Contents

# The Magic Manual

## Section 1

# Multiplication

**A guide for fabricators
and users to stations from the
Magic Mathworks Travelling Circus**

# 1. MULTIPLICATION

The **commutative** nature of multiplication: 4 x 3 = 3 x 4.
12 as a **common multiple** of 3 and 4: **4** x 3 = **3** x 4 = **12**.

## 1.1 HUMAN SENTENCES

**c** Addition and multiplication as **commutative** operations; their inverses, subtraction and division, as **non**-commutative.

**p** Whole-body enaction.

## 1.2 MULTIPLICATION SQUARE JIGSAW

**c** The following features of the operation table for multiplication:

1st jigsaw: alternative factorisations of a product: 24 = 4 x 6 = 3 x 8.
2nd jigsaw: the commutative property represented by the diagonal symmetry.

**p** 1st jigsaw: the puzzle makes the experimenter aware of the above ambiguity, which is resolved by examining surrounding cells and matching outlines: spatial ability can therefore supplement an arithmetic deficiency.
2nd jigsaw: those who know their products need not use the hinged flap; those who do not and who use the flap learn the commutative property by enactment.

## 1.3.1 TABLES TETRAHEDRA

**c** The small number of distinct products on the multiplication square resulting from the 2 properties listed under **1.2**.

**p** The tetrahedral dice must bear only 9 x 4 = 36 products, yet the table contains 9 x 9 = 81 cells.

## 1.3.2 TABLES RACE

**c** As **1.3**.

**p** An expert racer knows that every tetrahedron bears a product s/he needs. Having found what may be, say, the 7th multiple, s/he slots it into place immediately, even though the preceding 6 slots may be empty.

## 1.4 SEESAW

**c** Common multiples.

**p** Rule 2 of the game forces players to recognise that alternative factorisations of their opponents' products are possible.

## 1.5 TIMES CHIMES

**c** Common multiples.

**p** To hear a chord is to have found a common multiple of the numbers the constituent notes represent.

## 1.6 GEAR RATIOS

**c** Common multiples.

**p** Instead of hearing a chord we see the coincidence of the marker lines. The subjects are asked to predict when this will occur.

## 1.6.1 GEAR TRAINS

**c** The product of ratios.

**p** The experimenter must recognise the experimental operation the gears perform.

## 1.7 TWIN TRAINS

**c** Common multiples.

**p** Here we see rod-trains of equal length. This length must be predicted.

## 1.8 A TABLE CLOCK

**c** Common multiples.

**p** Here, cubes of different colour are found on a radius.

## 1.9 MAGIC MASKS

**c** Common multiples

**p** Here, numbers transmitted by a sequence of masks must be common multiples of numbers printed on each. These must be predicted.

## 1.10 CUBOIDS

**c** Prime v. composite numbers.

**p** The successful player gives his/her opponent a prime number of cubes.

## 1.11 EQUIVALENT FRACTIONS

**c** On the multiplication square, cell pairs in corresponding positions yield equal ratios.

**p** An investigation of what 'corresponding' means in the above context. (The students are led to discover that the cell pairs must share the same rows or columns.)

## 1.12 TABLE WINDOWS

**c** Consider 4 cells defining a rectangle on the multiplication square. The product of those defining one diagonal is equal to the product of those defining the other. The same is true for the addition square if the word 'sum' is substituted for 'product'.

**p** The manipulatives employed give the merest hint of the above relation. The exercise therefore provides an opportunity to make conjectures and test them.

| SECTION | | AGE RANGE | | | | | |
|---|---|---|---|---|---|---|---|
| **MULTIPLICATION** | | Appropriate point of entry - not necessarily to the task set by the caption - and levels on which extension activities generated (some to be pursued off-site) | | | | | |
| STATION | | | | | | | |
| NUMBER | NAME | 4 | 7 | 10 | 13 | 16 | 19+ |
| 1.1 | HUMAN SENTENCES | | * | * | | | |
| | | | | | | | |
| 1.2 | MULTIPLICATION SQUARE JIGSAWS | | * | * | | | |
| | | | | | | | |
| 1.3.1 | TABLES TETRAHEDRA | | | * | * | | |
| 1.3.2 | TABLES RACE | | | * | * | | |
| | | | | | | | |
| 1.4 | SEESAW | * | * | * | * | * | |
| | | | | | | | |
| 1.5 | TIMES CHIMES | | | * | * | | |
| | | | | | | | |
| 1.6 | GEAR RATIOS | | | * | * | | |
| 1.6.1 | GEAR TRAINS | | | | * | * | |
| | | | | | | | |
| 1.7 | TWIN TRAINS | | * | * | * | | |
| | | | | | | | |
| 1.8 | A TABLE CLOCK | | | * | * | | |
| | | | | | | | |
| 1.9 | MAGIC MASKS | | | * | * | * | * |
| | | | | | | | |
| 1.10 | CUBOIDS | | | * | * | | |
| | | | | | | | |
| 1.11 | EQUIVALENT FRACTIONS | | | | * | | |
| | | | | | | | |
| 1.12 | TABLE WINDOWS | | | * | * | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |

| | INSTRUCTION NEEDED | | | | SUPERVISION NEEDED | | | SERVICING NEEDED | | |
|---|---|---|---|---|---|---|---|---|---|---|
| → | Needs no explanation or caption | Caption enough for most people | Needs aural prompt | Visitors must be talked through stages | None | Benefits from a helper's input | Session must be directed | None | A little | Much |
| 1.1 | | | * | | | | * | | * | |
| | | | | | | | | | | |
| 1.2 | * | | | | * | | | * | | |
| | | | | | | | | | | |
| 1.3 | | | * | | | * | | * | | |
| 1.3* | | * | | | * | | | * | | |
| | | | | | | | | | | |
| 1.4 | | | * | | | * | | * | | |
| | | | | | | | | | | |
| 1.5 | | | * | | | * | | * | | |
| | | | | | | | | | | |
| 1.6 | | | * | | | * | | | * | |
| 1.6.1 | | * | | | | * | | * | | |
| | | | | | | | | | | |
| 1.7 | | * | | | | * | | * | | |
| | | | | | | | | | | |
| 1.8 | | * | | | | * | | * | | |
| | | | | | | | | | | |
| 1.9 | | * | | | | * | | * | | |
| | | | | | | | | | | |
| 1.10 | | * | | | | * | | * | | |
| | | | | | | | | | | |
| 1.11 | | * | | | | * | | | * | |
| | | | | | | | | | | |
| 1.12 | | * | | | | * | | * | | |
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| | | | | | | | | | | |

| | NUMBER | TITLE |
|---|---|---|
| GROUP | 1 | MULTIPLICATION |
| STATION | 1.1 | HUMAN SENTENCES |
| TOPIC | | The nature of all 4 binary arithmetic operations |

# HUMAN SENTENCES

- This needs 6.

- 5 of you: choose a colour from , , and ; each take a board with that colour on it.
- Number 6: line up the board people correctly:


a
2 3 5 6
b
+ − × ÷
c
=

| PICTURE KEY | DESCRIPTION | TRADE NAME | U.K. SOURCE |
|---|---|---|---|
| **a** | caption board as described | | |
| **b** | home-made board,<br>colour code = 'operation',<br>size comparable to **a** | | |
| **c** | home-made board,<br>colour code = 'equals' verb in mathematical sentence,<br>size comparable to **a** | | |
| **a** | 150 mm numerals, self-adhesive | | local |

| | NUMBER | TITLE |
|---|---|---|
| GROUP | 1 | MULTIPLICATION |
| STATION | 1.2 | 2 MULTIPLICATION SQUARE JIGSAWS |
| TOPIC | | The operation table for multiplication |

