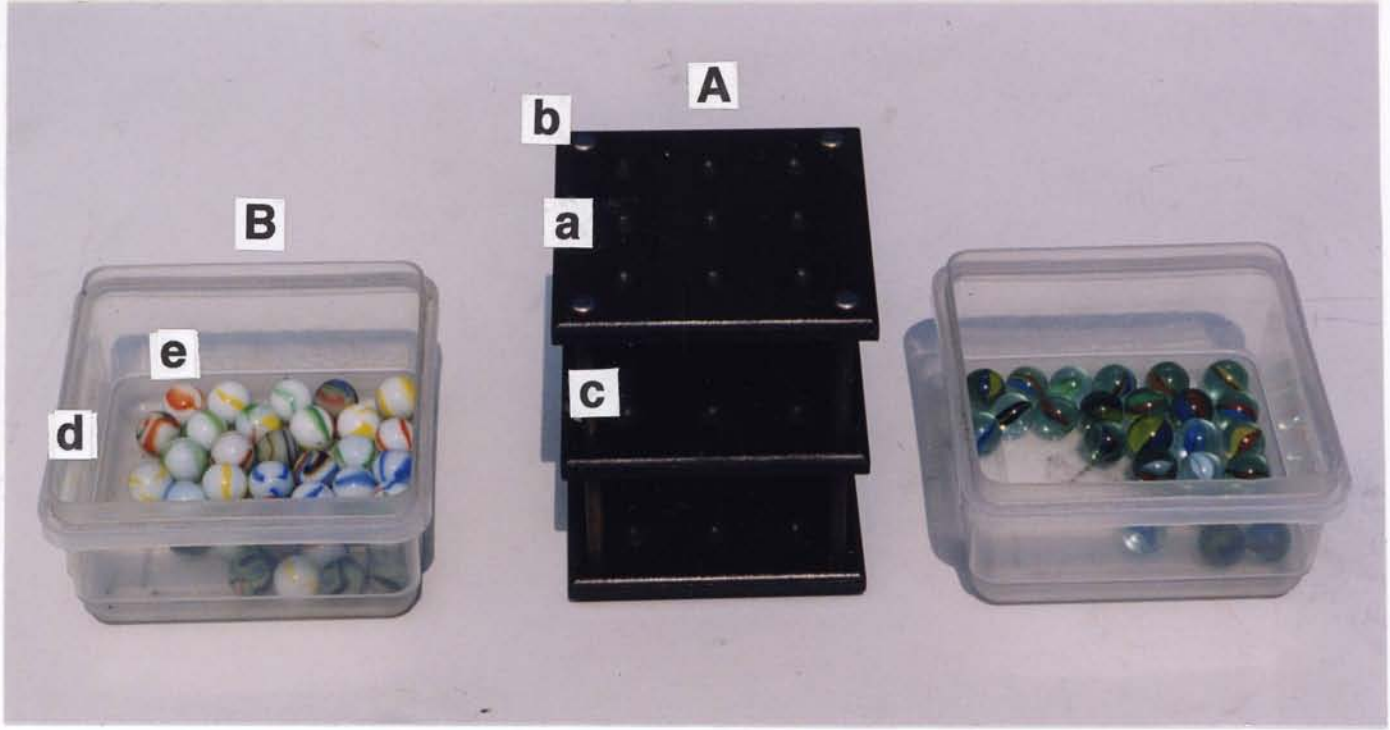
3-D

Os & Xs Three-in-a-Row Connect 3

With marbles of 2 colours, a game for 2
With marbles of 3 colours, a game for 3
With marbles of 4 colours, a game for 4

- **Each choose a colour.**
- **Take turns placing your marbles.**
- **The first to get 3 in a straight line - *in any direction* - wins.**



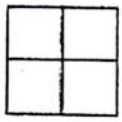

A
b
a
c
B
e
d

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
A	game board:		
a	varnished wood, 6 mm, 120 mm square, depressions where shown, x 3		local
b	bolt, 120 mm, + nut, x 4		local
c	tube as spacer, 50 mm long, x 8		local
B	container with balls, x 2, 3 or 4 as required:		
d	container: shallow tray or storage box without lid such as shown		local
e	ball (marble or bead), 15 mm diameter, preferably coloured uniformly*, x 12 (say)		local
	*For 2 players without sight, use 2 types of ball: one with a textured surface, one without.		

	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.1.6.1 10.1.6.2	TARTAN CHECKS: EDGES & CORNERS TARTAN CHECKS: TARTAN BLOCK
TOPIC		Chromatic tessellations

TARTAN CHECKS

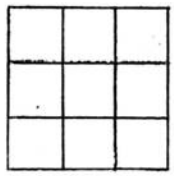
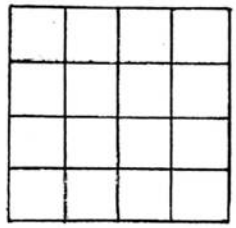
EDGES & CORNERS: An Investigation

- Can you fill this grid with cubes: 
  so that the same colours
  do not touch

  edge-to-edge

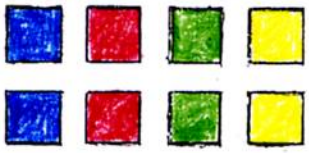
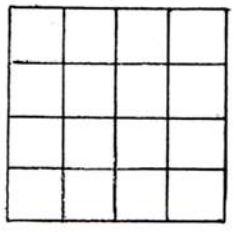
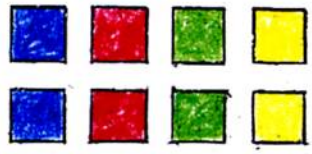
  or

  corner-to-corner?

- How many colours do you need?
- What about this grid?  or this one? 

TARTAN BLOCK: A Game for 2

- Follow the edge-&-corner rule.
- This is the board and these are your cubes:

- Take turns placing all 8 of your cubes at once. Your opponent must fill as many of the remaining squares as s/he can.

	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.1.6.3	THE CHECKS CHALLENGE
TOPIC		The lessons of 10.1.6.2 applied

THE CHECKS CHALLENGE

▶ **Here the rules are the same but the board is as large as you please.**

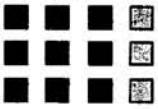
● **See what you can score with ...**

The record (14.1.02):

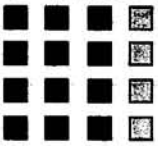
1 tile of each colour **2**

2 tiles of each colour **5**

3 tiles of each colour **10**



4 tiles of each colour **15**



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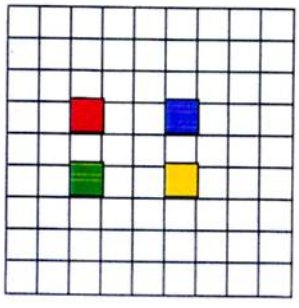
. .
. .

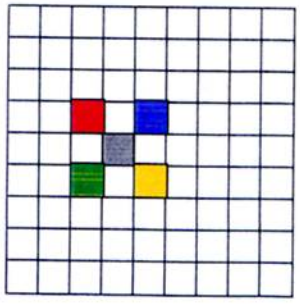
	NUMBER	TITLE
GROUP		
STATION		(Above continued)
TOPIC		

EXAMPLE

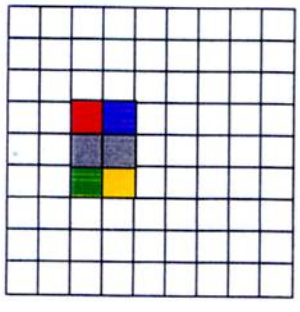
1 tile of each colour



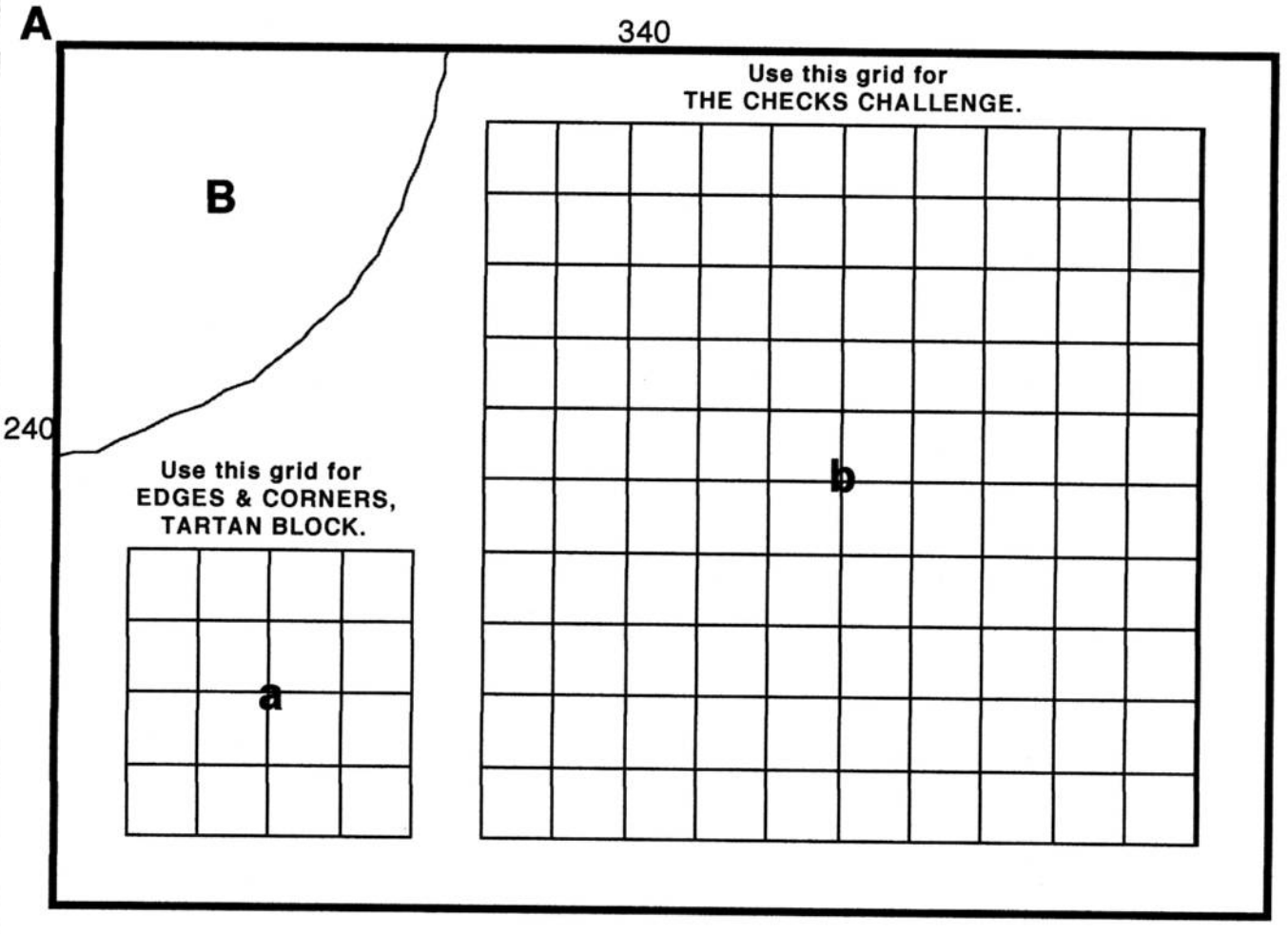
Hmmm **Score: 0**



Better **Score: 1**



Best! **Score: 2**


A
340
B
240
Use this grid for
THE CHECKS CHALLENGE.
b
Use this grid for
EDGES & CORNERS,
TARTAN BLOCK.
a

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
A	game board, dimensions marked: 6 mm plywood tray, white-faced, 10 mm lip, graphics as shown		local
a	20 mm grid, 4 x 4, (for **10.1.6.1**, **10.1.6.2**)		
b	20 mm grid, 10 x 10, (for **10.1.6.3**)		
	The tray also stores:		
B	20 mm square plastic tiles in 4 colours, (say) 25 of each		NES Arnold Ltd (address above)

	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.2.1	SAFE QUEENS
TOPIC		Systematic v. exhaustive search methods

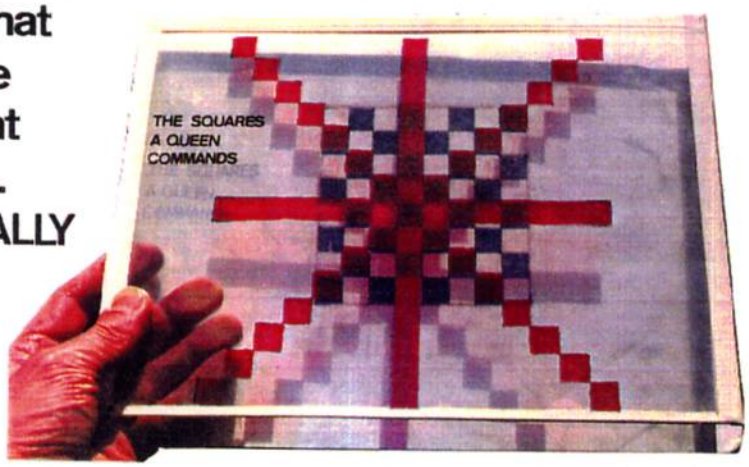
SAFE QUEENS

- A chess queen at the centre can take a piece anywhere on this red cross:

VERTICALLY

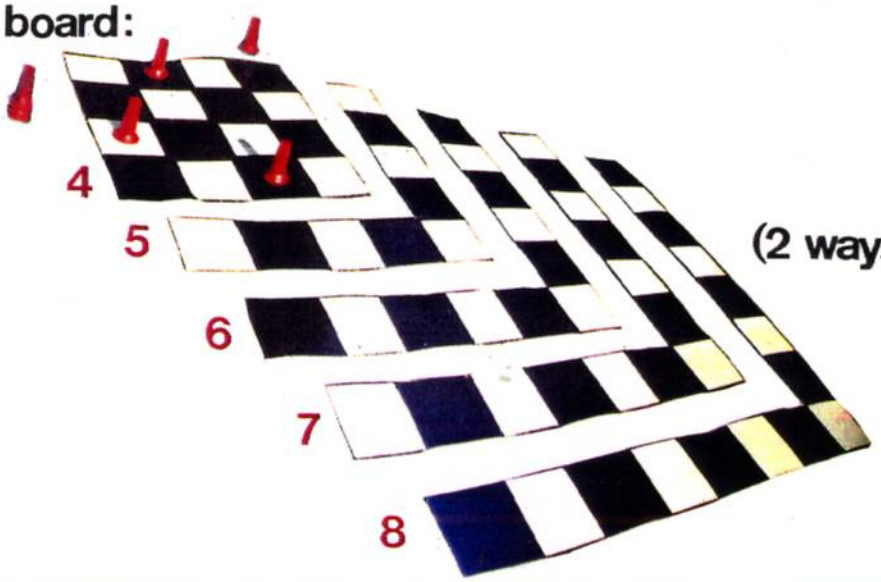
- Place queens so that no queen can take another, i.e. so that no 2 are in line ...

HORIZONTALLY



DIAGONALLY

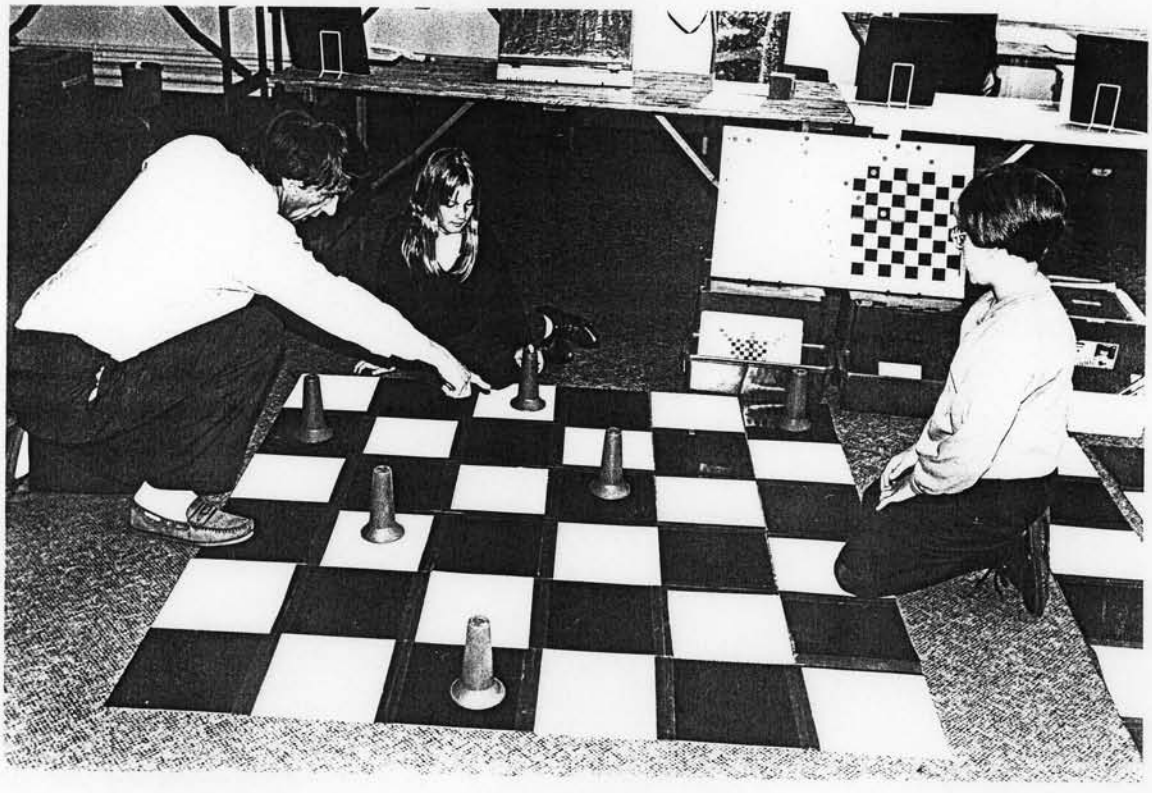
- You can make the board bigger by adding 'L's.
- Try to match the number of queens and the size of the board:

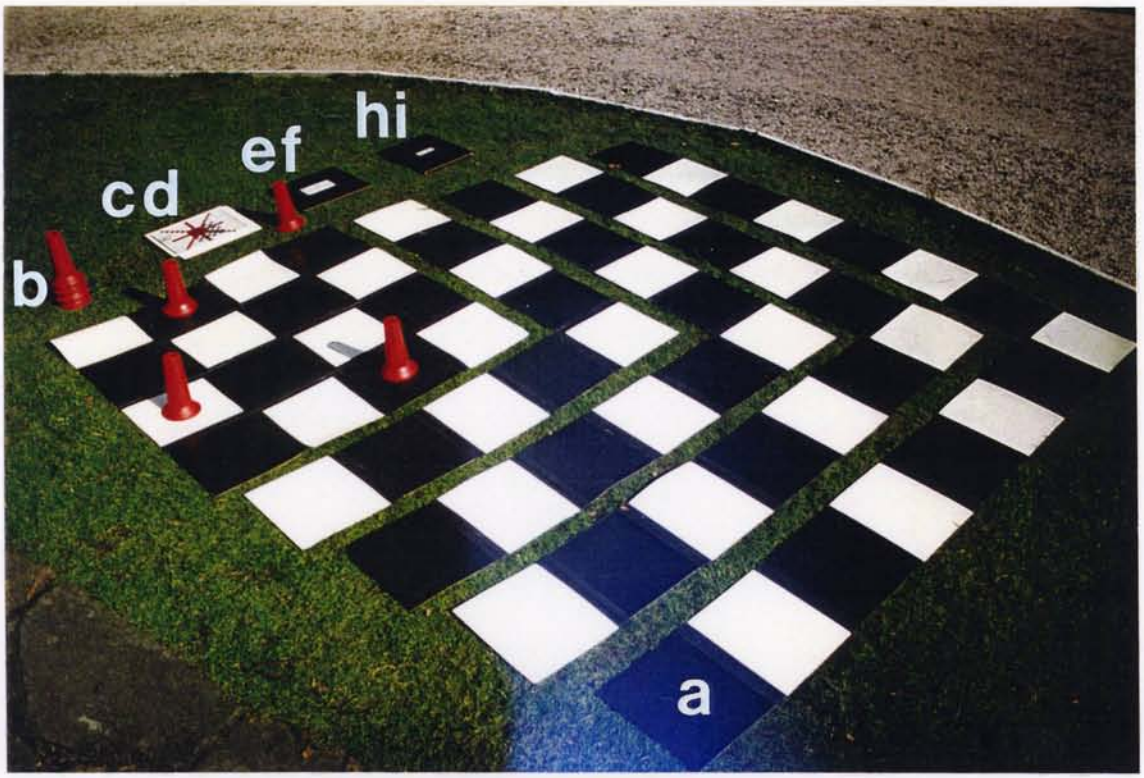


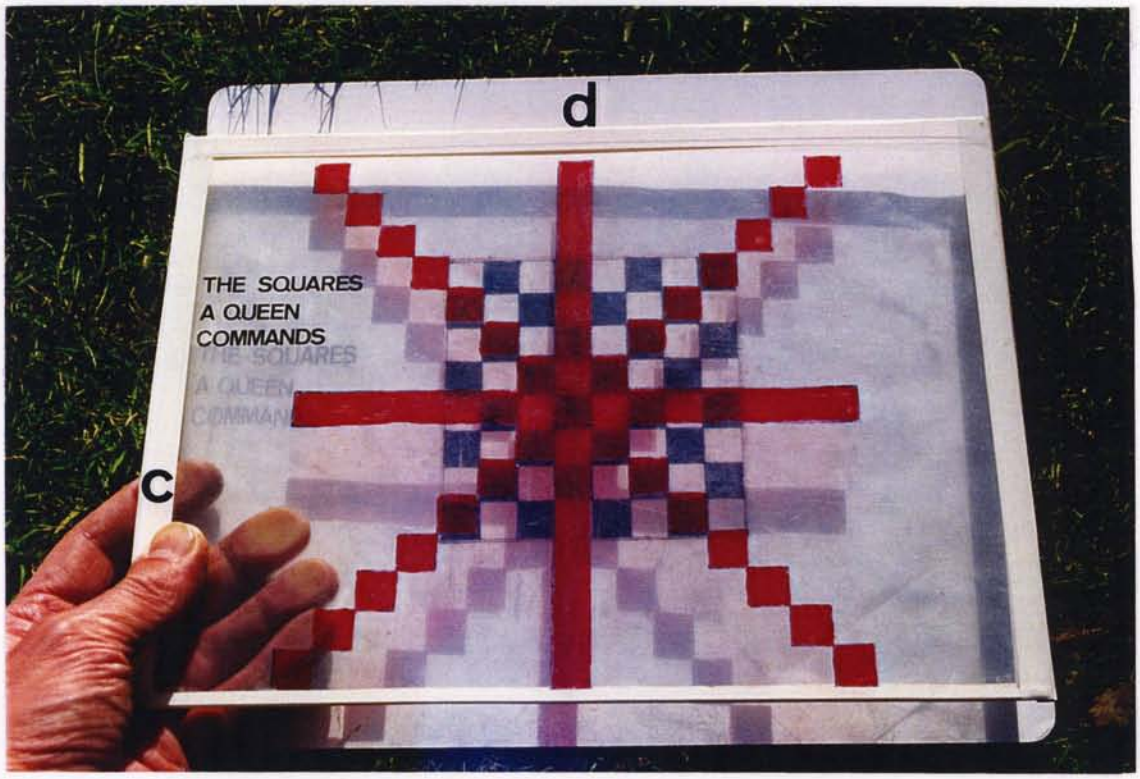
(2 ways)

(6 ways)

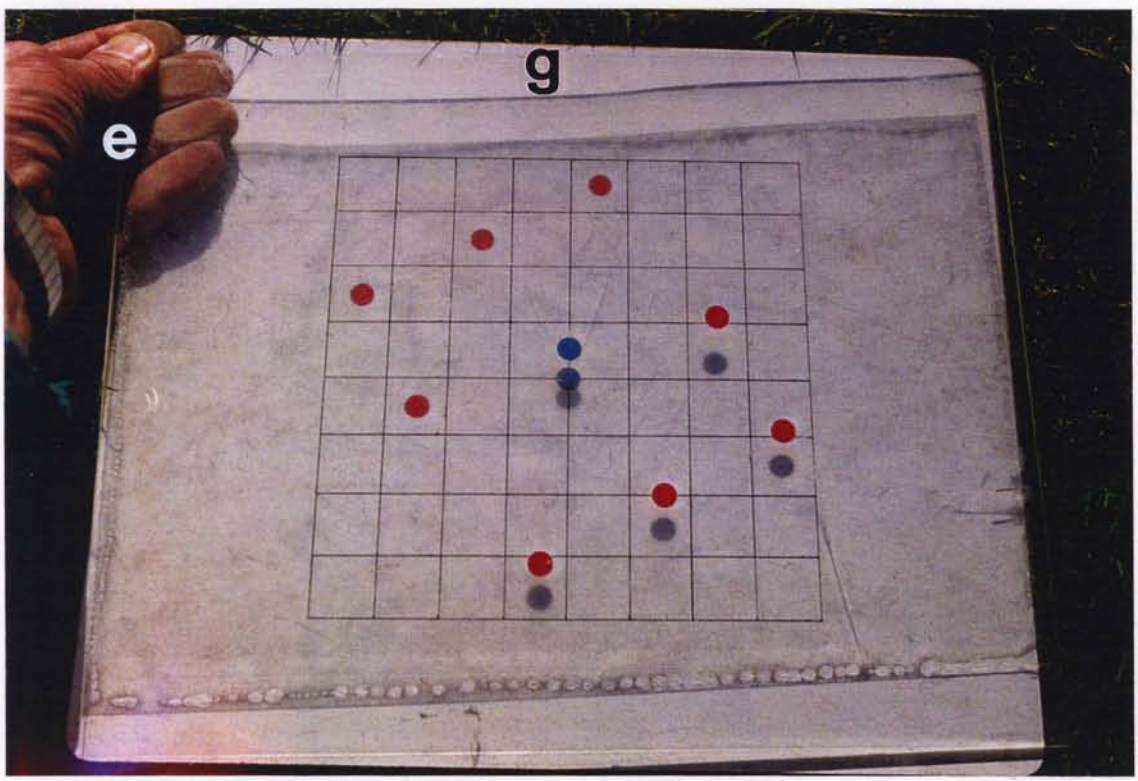
(12 ways)




hi
ef
cd
b
a


d
THE SQUARES
A QUEEN
COMMANDS
c

f
8 QUEENS

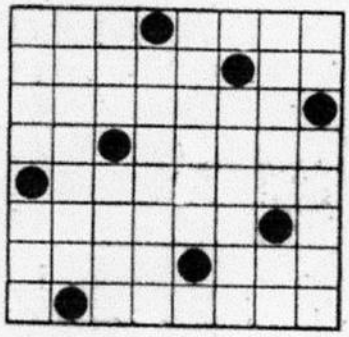

e
g

h

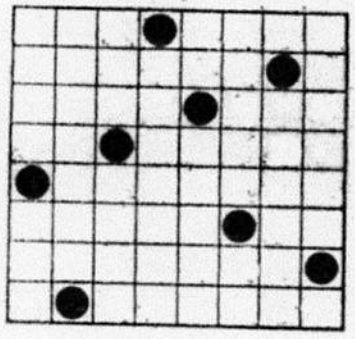
INSIDE THIS ENVELOPE
ARE THE SOLUTIONS
FOR THE 8 X 8 BOARD.