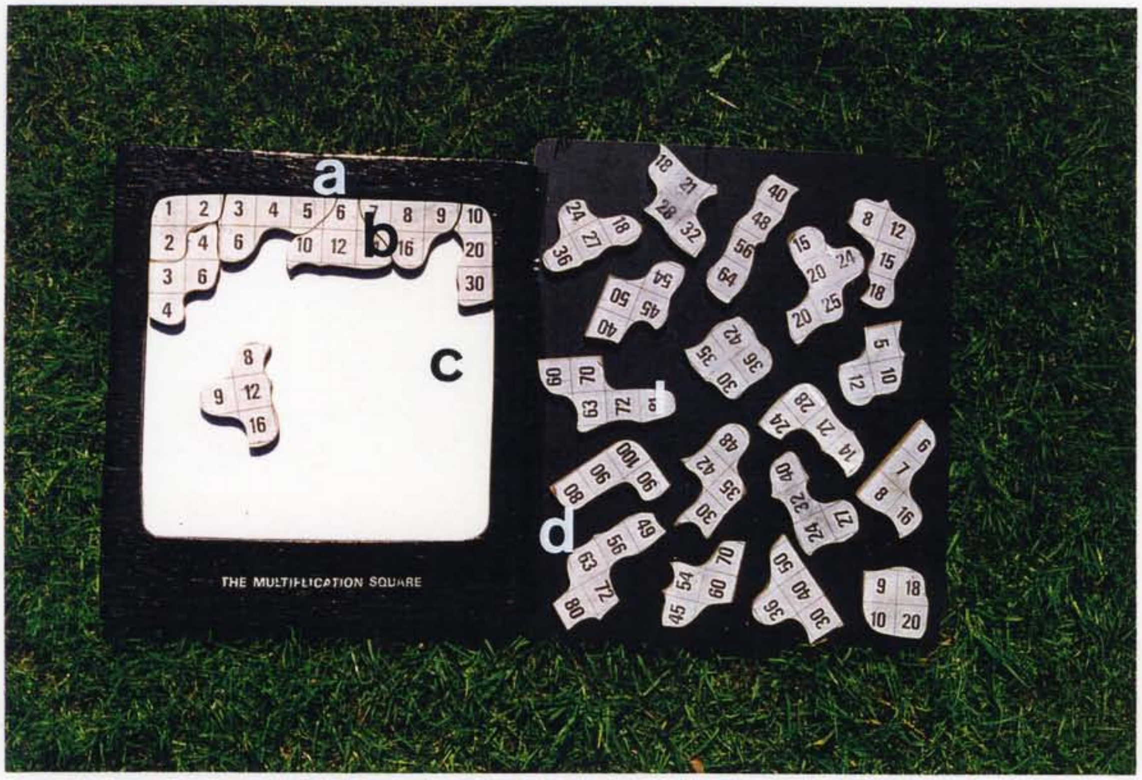
# 2 MULTIPLICATION SQUARE JIGSAWS

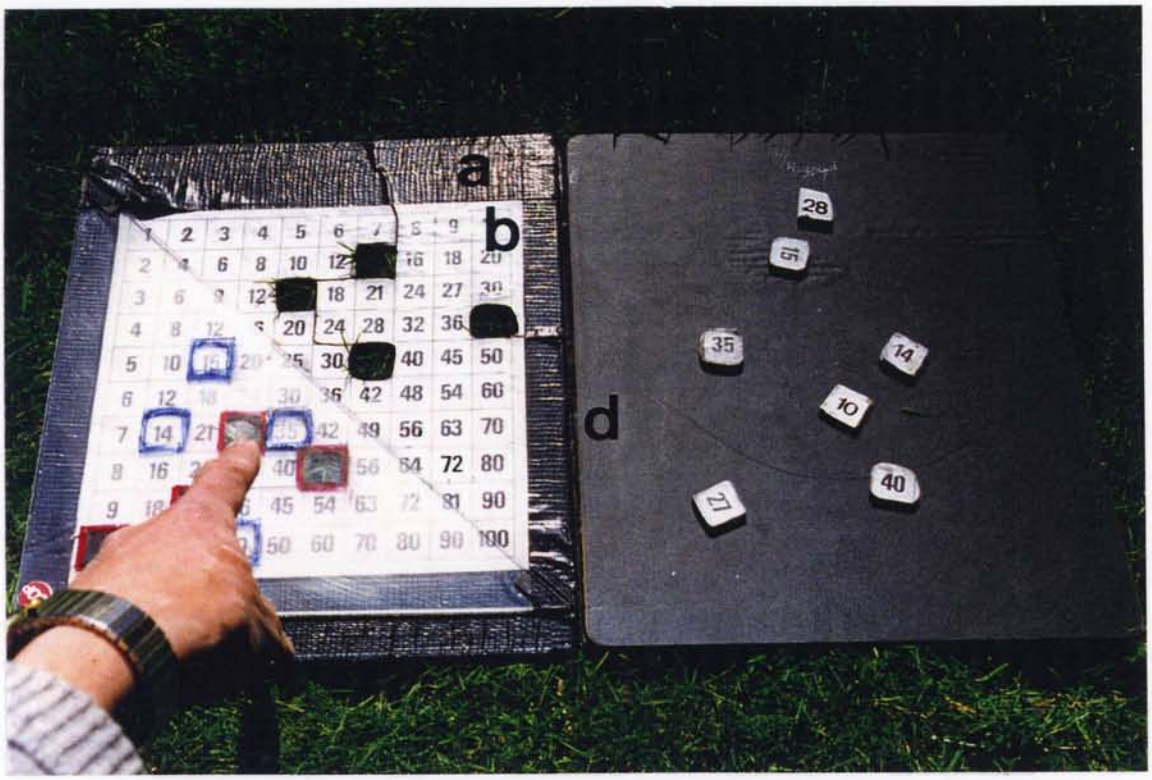
**An ordinary one - HARD**

**The FLIP-FLOP - EASY !**

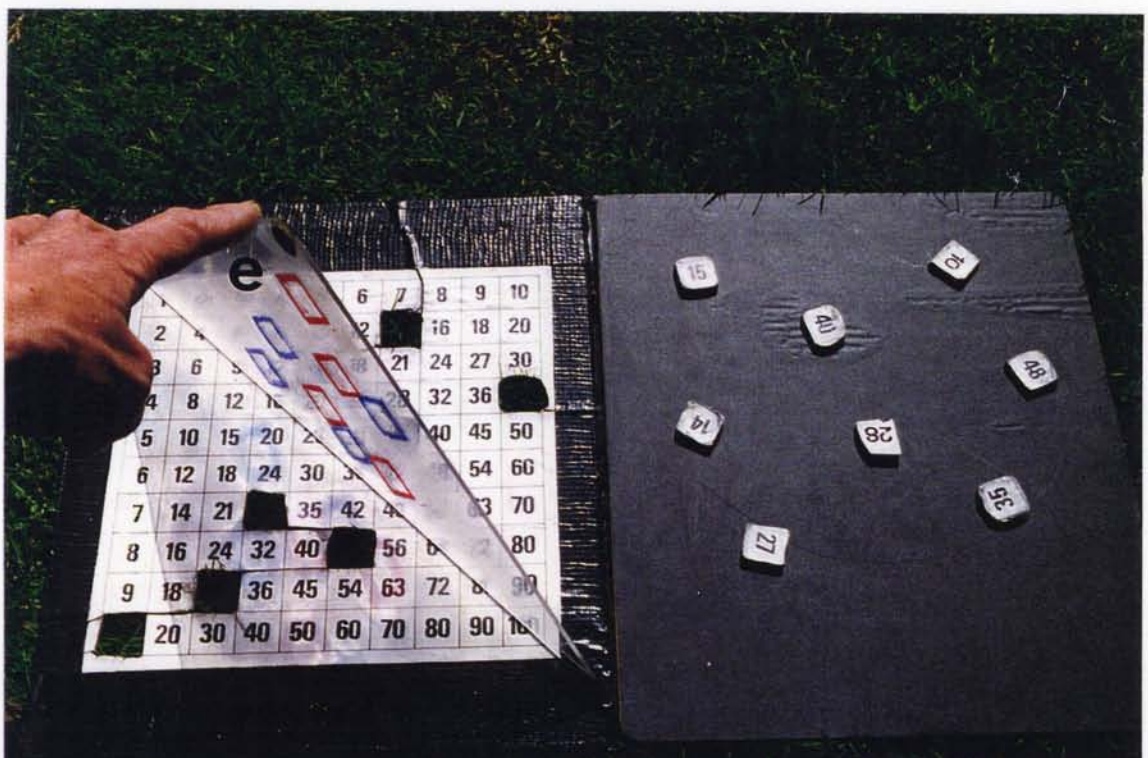
| × | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | ? | | | | | |
| 6 | | | | | | | | | | |
| 7 | | | | | | | | | | |
| 8 | | | | | | | | | | |
| 9 | | | | | | | | | | |
| 10 | | | | | | | | | | |


TIMES
SQUARE


a
b
c
d
THE MULTIPLICATION SQUARE

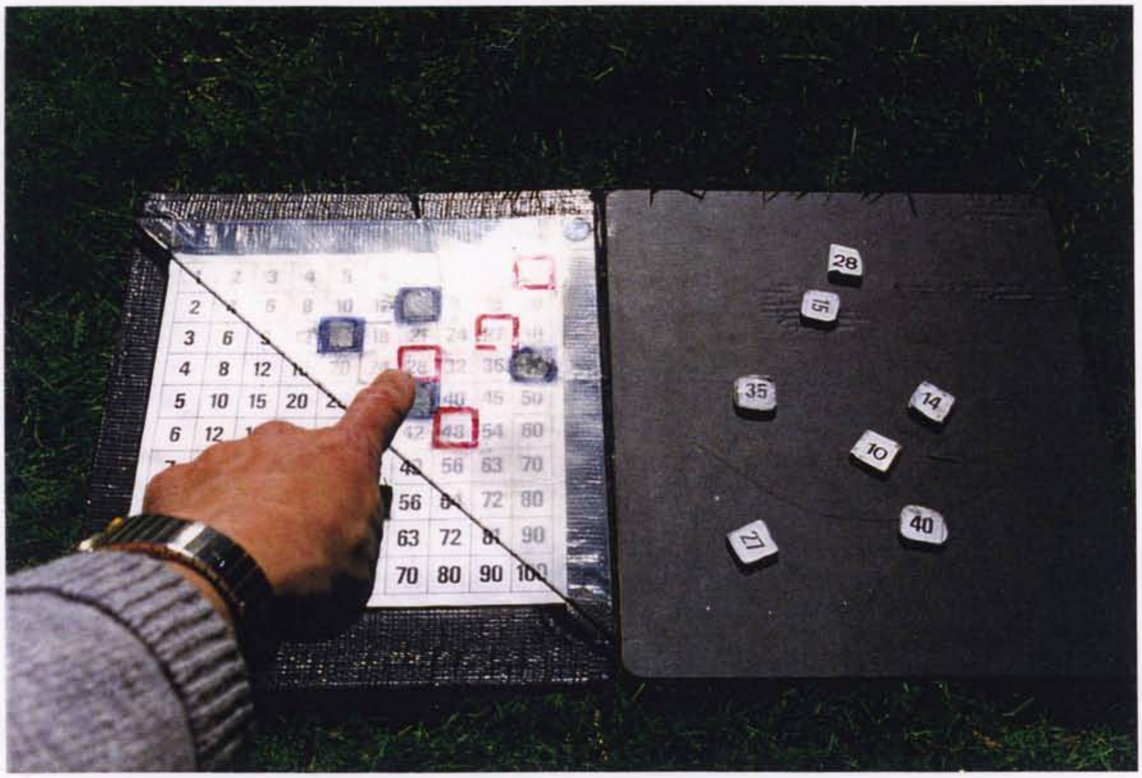



1

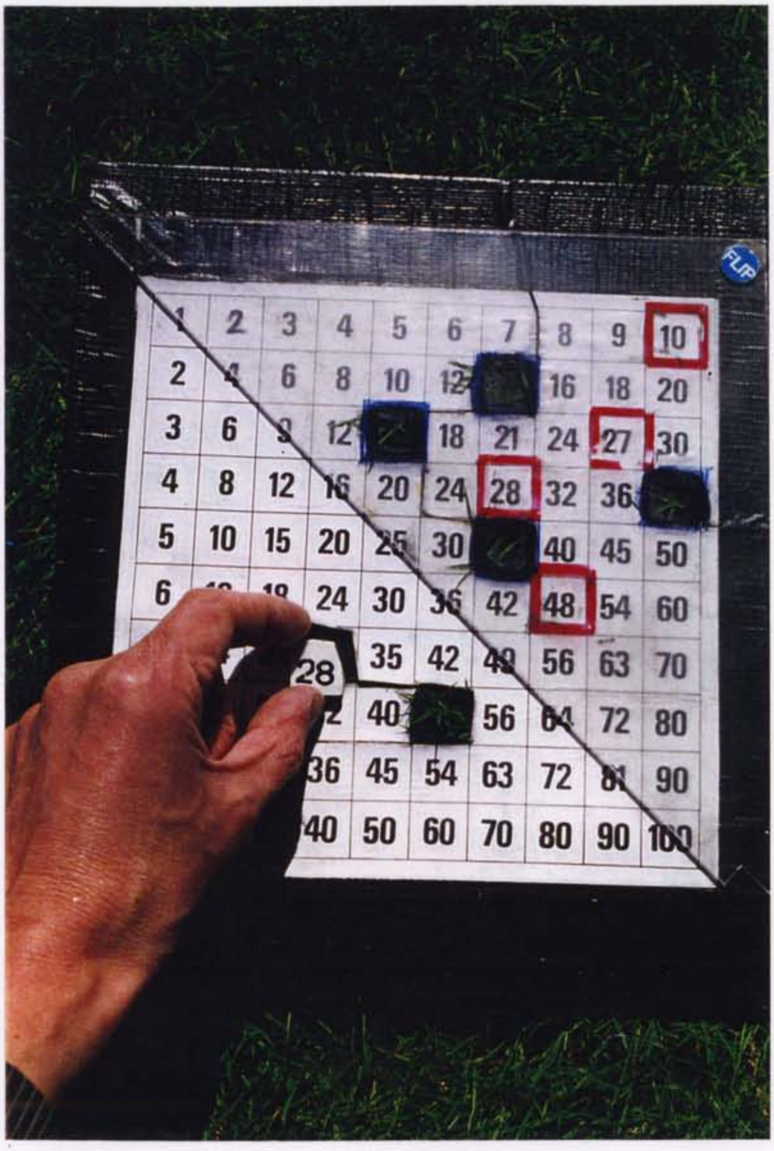



2

3




4



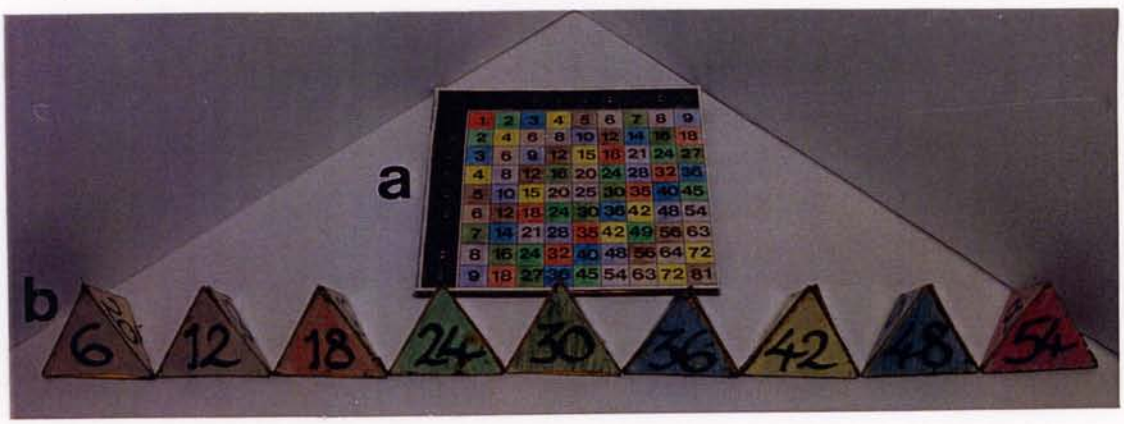
| PICTURE KEY | DESCRIPTION | TRADE NAME | U.K. SOURCE |
|---|---|---|---|
| **a, b** | caption board as described | | |
| **c** | v.s. | | |
| **d** | v.s. | | |
| **a, b** | multiplication square with 20 mm cells photocopied on to white card, stuck on black face of **a/b;** fretwork done; pieces varnished | | |
| **c** | white face stuck to white face of **a** | | |
| **d** | board hinged with fibrous black tape so that, when closed for storage, board order is: **a/b, c, d** | | |
| **e** | Glodex flap hinged to **a** with fibrous black tape | | |
| **1** | product sought for selected square framed on flap in 'flop' position | | |
| **2** | flap pivoted into 'flip' position | | |
| **3** | product sought identified because now framed on **b** | | |
| **4** | corresponding jigsaw piece found and located | | |
| | The sequence of operations is performed in reverse, with the substitution of blue frames for red. | | |

| | NUMBER | TITLE |
|---|---|---|
| GROUP | 1 | MULTIPLICATION |
| STATION | 1.3.1 | TABLES TETRAHEDRA |
| TOPIC | | The multiplication table: how few different products |

# TABLES TETRAHEDRA

- Make all the tables up to 9 x 9.