i

The twelve solutions to the classic problem of eight queens



1



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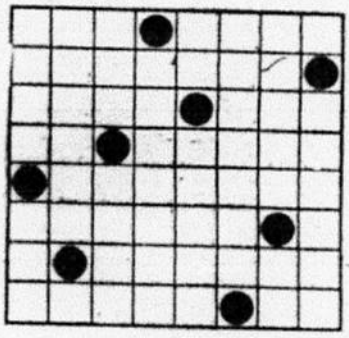

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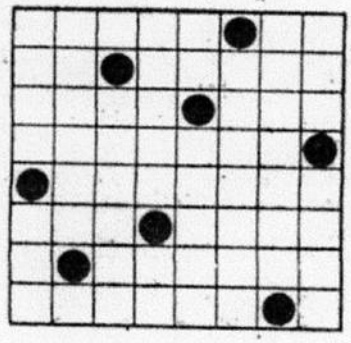
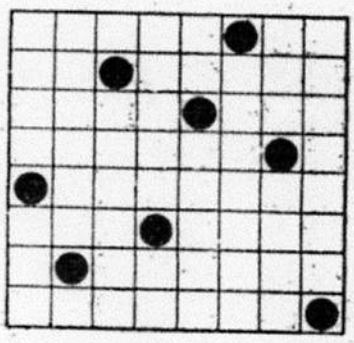
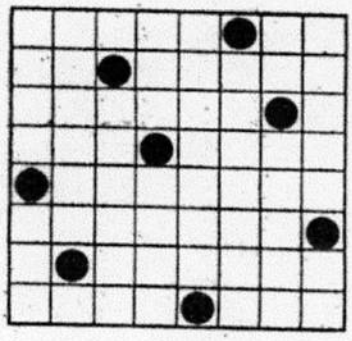


4



5



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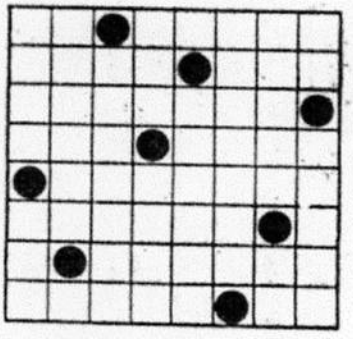
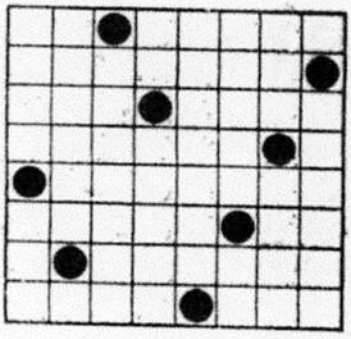
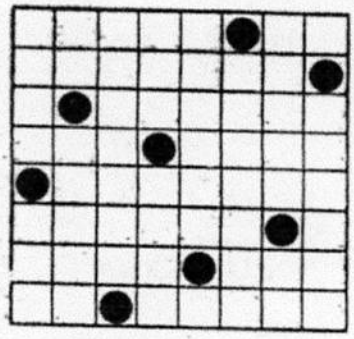


7



8



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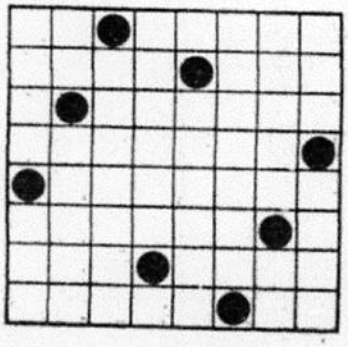

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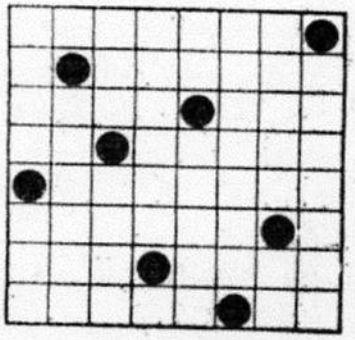
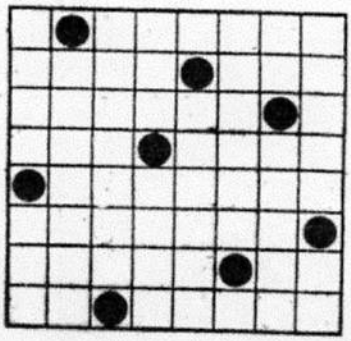


11



12

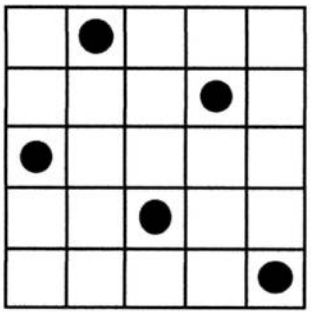
PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	as **7.6.1 a,f,** but Ls of sides 5,6,7,8 squares surround a 4 x 4 square		
b	'queens': polymer marker cones, 200 mm high, 100 mm diameter	Mini-cones NES Arnold catalogue: N 6236/3	NES Arnold Ltd (address above)
c,d	visual aid to define a queen's permitted moves, square size: 12.5 mm:		
c	Glodex sheet, A4, marked in permanent felt-tip as shown and labelled: **THE SQUARES** **A QUEEN** **COMMANDS**		
d	caption board as described, with chess board represented centrally, darker squares in blue, covered with Transpaseal		
e,f	manipulative providing clue to the 1 '8 x 8' solution with rotation symmetry:		
e	Glodex sheet, A4, bearing 4 red and 1 blue sticker placed to match same on **g**		
f	caption board as described, bearing caption: **8 QUEENS** **There are 12 solutions on the full 8 x 8 board. 1 has half-turn symmetry. On the back of this board 4 queens are in position. Can you place the rest?**		
g	= **f** verso, with 8 x 8 grid of 20 mm squares, on which 4 red and 1 blue sticker are placed as shown		
h,i	solution sheet and containing sleeve, labelled as shown:		
h	2 caption boards as described, black faces outwards, spaced by 2 strips of same stuck to their longer edges, to form a sleeve to contain:		
i	sheet shown, faced with Glodex		

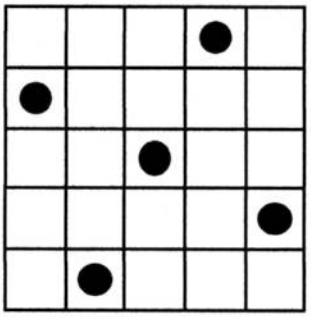
	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.2.2	SAFE QUEENS ON A 5 x 5 BOARD
TOPIC	Solutions rendered equivalent by transformations	

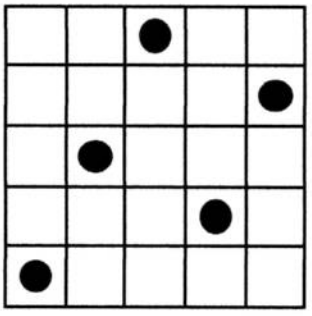
SAFE QUEENS ON A 5 X 5 BOARD

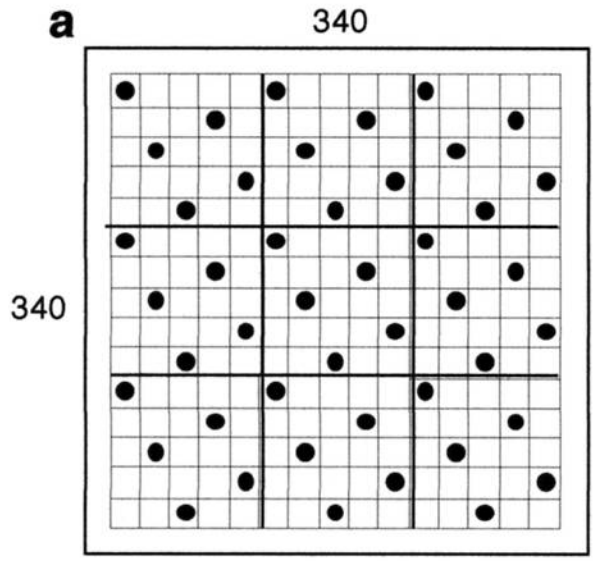
- **Turn, flip or slide the big plastic sheet on the empty 5 x 5 grid.**