- How many different numbers are there?


a
1 2 3 4 5 6 7 8 9
2 4 6 8 10 12 14 16 18
3 6 9 12 15 18 21 24 27
4 8 12 16 20 24 28 32 36
5 10 15 20 25 30 35 40 45
6 12 18 24 30 36 42 48 54
7 14 21 28 35 42 49 56 63
8 16 24 32 40 48 56 64 72
9 18 27 36 45 54 63 72 81
b
6
12
18
24
30
36
42
48
54

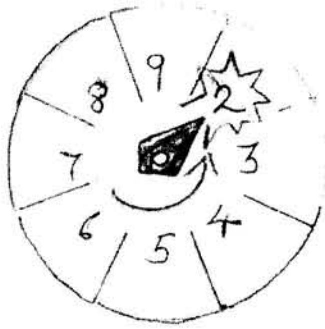
| | | | |
|---|---|---|---|
| **a** | caption board as described with multiplication square - but only up to 9 x 9 - added as above | | |
| **b** | 9 tetrahedra, each coded with a colour corresponding to that used for its 9 cells on **a**; | | |
| | tetrahedra made from triangles of 58 mm edge, in 1.1 mm card; triangles used double-thickness, pairs stuck with white faces outwards then bevelled; numbers added in transfer lettering; colours added in pencil crayon; tetrahedra assembled from same and varnished; | MAT<br>ATM code: MAT 003 | (= Mathematical Activity Tile)<br>Association of Teachers of Mathematics<br>7 Shaftesbury Street<br>Derby DE23 8YB<br>T +44 1332 346599<br>F +44 1332 204357 |
| | tetrahedra numbered thus: | | |

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 18 | 24 | 14 | 15 | 12 | 20 | 16 | 21 | 10 |
| 32 | 45 | 36 | 42 | 56 | 63 | 27 | 25 | 28 |
| 35 | 49 | 40 | 72 | 81 | 64 | 30 | 54 | 48 |

| | NUMBER | TITLE |
|---|---|---|
| GROUP | 1 | MULTIPLICATION |
| STATION | 1.3.2 | TABLES RACE |
| TOPIC | | As 1.3 |

# TABLES RACE

- **Sit opposite each other.**
  - **Each of you has 9 blocks.**
    **Arrange them in the right order in the 9 grooves and you can make each of the 'times tables' up to 9 x 9.**
- **Flick the spinner:**




- **Make that 'times table'.**

# GO!

# Magic Mathworks Information Sheet

## Tables Race

A 9 x 9 multiplication square has 81 cells. But these contain only 36 different numbers. A maths teacher called Jon Millington found that these could be shared between 9 objects in such a way that, by putting them in the correct order and setting them in a row, you could make each row of the 9 x 9 multiplication square - that is to say, each of the 'times tables' up to 9 x 9.

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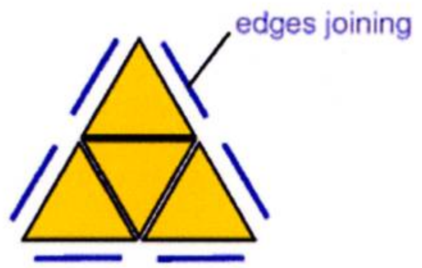
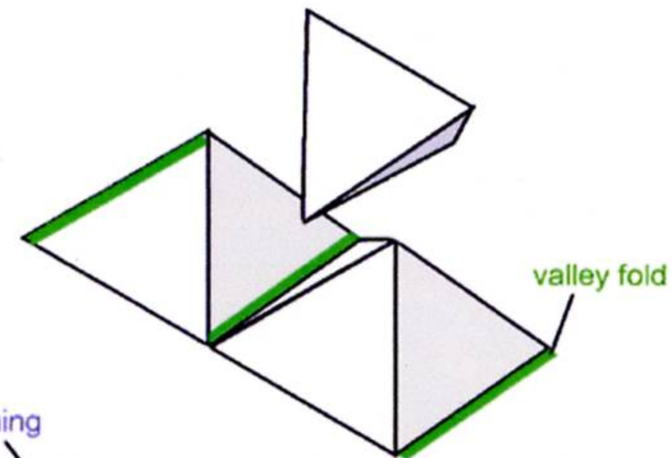
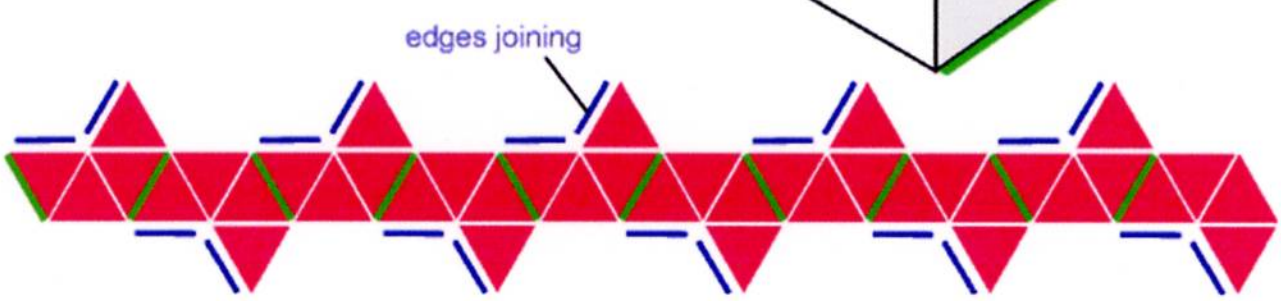
## Making Tables Tetrahedra

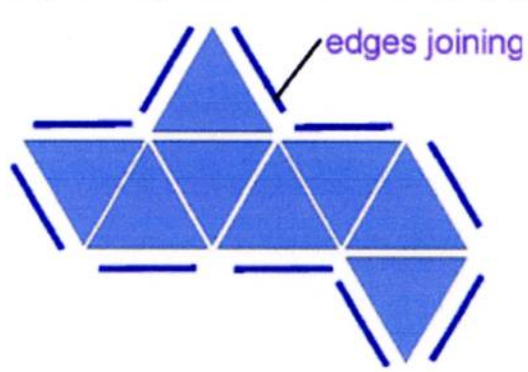
To set up a tables race on a standard school double desk, use tetrahedra (triangular pyramids) and the spaces between 10 square-based pyramids:




Only equilateral triangles are needed. The easiest way to make the solids is from Polydron (or other kit of interconnecting tiles). Use yellow for the tetrahedra so that the numbers show up. Use red (say) for the pyramid chain. Make 2 sets so that, instead of racing against the clock, you can race against an opponent. Sit at the long sides of the desk, facing each other across it. You each make your times table so that it faces you. In place of the spinner, make a die with 8 faces. The die is a regular octahedron, needing 8 more triangles - blue (say). Sellotape the edges so that it doesn't break up when thrown:




Altogether therefore you will need:

2 x 9 x 4 = 72 yellow triangles,

2 x 10 x 4 = 80 red triangles,
1 x 8 = 8 blue triangles.

Use dri-wipe pen for the numbers.

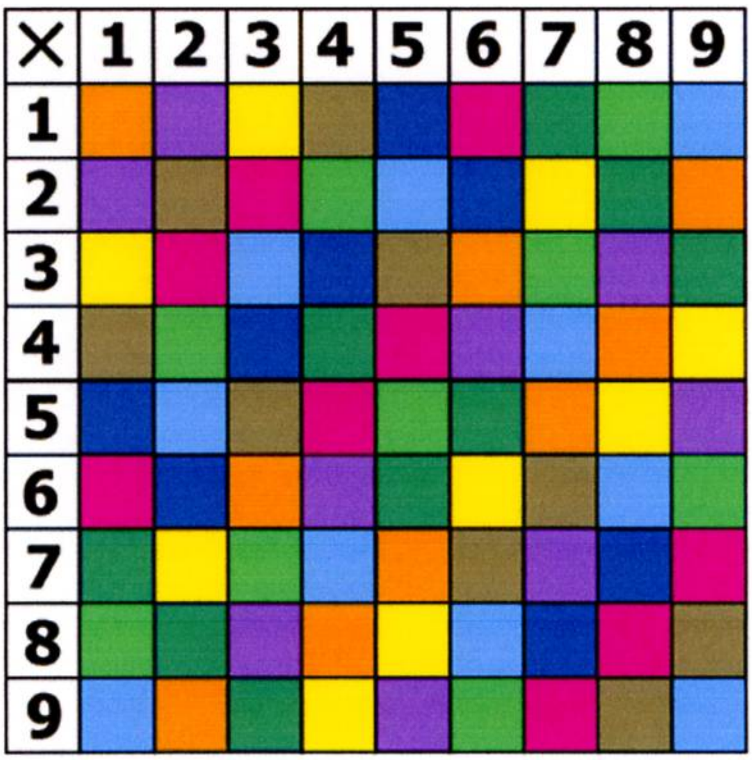
Here is one way to distribute the numbers between the tetrahedra:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 18 | 24 | 14 | 15 | 12 | 20 | 16 | 21 | 10 |
| 32 | 45 | 36 | 42 | 56 | 63 | 27 | 25 | 28 |
| 35 | 49 | 40 | 72 | 81 | 64 | 30 | 54 | 48 |

## Designing Tables Tetrahedra

How can the 36 numbers be allocated to their 9 tetrahedra?
What conditions must be satisfied?