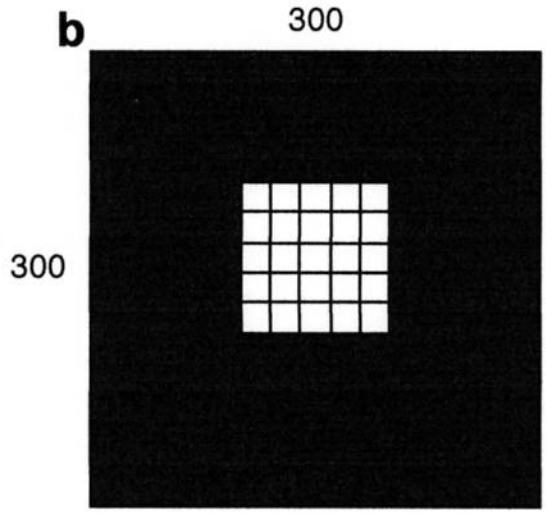
Try to obtain these solutions:








a
340
340


b
300
300

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	square plate, 340 mm edge: graphic shown printed on 300 micron acetate, sandwiched between sheets of 3 mm Glodex		local
b	square plate, 300 mm edge: graphic shown printed on paper, stuck to 12 mm MDF, faced with 2 mm Glodex		local

	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.3.1	GRANDPA'S ARMCHAIR
TOPIC		A question of parity

GRANDPA'S ARMCHAIR

▶ **Here's the problem:**

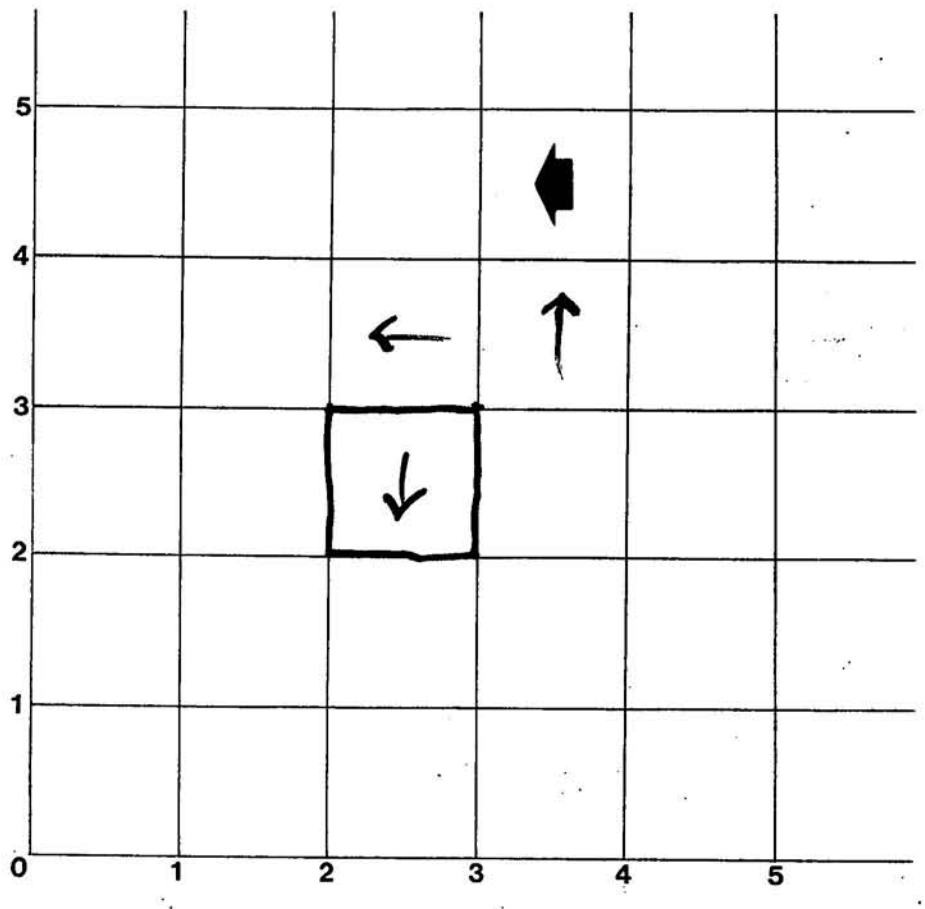
This is Grandpa's lounge:

Grandpa's armchair is directly under the loft trap door so we decide to move the T.V. and the armchair as shown. Both are on castors. There's no problem with the T.V. but the armchair's so heavy you can only swing it about its corners.

■ **Allowing only $\frac{1}{4}$- turns, is this possible or not?**

	NUMBER	TITLE
GROUP		
STATION		(Above continued)
TOPIC		

- **Use the yellow chair and a black pen on this board to study the pattern of moves.**



Use the yellow chair and a black pen
on this board
to study the pattern
of moves.
b
a
0
1
2
3
4
5

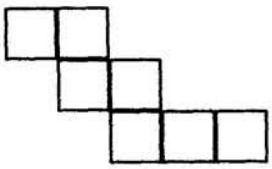
PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
	(only on caption:)		
	doll's house furniture:		local
	T.V. (optional)		
	armchair:		
	must be square in plan - say 40 mm - or, if not, should be mounted on a square plate of plasticard/ perspex/ ...		local
	checkerboard, 6 x 6, square size to match armchair, as **7.5.1 a/b**		
	It is easier to match the armchair to the checkerboard than vice versa.		
a	caption board as described, bearing grid of 40 mm squares, 6 x 6, i.e. to correspond to checkerboard, coordinates marked, faced with Glodex		
b	square prism or cube, 40 mm, wood/MDF, painted as caption, marked to suggest armchair		local
	The photograph shows the board in use.		
	(N.B.: the caption is now as preceding, not as prototype shown.)		

	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.3.2	DOMINO TILINGS
TOPIC		Necessary v. sufficient conditions

DOMINO TILINGS

First investigations

- **A polyomino has any number of squares joined edge-to-edge:**



- **A domino is a polyomino of 2 squares:**

- **So a polyomino can only be covered exactly with dominoes ('tiled') if it has an even number of squares.**
- **On a checkerboard a domino must cover exactly 1 black square and 1 white square:**

- **So a checkered polyomino can only be tiled with dominoes if it has equal numbers of black and white squares.**
- **The face has 20 squares, an even number, but 9 are white and 11 are black.**

■ **Can it be tiled with dominoes?**

	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.3.2	DOMINO TILINGS
TOPIC		Necessary v. sufficient conditions

What we have proved so far

'A polyomino which tiles with dominoes must have:
(1) an even number of squares,
(2) equal numbers of black and white squares.'

We don't need rule (1) as well as rule (2) because, if the numbers of black and white squares are equal, their total must be even. So all we need write is the black-&-white theorem:

'A polyomino which tiles with dominoes must have equal numbers of black and white squares.'

The next challenge

The black-&-white condition is *necessary*.
Is it also *sufficient?*

If it is, the *converse* of the black-&-white theorem is true:

'If a polyomino has equal numbers of black and white squares, it can be tiled with dominoes.

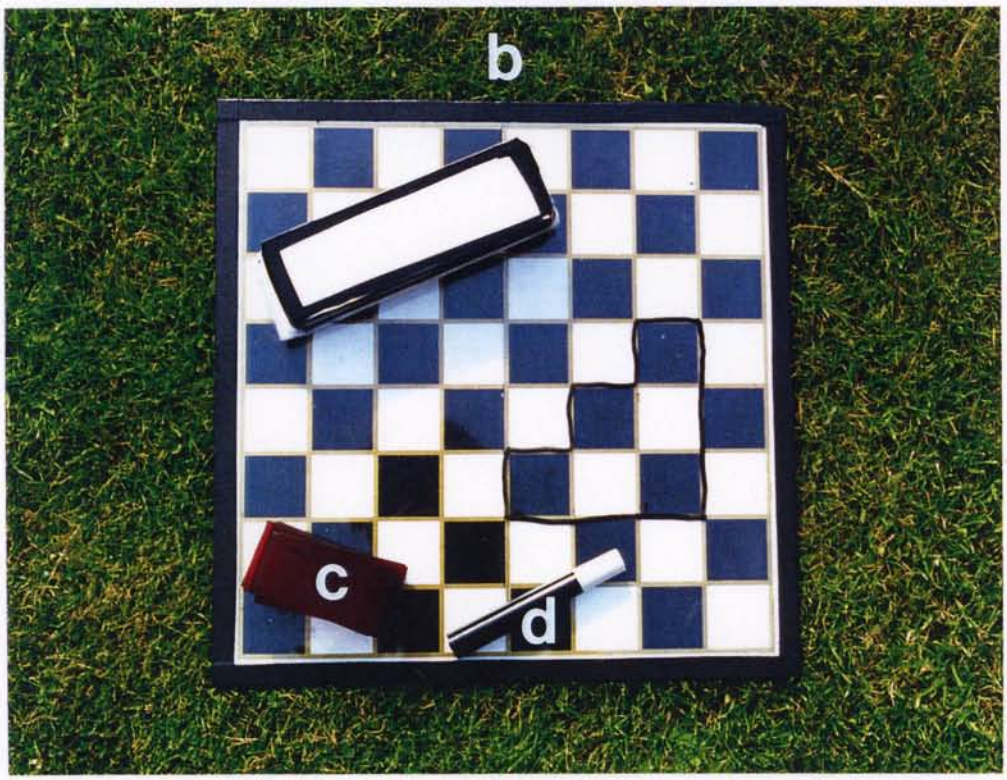
If it is not, the converse is false.
This means you can draw a polyomino which has equal numbers of black and white squares but which does *not* tile with dominoes.