Give each tetrahedron a colour:



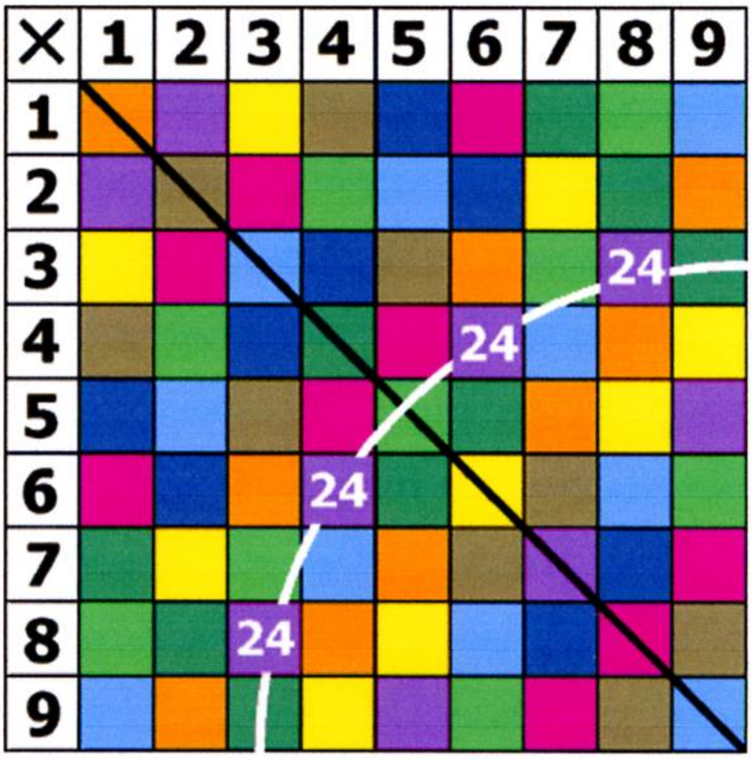

On the multiplication square each tetrahedron must occur exactly once in every row and column: in other words, the coloured cells must form a 'Latin' square. As a consequence, each tetrahedron is represented 9 times on the square. The 4 distinct face account for 4 of these cells. The remaining 5 are accounted for in two ways:

1. the commutative property means that every cell is reflected in the main diagonal: 4 x 6 = 6 x 4.

2. some numbers factorise in more than one way: 4 x 6 = 3 x 8.

The number 24, then, occurs 4 times in different guises: as 4 x 6, 6 x 4, 3 x 8, 8 x 3.

And these cells lie on a hyperbola:




Squares lie on the main diagonal of the multiplication square, which is to say that the commutative property reflects a square on to itself. This has an important consequence:
each non-square occurs an **even** number of times;
each square occurs an **odd** number of times.

Because each tetrahedron is repeated 9 times - an odd number - and only squares can contribute odd numbers, there must be exactly one square on each tetrahedron.

The squares fall into 2 sets: those with just 1 entry on the multiplication square and those with 3.

For those with 1 entry, the remaining 3 faces of the tetrahedron must account for 9 - 1 = 8 entries, each of which must be an even number. The only solution in positive integers to that equation is: 8 = 2 + 2 + 4. 2 numbers must therefore occur twice but one 4 times.

For those with 3 entries, the remaining 3 faces account for 9 - 3 = 6 entries, splitting 2-2-2.

The numbers which occur 4 times are: 6, 8, 12, 18, 24. These must be paired in some order with the corresponding squares: 1, 25, 49, 64, 81. Some pairings fail: the numbers lie in the same row or column and must therefore be given different colours: 1-6, 1-8, 8-64, 24-64, 18-81. Even so, many possibilities remain open at this stage. But every pair of numbers you put on a tetrahedron must pass the 'colour' test. So, the more numbers you add to each, the harder the task becomes. This difficulty is compounded by the fact that you have fewer left to choose from.

The problem is so difficult you may decide to program a computer to find the 9 sets of numbers for you. This is what Jon Millington did.

If you want to tackle it by hand, write each number on a separate card and make a 9 x 4 grid to put them on. You can fix one number on each tetrahedron by choosing one of the 9 tables - say the 1s: 1, 2, 3, 4, 5, 6, 7, 8, 9 - or the set of squares.

Surprisingly, there is more than one solution. One has been chosen for the Tables Race. Can you find another?

---


a
TABLES RACE
Sit opposite each other.
Each of you has 9 blocks. Arrange them in the right order in the 9 grooves and you can make each of the 'times tables' up to 9 x 9.
Flick the spinner.
Make that 'times table'.
GO!

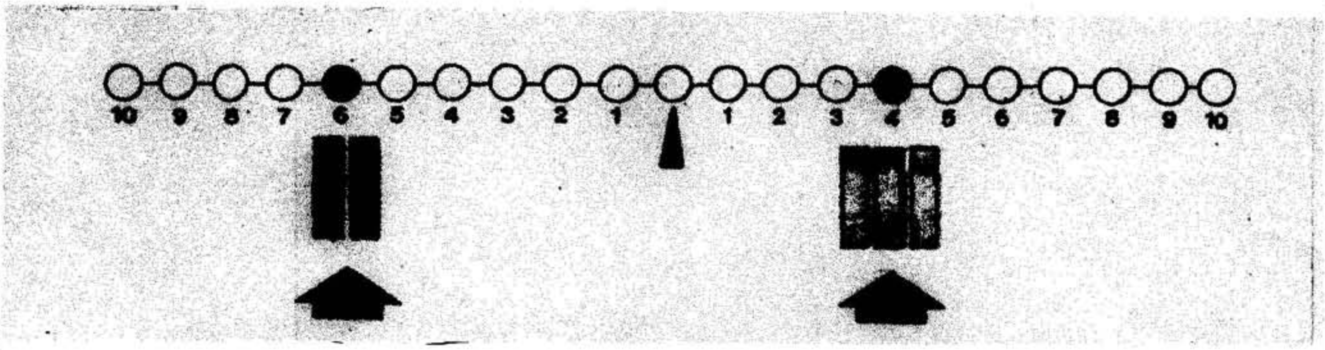
| PICTURE KEY | DESCRIPTION | TRADE NAME | U.K. SOURCE |
|---|---|---|---|
| **a** | interlocking plastic triangles | Polydron | Polydron International Ltd (address above) |

| | NUMBER | TITLE |
|---|---|---|
| GROUP | 1 | MULTIPLICATION |
| STATION | 1.4 | SEE-SAW |
| TOPIC | | Identifying common multiples revealed by 1.3 |

# SEE-SAW

- This is a game for 2:
  BLUE on the left, YELLOW on the right.

- An example:




BLUE's challenge

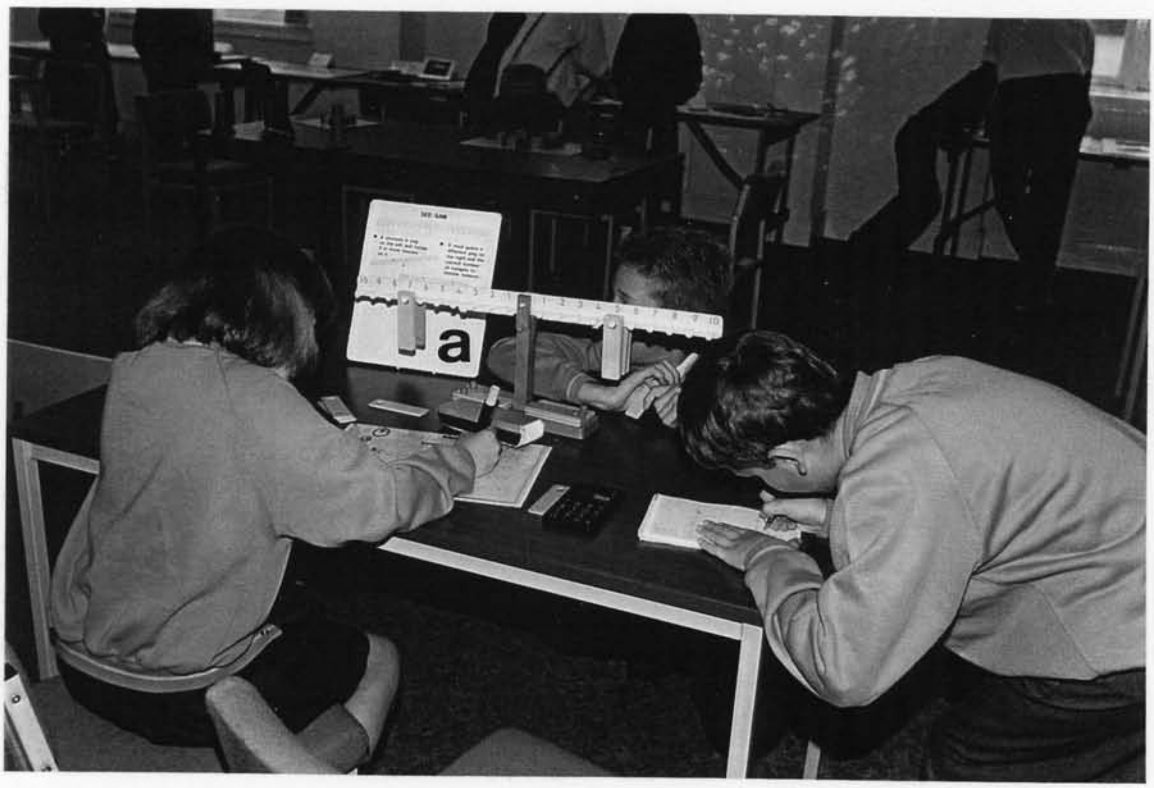
YELLOW's reply

The beam balances.

YELLOW wins a point.

Now YELLOW issues a challenge...

- Suggested rules:
  1. The challenger must use 2, 3 or 4 hangers, 1 peg.
  2. Her opponent must use a different peg.
  3. She only has 1 chance to balance the beam!