- **Try!**

  Draw polyominoes on the squared board which contain equal numbers of black and white squares.

  Try to tile them with the dominoes.

a


b
c
d

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	'face' The design should distinguish clearly the squares which are to be tiled - those in black or white - and those which are not - the 'eyes' and 'mouth'. Here, the piece of checkerboard shown - squares: 35 mm - is set in a matrix of blue expanded polyethylene, 'mouth' and 'eyes' indicated as shown, and the whole sandwiched between A4 Glodex sheets.		local
b	checkerboard, 35 mm squares, 8 x 8, 300 mm square overall, faced with Glodex		local
c	dominoes, 35 mm x 70 mm, here in red perspex but 3 mm PVC easier to obtain Perspex has the pedagogic advantage that one can see that a domino covers exactly one black square and one white one.	Amari Foam	Amari Plastics (address above)
d	In preference to the **black** Pilot pen WBMA-M shown in use here, use the same in **red*** for better contrast.		(see **THE CAPTION BOARDS**)
d*			

	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.4	WEAVES
TOPIC		Defining conditions, choosing a notation, creating an algebra

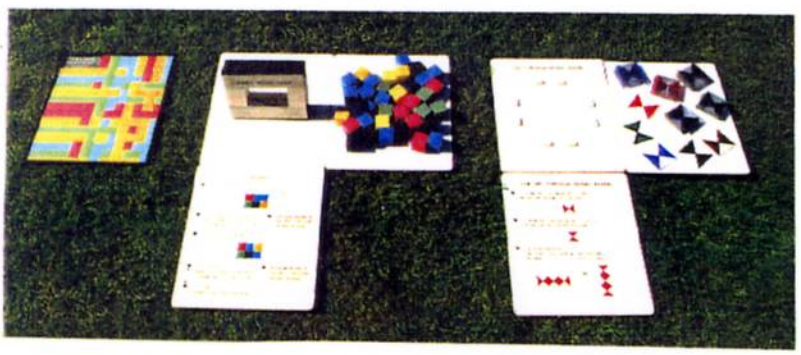
WEAVES

- There are 3 pieces of apparatus for you to use here.
- Read the instructions with each.

The SPECIMEN WEAVE

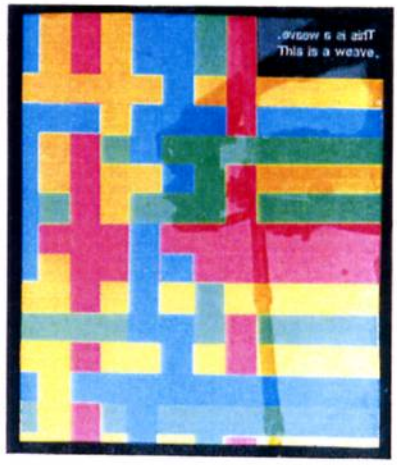
The CHUNKY WEAVE HOLE

The SEE-THROUGH WEAVE BOARD

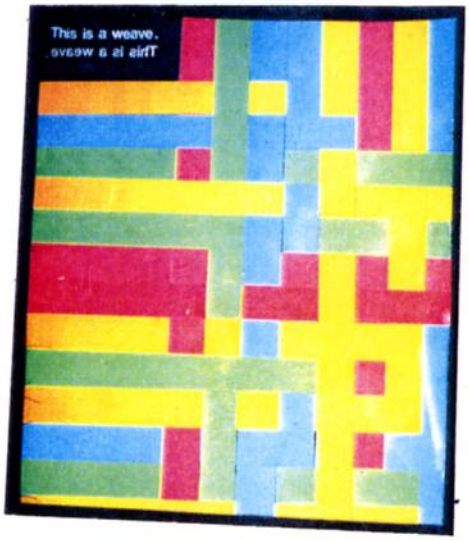


	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.4.1	WEAVES: THE SPECIMEN WEAVE
TOPIC		As 10.4 introduction

THE SPECIMEN WEAVE



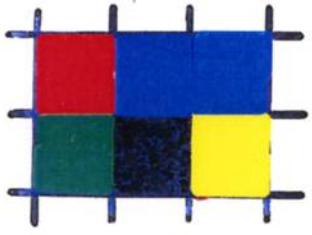
- **Read the board from both sides.**



	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.4.2	WEAVES: THE CHUNKY WEAVE HOLE
TOPIC		As 10.4 introduction

THE CHUNKY WEAVE HOLE

▶ This is a weave:



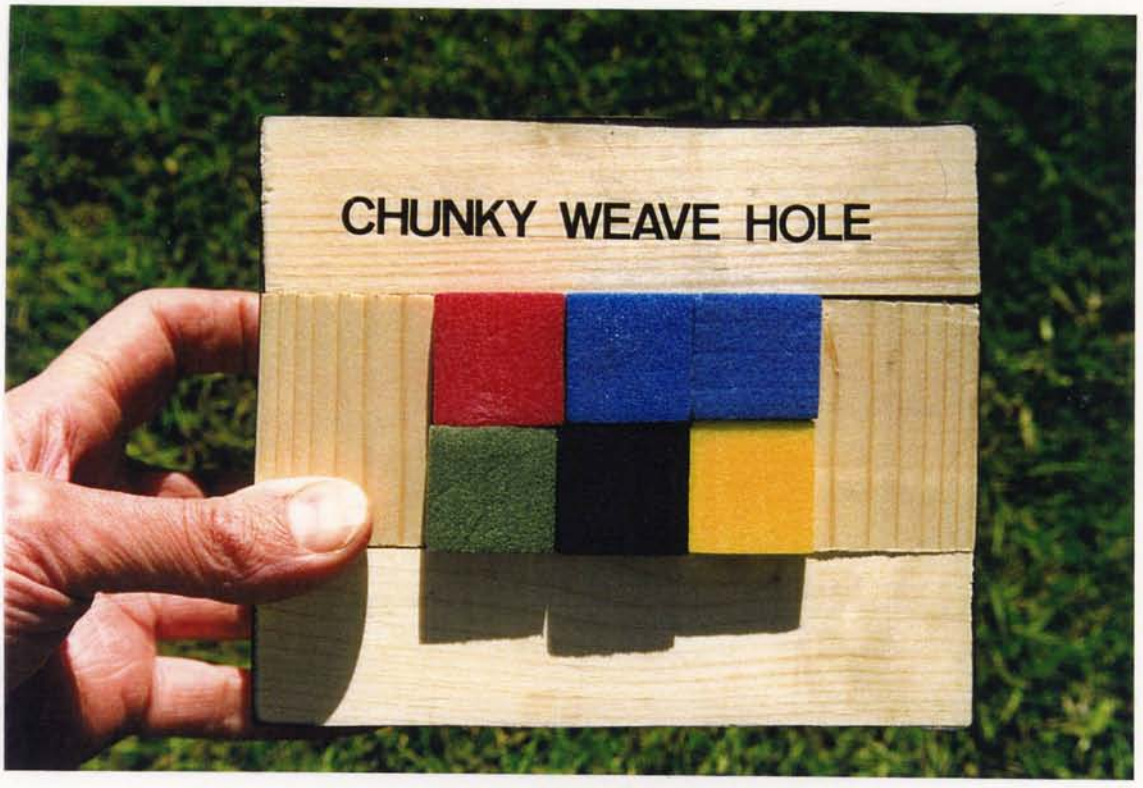
- Put cubes in both sides of the CHUNKY WEAVE HOLE to show the front and back.
- Test your weave on the SEE-THROUGH WEAVE BOARD.

■ Could this be a weave?

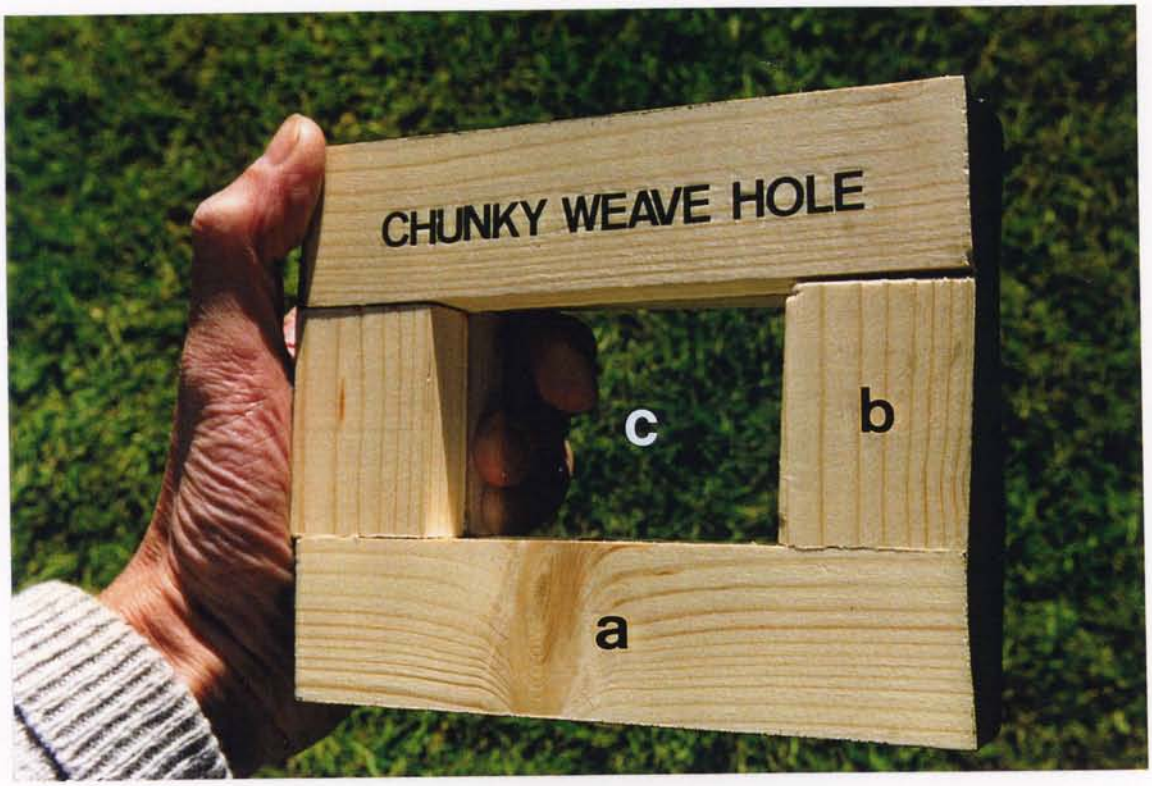
- If it could be, again put cubes in the HOLE to show the front and back.
- Test your weave on the SEE-THROUGH WEAVE BOARD.

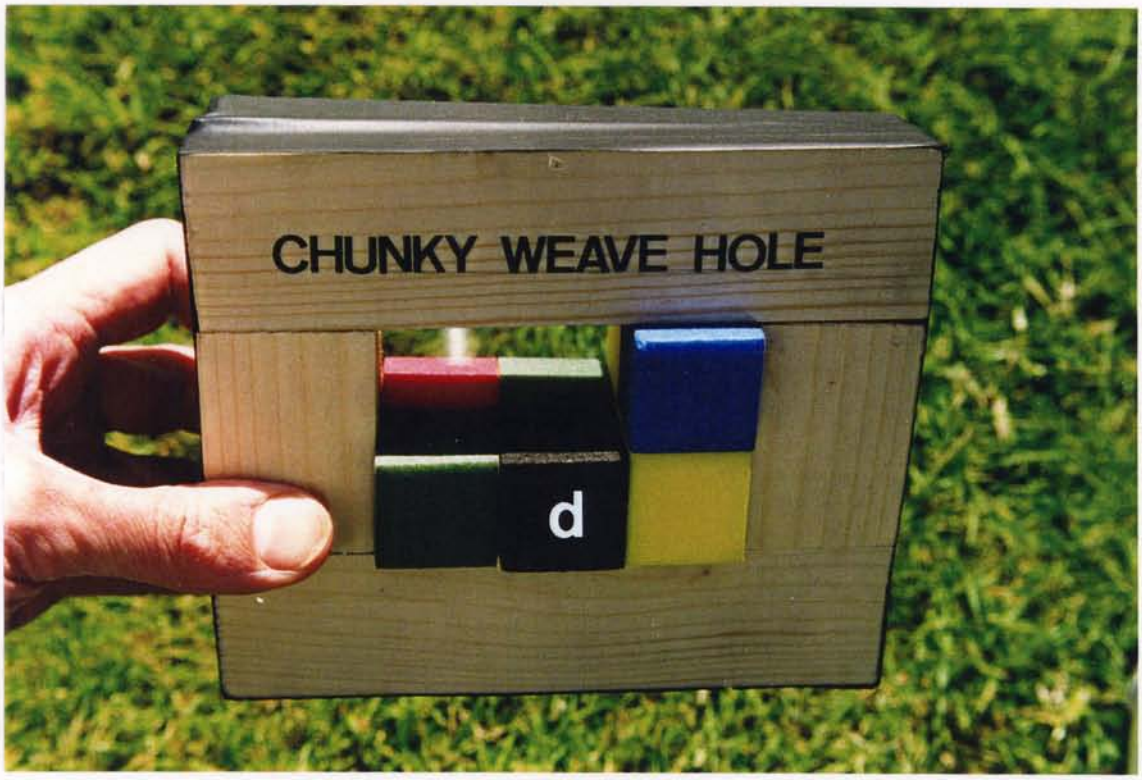
▼

- If it could not be, explain to your partner why.


CHUNKY WEAVE HOLE

CHUNKY WEAVE HOLE


CHUNKY WEAVE HOLE
c
b
a


CHUNKY WEAVE HOLE
d

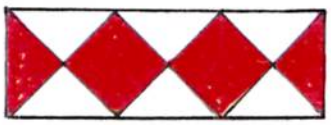
	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.4.3	WEAVES: THE SEE-THROUGH WEAVE BOARD
TOPIC		As 10.4 introduction

THE SEE-THROUGH WEAVE BOARD

- To show the strips going left-to-right, set the see-through tiles this way:

- To show the strips going top-to-bottom, set the see-through tiles this way:

- If your weave works, the same colours will go right through this way: ... and this way:



e

SEE-THROUGH WEAVE BOARD

f

SEE-THROUGH WEAVE BOARD

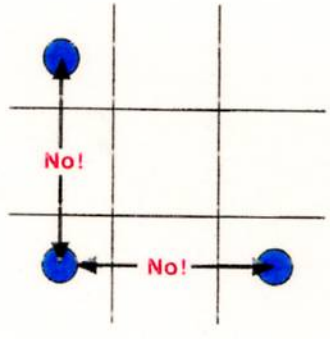
PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
10.4.1	as caption:		
	20 mm strips of coloured paper, woven in a random way, sandwiched between 2 sheets of Glodex, 260 mm (wide) x 300 mm (deep)		local
	The caption		
	This is a weave.		
	is printed twice, so as to be read from both sides.		
10.4.2			
a,b	2 wood frames in '2-by-1 PAR' (44 mm x 22 mm), smaller dimension as thickness:		local
a	4 lengths required x 172 mm,		
b	4 lengths required x 56 mm,		
	labelled		
	CHUNKY WEAVE HOLE,		
	sandwich		
c	a Glodex sheet, 172 mm (wide) x 144 mm (deep).		
	The frame size is such that		
d	DIME cubes	(same)	Tarquin Publications (address above)
	are lightly gripped in a 3 x 2 rectangle, i.e. one 84 mm (wide) x 56 mm (deep).		
	Provide 12 - i.e. 2 x (3 x 2) - in each of the 5 colours.		
10.4.3			
e	caption board as described, labelled		
	SEE-THROUGH WEAVE BOARD		
f	stops, cut from caption board as described, to define a 150 mm square as shown, i.e. one to contain a 3 x 3 array in double thickness (4 mm) of		
g	50 mm square Glodex tiles, coloured in permanent felt-tip as shown		
	Provide 18 - i.e. 2 x (3 x 3) - of each of the 5 colours.		
	These colours should match those of **d** as closely as possible.		

	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.5.1	LATIN SQUARES
TOPIC		Latin Squares

LATIN SQUARES

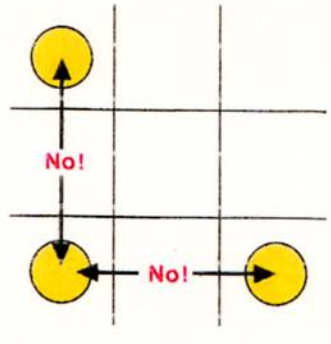
The 'cup' rule:

- **Arrange the cups in a 3 x 3 square so that the same colour never repeats in a row or a column:**



The 'saucer' rule:

- **Arrange the saucers in a 3 x 3 square so that the same colour never repeats in a row or a column:**



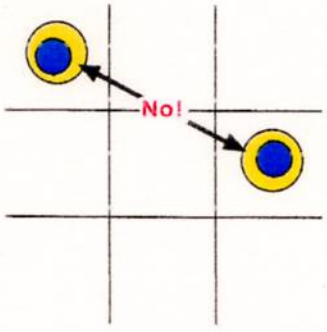
	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.5.2	GRAECO-LATIN SQUARES 1
TOPIC		Graeco-Latin Squares (order 3)

GRAECO-LATIN SQUARES 1

- Keep the 'cup' rule.
  Keep the 'saucer' rule.

The 'cup-&-saucer' rule:

- Put the cups in the saucers in such a way that no pair repeats anywhere:



- In how many different ways can this be done?

	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.5.3.1	GRAECO-LATIN SQUARES 2 (1st version)
TOPIC		Graeco-Latin Squares (order 4)

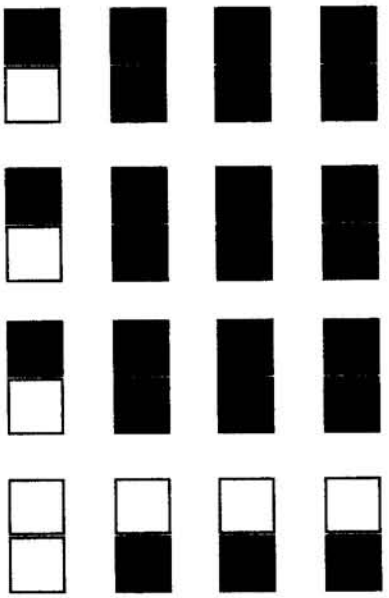
GRAECO-LATIN SQUARES 2

- **Now use the Multilink cubes and pegboard to investigate the 4 x 4 case.**

A tower of 2 cubes is your cup-on-saucer:

Space them out so that you can see them properly.