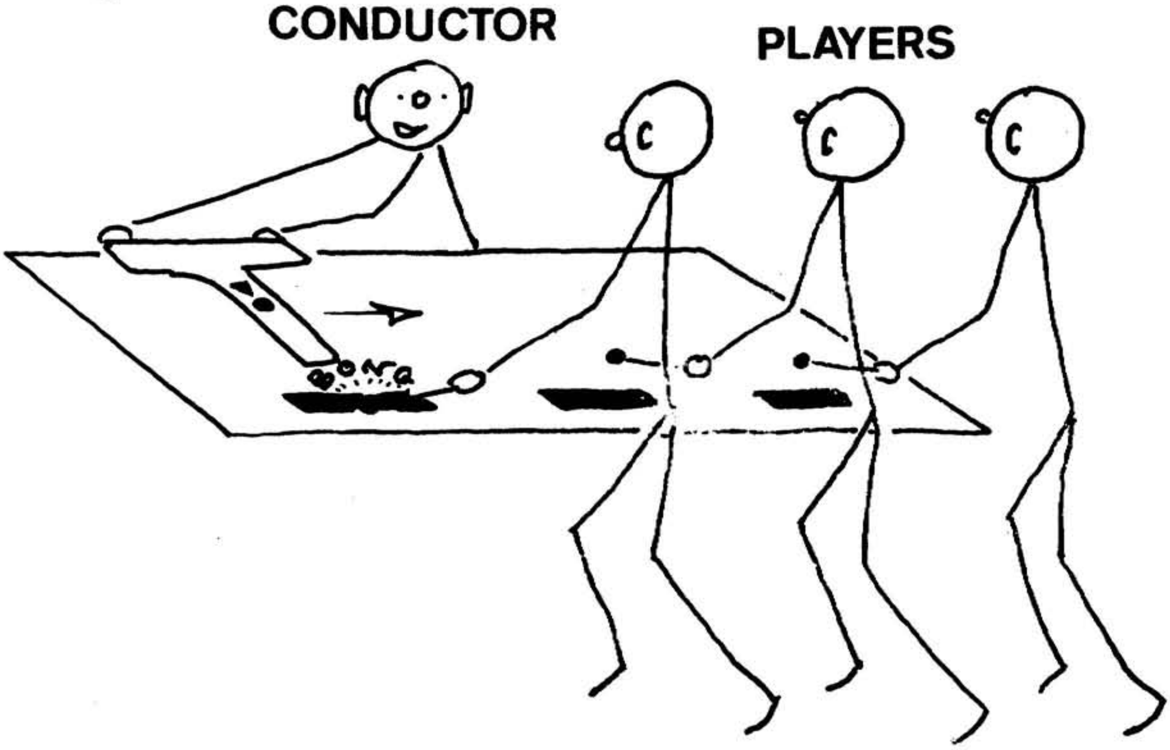

a

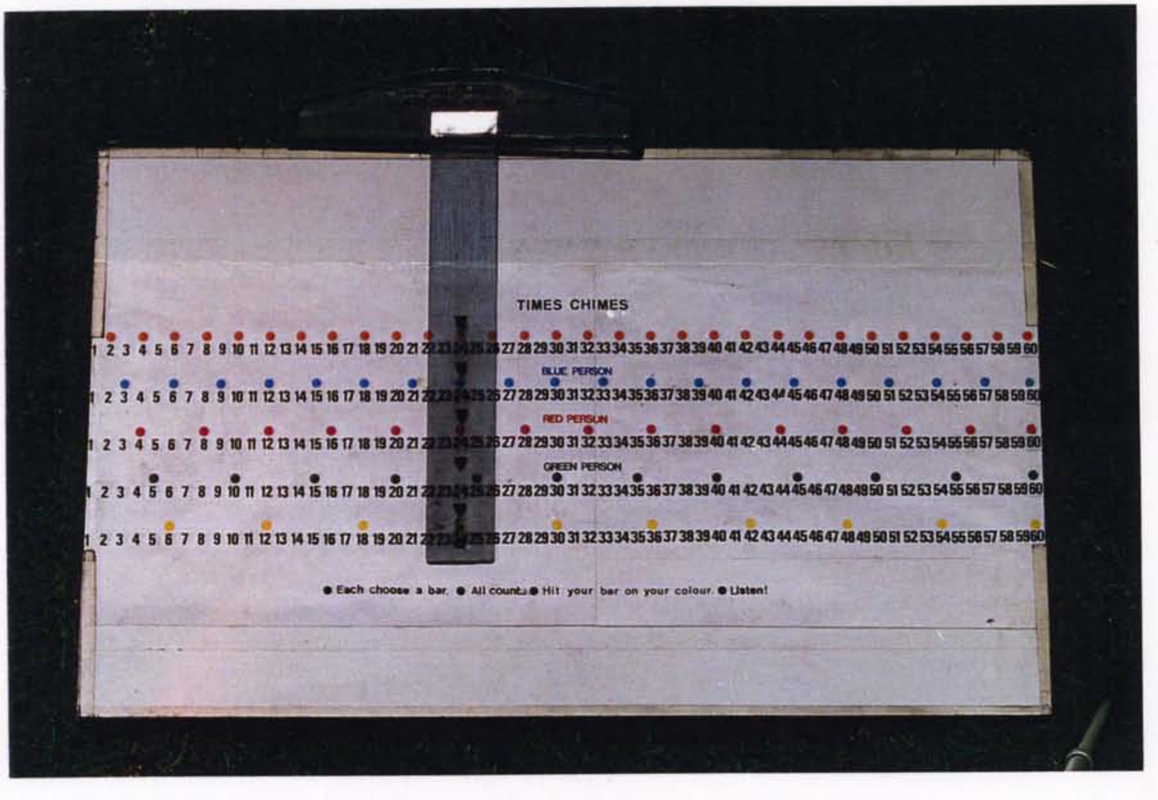
| PICTURE KEY | DESCRIPTION | TRADE NAME | U.K. SOURCE |
|---|---|---|---|
| a | mathematical balance<br>The hangers supplied with the model cited are in 1 colour. If you wish to distinguish the opponents in the game by colour, (as suggested by the caption, though not strictly necessary), you must therefore buy 2. | Equalising balance<br>NES Arnold catalogue: SX055/5 | NES Arnold Ltd (address above) |

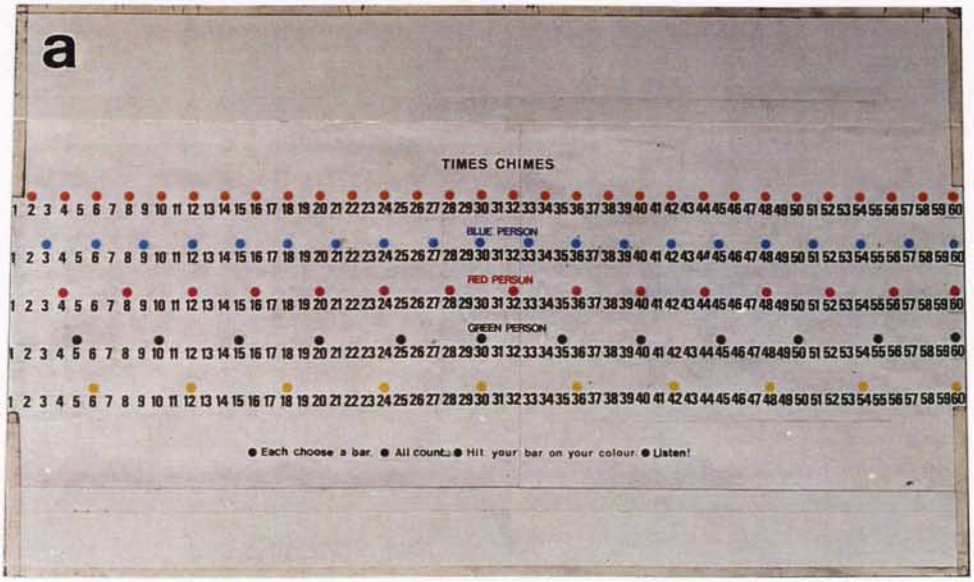
| | NUMBER | TITLE |
|---|---|---|
| GROUP | 1 | MULTIPLICATION |
| STATION | 1.5 | TIMES CHIMES |
| TOPIC | | As 1.4 |

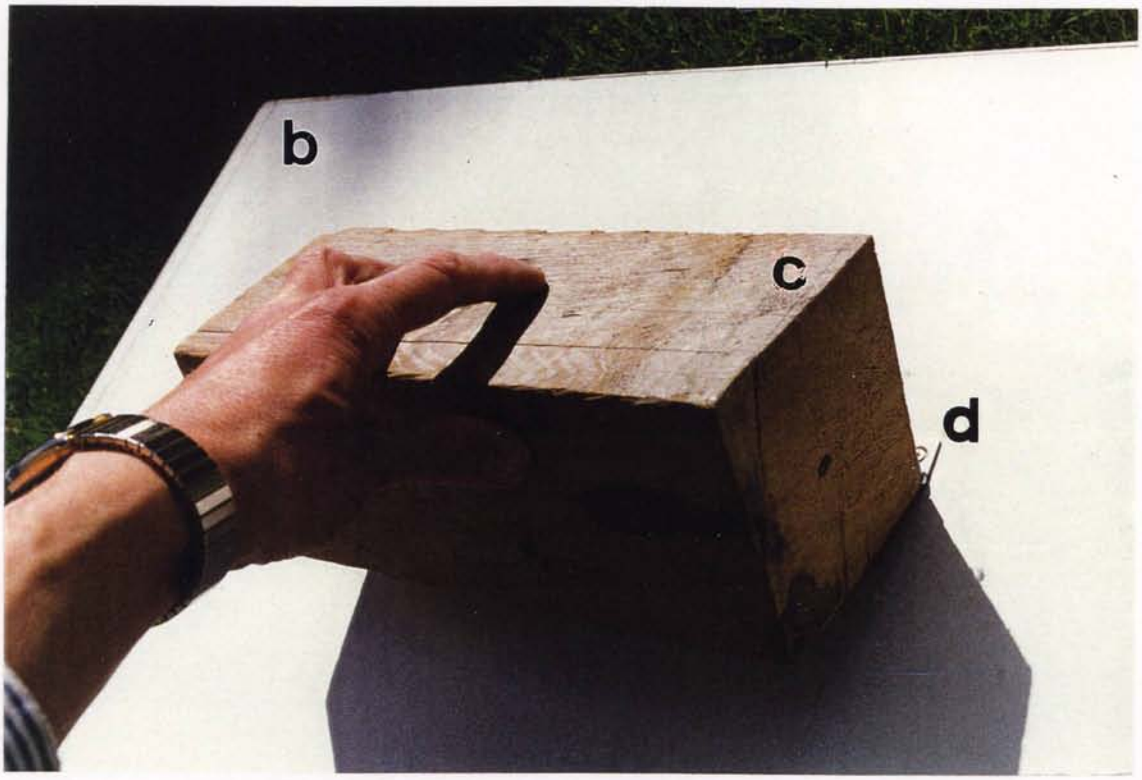
# TIMES CHIMES

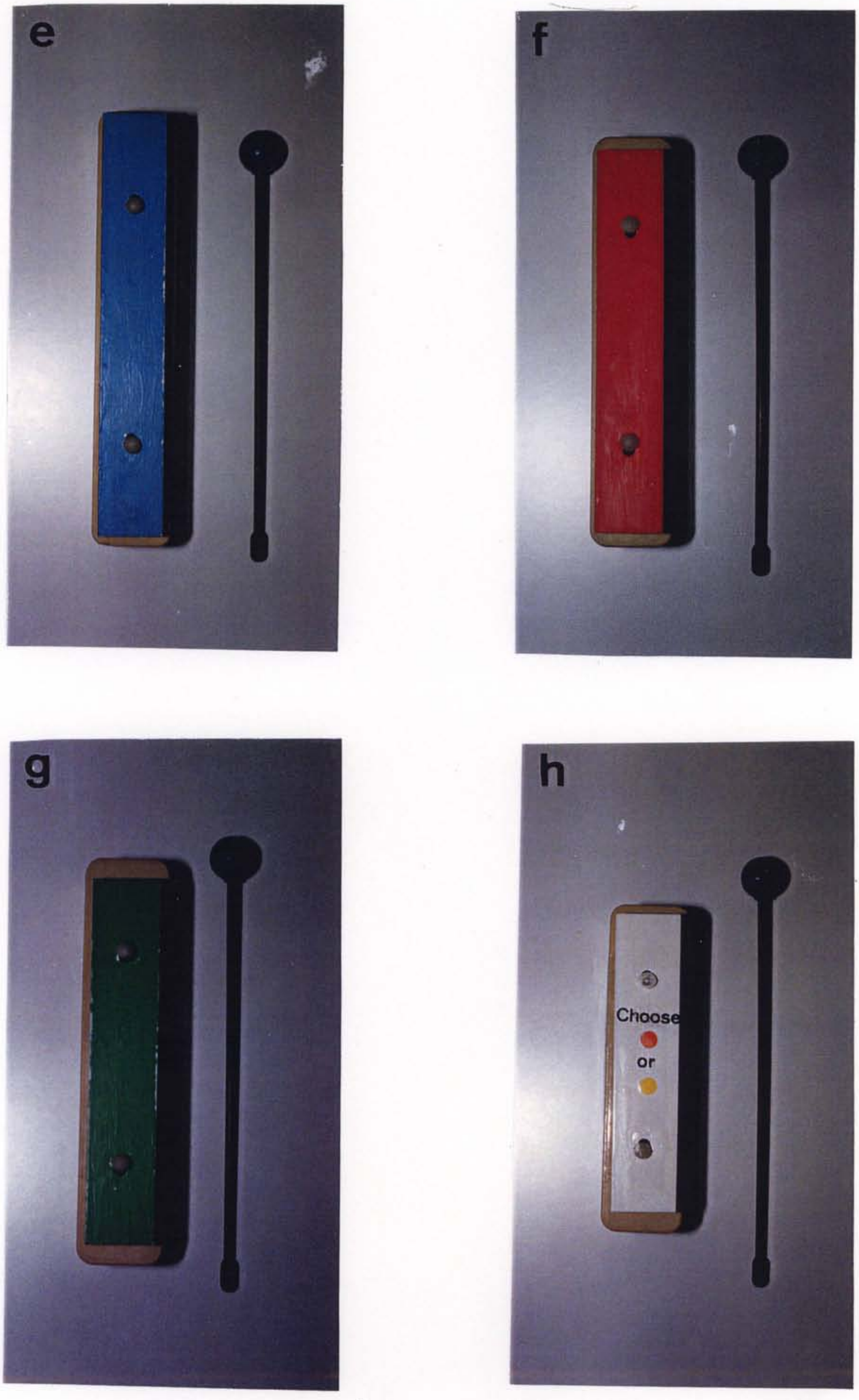
- **CONDUCTOR,**
  **drag the counting stick from 1 to 60, counting aloud.**
- **PLAYERS,**
  **when the stick points to your colour, hit your bar.**