You can either lay out all the saucer cubes, then add the cups, or build all 16 pairs first and move them around the board.



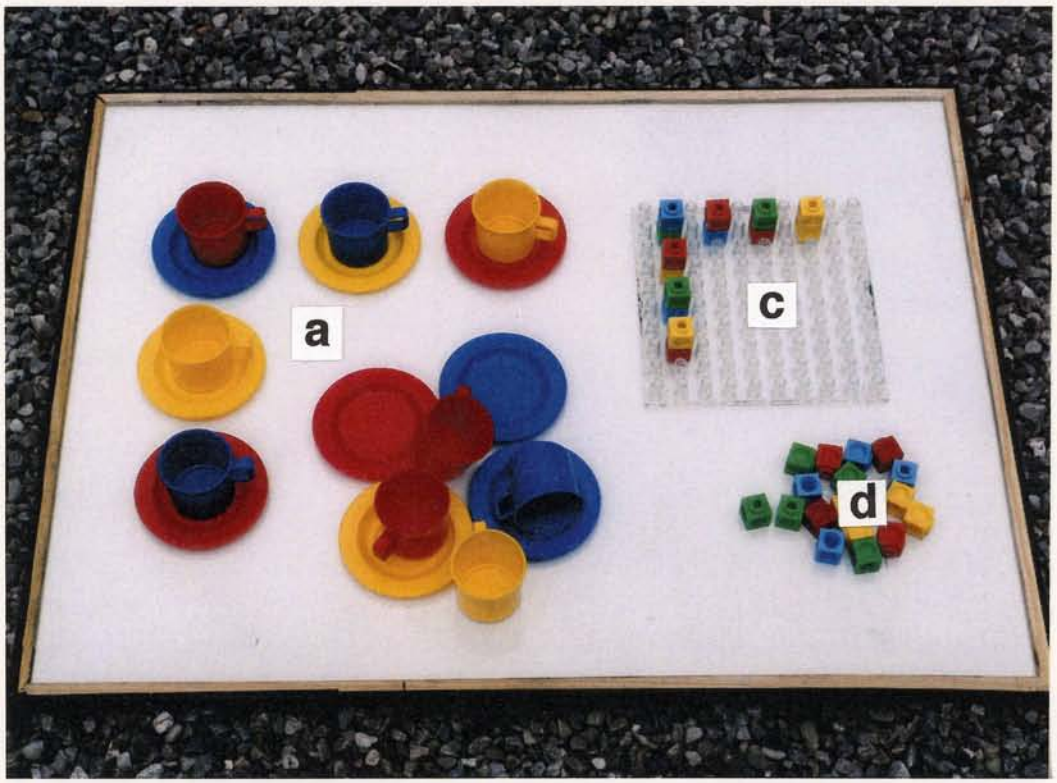
▶ **Here are the 16 combinations as a check-list:**

	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.5.3.2	GRAECO-LATIN SQUARES 2 (2nd version)
TOPIC		Graeco-Latin Squares (order 4)

GRAECO-LATIN SQUARES 2

(A PLAYING CARD ALTERNATIVE)

- **Arrange the 16 cards in a 4 x 4 square so that in each row and in each column you have all four face cards (jack, queen, king, ace) and all 4 suits (clubs, spades, diamonds, hearts).**



b

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
	10.5.1, 10.5.2:		
a	set of plastic cups & saucers of the kind shown		NES Arnold Ltd (address above)
	to be arranged on:		
b	dri-wipe board, 1000 mm x 650 mm, marked as shown		local
	10.5.3.1:		
c	10 x 10 pegboard, specific to type of interlocking cube shown:	Multilink pegboard	NES Arnold Ltd (address above)
d		Multilink	
	10.5.3.2:		
	Use 16 cards from a standard card pack - i.e. without additional lamination. (It will be necessary to replace the originals periodically.)		local
	The cards are most effectively contained and displayed on a pinboard covered with black muslin, 650 mm x 450 mm		

	NUMBER	TITLE
GROUP	10	HEURISTIC
STATION	10.6	A DOMINO PUZZLE
TOPIC		Proceeding according to drawn inferences, avoiding guesswork

A DOMINO PUZZLE

- **The grid shows where the numbers go - but not which dominoes they belong to!**

Try to complete the puzzle.

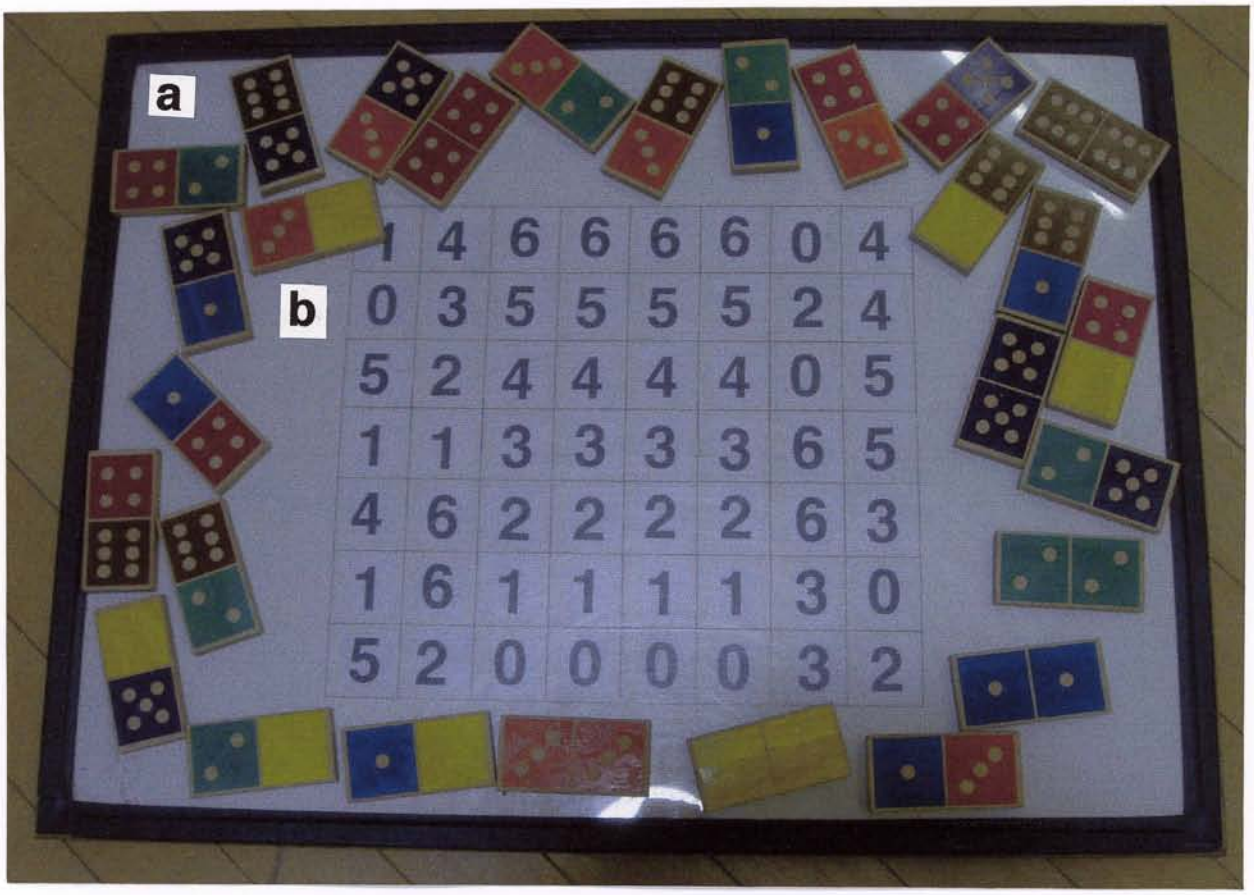
The 'double 4' goes in the top righthand corner.

(Solution on the back of this board)

David Fielker's domino dissection

1	4	6	6	6	6	0	4
0	3	5	5	5	5	2	4
5	2	4	4	4	4	0	5
1	1	3	3	3	3	6	5
4	6	2	2	2	2	6	3
1	6	1	1	1	1	3	0
5	2	0	0	0	0	3	2

Mathematics Teaching Sept. 2004


a
b
1 4 6 6 6 6 0 4
0 3 5 5 5 5 2 4
5 2 4 4 4 4 0 5
1 1 3 3 3 3 6 5
4 6 2 2 2 2 6 3
1 6 1 1 1 1 3 0
5 2 0 0 0 0 3 2

PICTURE KEY	DESCRIPTION	TRADE NAME	U.K. SOURCE
a	dri-wipe board, 1000 mm x 650 mm, marked as shown, faced with 2 mm Glodex		local
b	dominoes, full set, 50 mm edge	Giant Dominoes	Plan Toys (via local toyshop)