TIMES CHIMES
BLUE PERSON
RED PERSON
GREEN PERSON
● Each choose a bar. ● All count. ● Hit your bar on your colour. ● Listen!


a
TIMES CHIMES
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60
BLUE PERSON
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60
RED PERSON
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60
GREEN PERSON
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60
● Each choose a bar. ● All count ● Hit your bar on your colour ● Listen!


b
c
d

e
f
g
h
Choose
or

i

| PICTURE KEY | DESCRIPTION | TRADE NAME | U.K. SOURCE |
|---|---|---|---|
| **a** | | | |
| **b** | back of **a** | | |
| | **a**, **b**: 2 sheets of white-faced hardboard stuck back-to-back, 600 mm x 450 mm, marked with number lines 1 to 60 as shown | | local |
| **c** | wooden block | | local |
| **d** | 1 of 2 hinges in corresponding positions at opposite ends of block | | local |
| | **c/d** removed from **b** for transport | | |
| **e - h** | chime bars, painted/marked as shown; suggested pitches: | NES Arnold catalogue: PP 932/8 | NES Arnold Ltd (address above) |
| | **e**: C | | |
| | **f**: E | | |
| | **g**: G | | |
| | **h**: C' | | |
| **i** | T-square, marked as shown: the arrows are positioned to point to the coloured dots on **a**; the circles, to ring the corresponding numbers. | | local |
| | **c/d** supports **a/b** at a shallow angle (around 30°), thus: | | |
| | 1. improving visibility of **a** for 'players', | | |
| | 2. enabling the 'conductor' to rest **i** on the upper edge as s/he drags it along. | | |

| | NUMBER | TITLE |
|---|---|---|
| GROUP | 1 | MULTIPLICATION |
| STATION | 1.6 | GEAR RATIOS |
| TOPIC | | As 1.4 |

# GEAR RATIOS

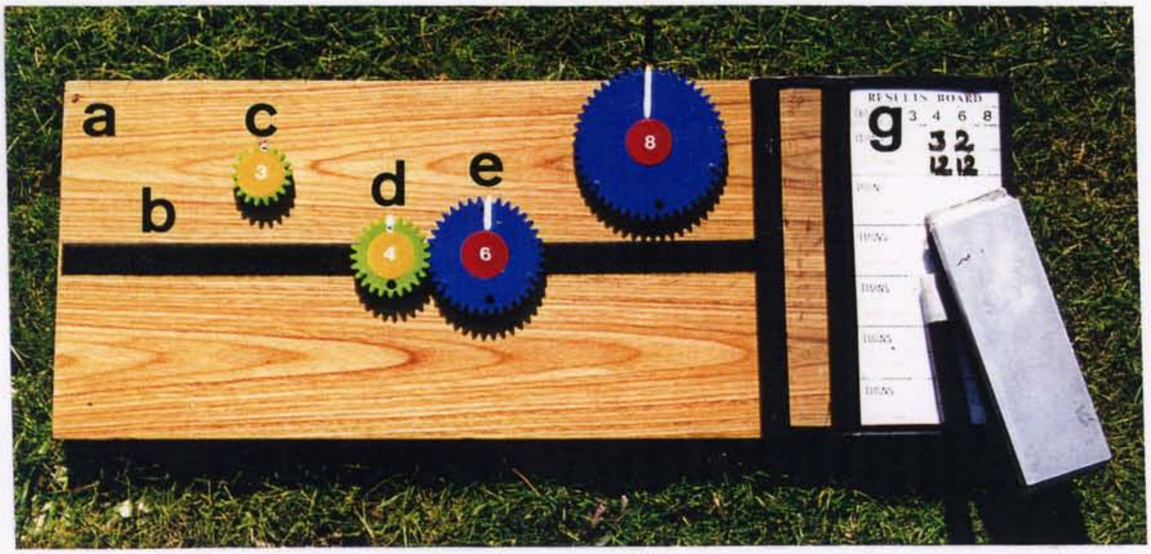
- Choose 2 wheels.
- Mesh them on the velcro with their white lines pointing upwards.

■ How many turns will each make before both white lines point upwards again?

- Test your guess.

- Choose another pair.

- Use the RESULTS table to help you.




g

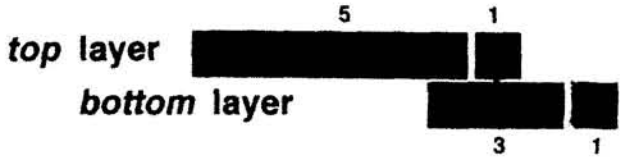
| RESULTS BOARD | | | | |
|---|---|---|---|---|
| TEETH (scaled down x 6) | 3 | 4 | 6 | 8 |
| TURNS | | | | |
| TEETH X TURNS | | | | |
| TURNS | | | | |
| TEETH X TURNS | | | | |
| TURNS | | | | |
| TEETH X TURNS | | | | |
| TURNS | | | | |
| TEETH X TURNS | | | | |
| TURNS | | | | |
| TEETH X TURNS | | | | |

| PICTURE KEY | DESCRIPTION | TRADE NAME | U.K. SOURCE |
| --- | --- | --- | --- |
| **a** | baseboard (veneered 18 mm Contiboard = reconstituted wood), 450 mm x 200 mm | | local |
| **b** | 'loop' velcro | | local |
| **c - f** | gears: outer part pivots about hub backed with 'hook' velcro;<br>markings as shown;<br>numbers of teeth as follows:<br>**c**: 18 (= 6 x **3**)<br>**d**: 24 (= 6 x **4**)<br>**e**: 36 (= 6 x **6**)<br>**f**: 48 (= 6 x **8**) | | from toy no longer available in the U.K. designed by Israeli Ivan Moscovich;<br>it is available, however, from:<br>Regev Games<br>Kibbutz Regavim<br>D N Menashe<br>37820 Israel<br>T +972 6 307826<br>F +972 6 389929<br>under the name 'Gob's Gears'.<br>Only bulk orders are taken.<br>The Magic Mathworks has a few sets from this source which it is prepared to sell at the purchase price + p & p. |
| **g** | Glodex-covered sheet, marked as shown | | |

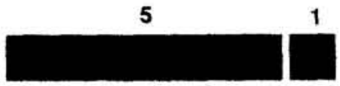
|  | NUMBER | TITLE |
|---|---|---|
| GROUP | 1 | MULTIPLICATION |
| STATION | 1.6.1 | GEAR TRAINS |
| TOPIC |  | Application of 1.6 |

# GEAR TRAINS

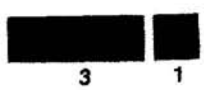
- **Look at the gear train from the side:**




**The gears in the *top* layer work like the gears in 'GEAR RATIOS';**

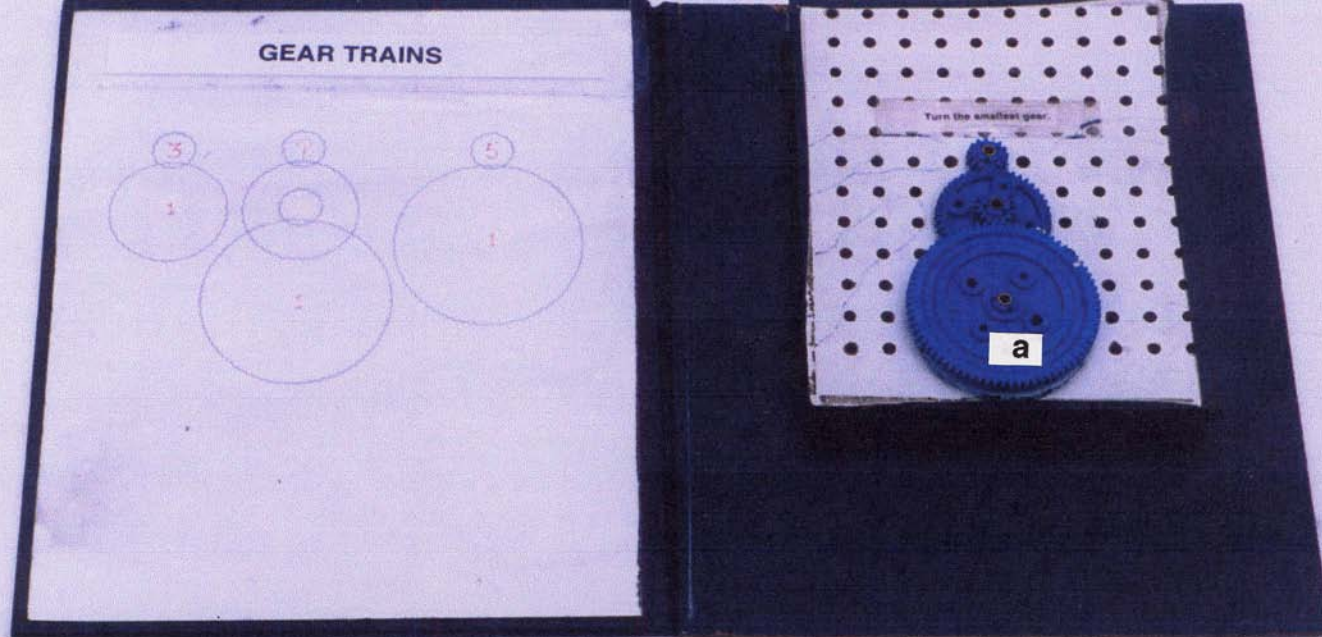


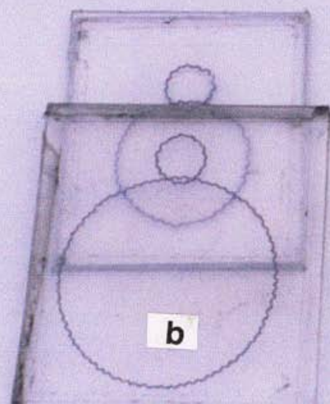

**the gears in the *bottom* layer work like the gears in 'GEAR RATIOS'.**



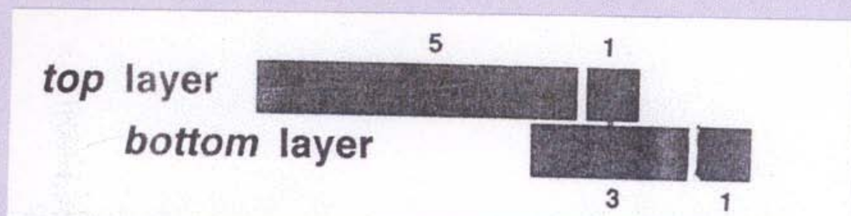

**But in a gear train a gear in the *top* layer is fixed to a gear in the *bottom* layer. In this one, the '1' gear in the top layer is fixed to the '3' gear in the bottom layer.**

- **Experiment with the gears, experiment with the perspex overlays with diagrams of the gears and decide what number should be written in place of the '?', i.e. what the *overall* gear ratio is.**




the gear train from the side:



| PICTURE KEY | DESCRIPTION | TRADE NAME | U.K. SOURCE |
| --- | --- | --- | --- |
| a | gears (ratios as fig.) from kit (designed specially for the Mechanics in Action project) | | Unilab (address above) |
| b | overlays matching respectively the upper, right gear pair and the lower, left gear pair | | |

| | NUMBER | TITLE |
|---|---|---|
| GROUP | 1 | MULTIPLICATION |
| STATION | 1.7 | TWIN TRAINS |
| TOPIC | | As 1.4 |

# TWIN TRAINS

▶ Here is an example to show you what to do.

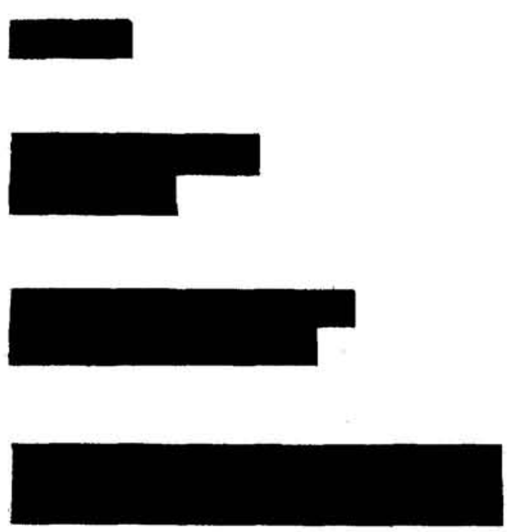
Your partner:

"A pale green and a purple train are the same length but their coaches are not. How many coaches long must each be?"

You:

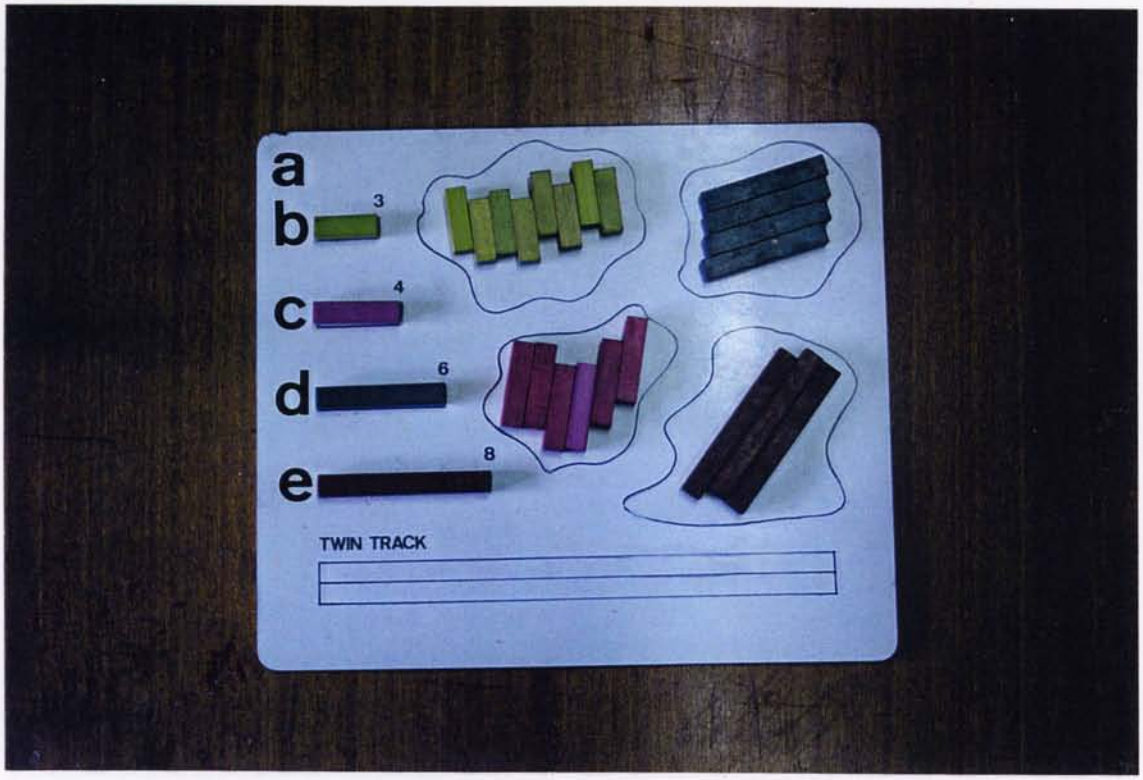
"Pale green: 4. Purple: 3."

The TWIN TRACK test:



● Now try other colour pairs.

■ (How many are possible?)

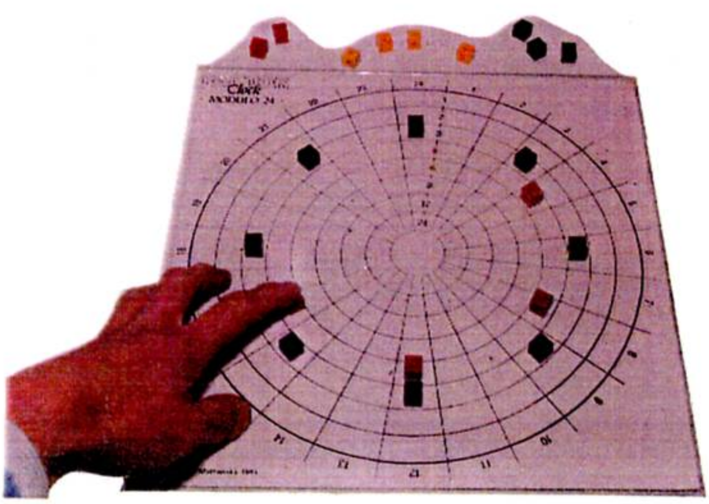

a
b
3
c
4
d
6
e
8
TWIN TRACK

| PICTURE KEY | DESCRIPTION | TRADE NAME | U.K. SOURCE |
|---|---|---|---|
| a | caption board as described, marked as shown | | |
| b - e | Cuisenaire rods, lengths in cm as marked, stuck to board;<br>to right, same loose in their Venn circles for use on TWIN TRACK | NES Arnold catalogue: SY 358/4 | NES Arnold Ltd (address above) |

| | NUMBER | TITLE |
|---|---|---|
| GROUP | 1 | MULTIPLICATION |
| STATION | 1.8 | A TABLE CLOCK |
| TOPIC | | As 1.4 |

# A TABLE CLOCK

- **Place:**
  - a blue cube every 3rd box in the '3' ring,
  - a red cube every 4th box in the '4' ring,
  - a yellow cube every 6th box in the '6' ring,
  - a green cube every 8th box in the '8' ring:

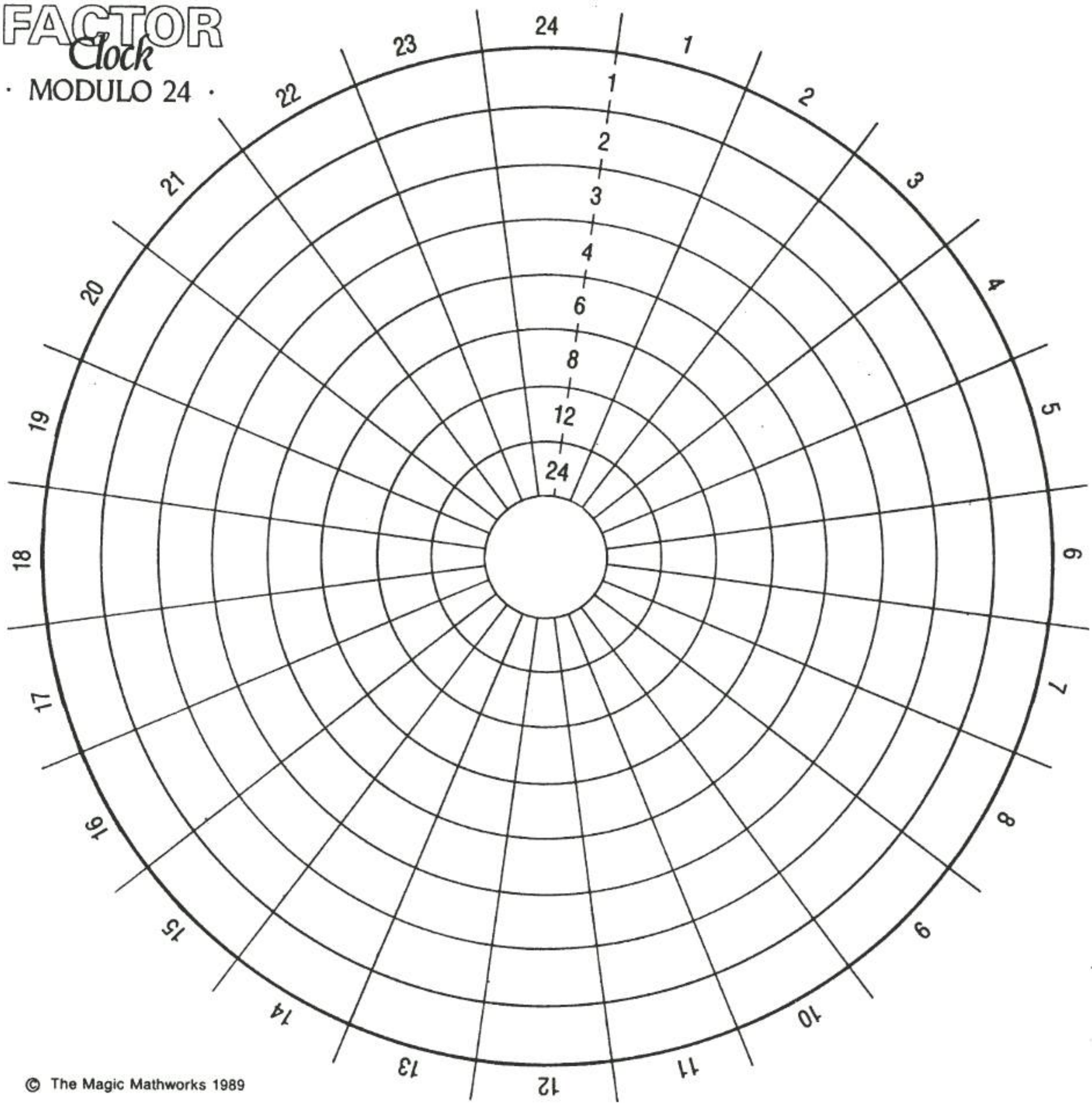


- **Where do they line up:**
  - in 2s?
  - in 3s?
  - in 4s?

**a**

FACTOR Clock

· MODULO 24 ·



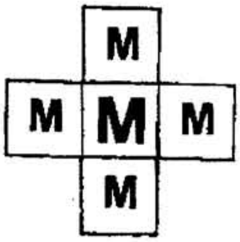
| PICTURE KEY | DESCRIPTION | TRADE NAME | U.K. SOURCE |
|---|---|---|---|
| a | same enlarged to 300 mm x 300 mm and sandwiched between Glodex squares of same size | | |
| | **Shown on caption only:**<br>1 cm cubes | Centicube<br>Economatics catalogue: 08200<br>(formerly Osmiroid catalogue: 8200) | Economatics Ltd<br>Epic House<br>Darnall Road<br>Attercliffe<br>Sheffield S13 9NP<br>T +44 1142 813344 |

| | NUMBER | TITLE |
|---|---|---|
| GROUP | 1 | MULTIPLICATION |
| STATION | 1.9 | MAGIC MASKS |
| TOPIC | | As 1.4 |

# MAGIC MASKS

● Start with all the flaps folded out:

**M** multiplication square
M multiple mask



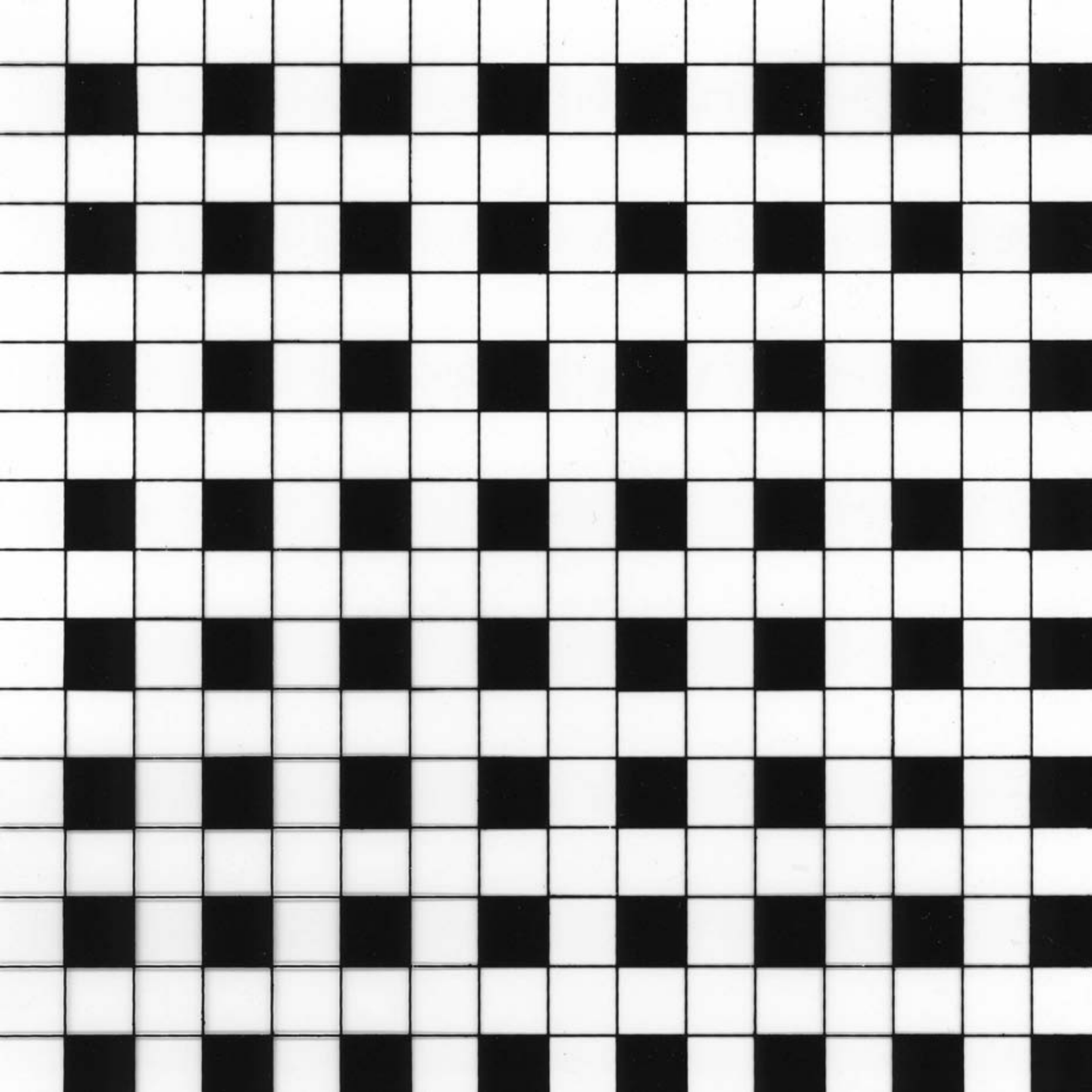

● Fold the flaps in one at a time.
● Notice the number on each.
■ What do you see through the windows?

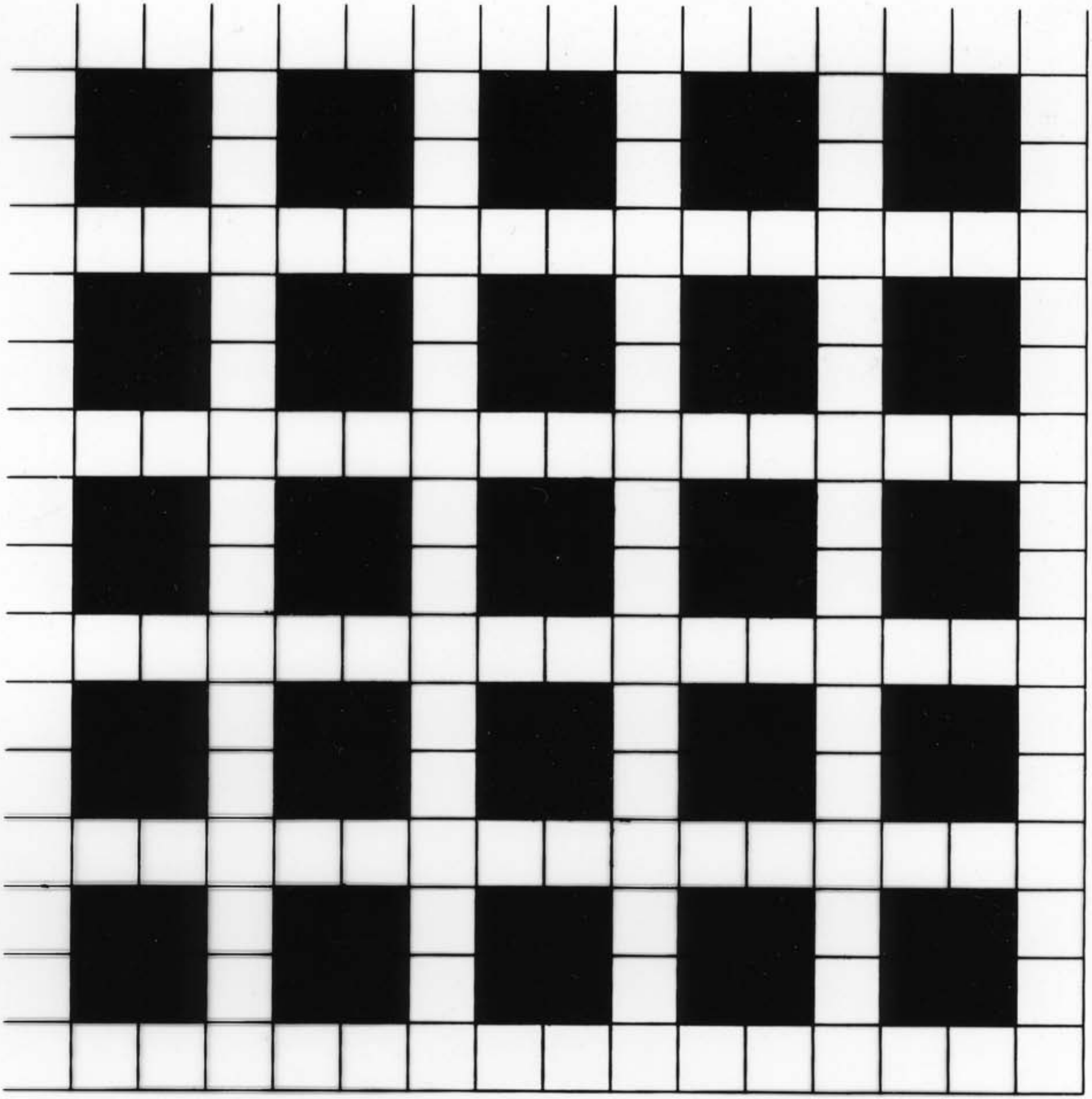
● Fold the flaps in two at a time.
■ (How many pairs can you try?)
● Predict what you will see through the windows.

● Fold the flaps in three at a time.
■ (How many threes can you try?)
● Predict what you will see through the windows.

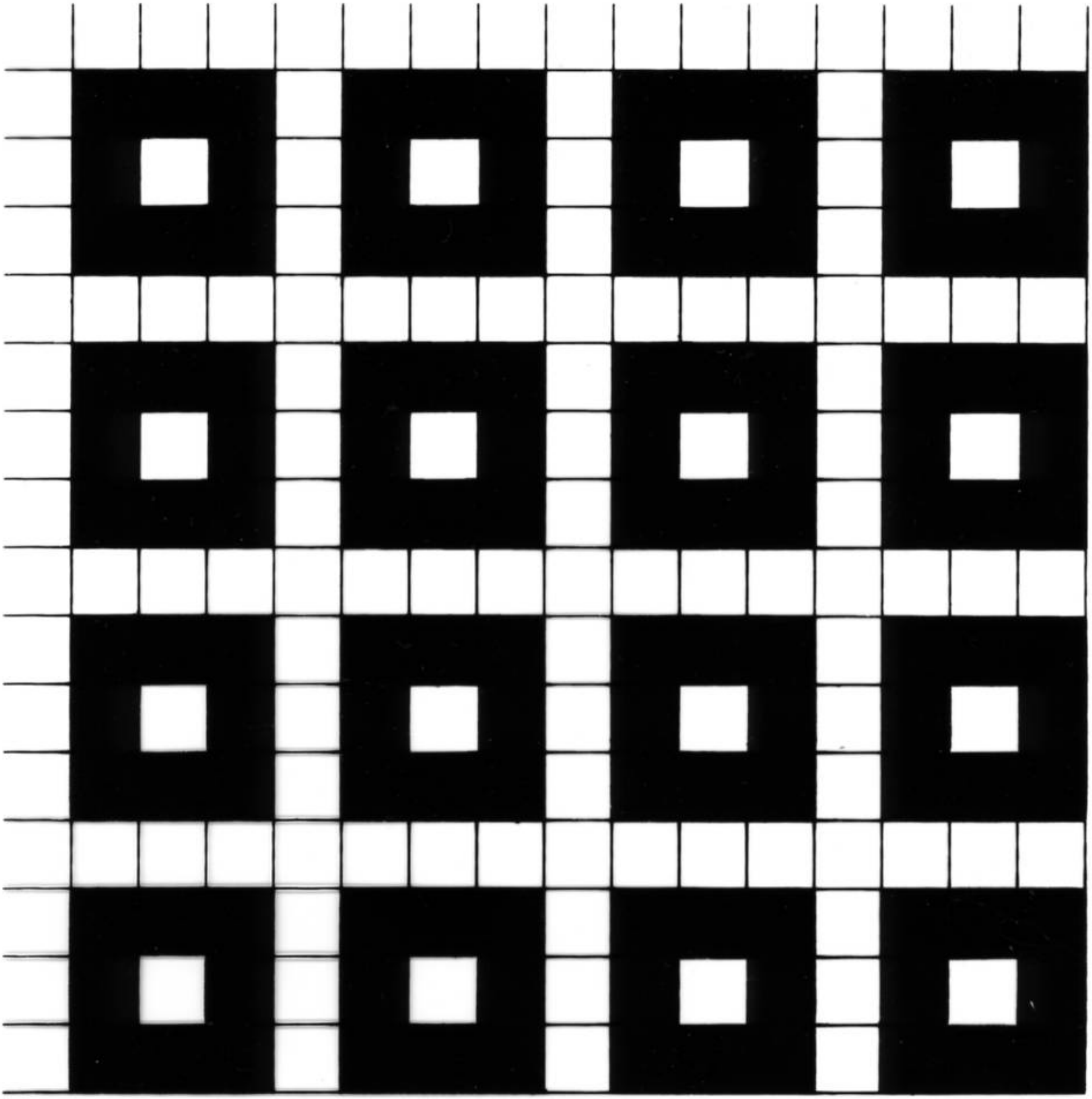
● Fold all four flaps in.
■ What will you see through the windows?

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 | 42 | 45 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 | 52 | 56 | 60 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 | 78 | 84 | 90 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 | 91 | 98 | 105 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 | 104 | 112 | 120 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 | 117 | 126 | 135 |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 |
| 11 | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 | 143 | 154 | 165 |
| 12 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 | 156 | 168 | 180 |
| 13 | 13 | 26 | 39 | 52 | 65 | 78 | 91 | 104 | 117 | 130 | 143 | 156 | 169 | 182 | 195 |
| 14 | 14 | 28 | 42 | 56 | 70 | 84 | 98 | 112 | 126 | 140 | 154 | 168 | 182 | 196 | 210 |
| 15 | 15 | 30 | 45 | 60 | 75 | 90 | 105 | 120 | 135 | 150 | 165 | 180 | 195 | 210 | 225 |

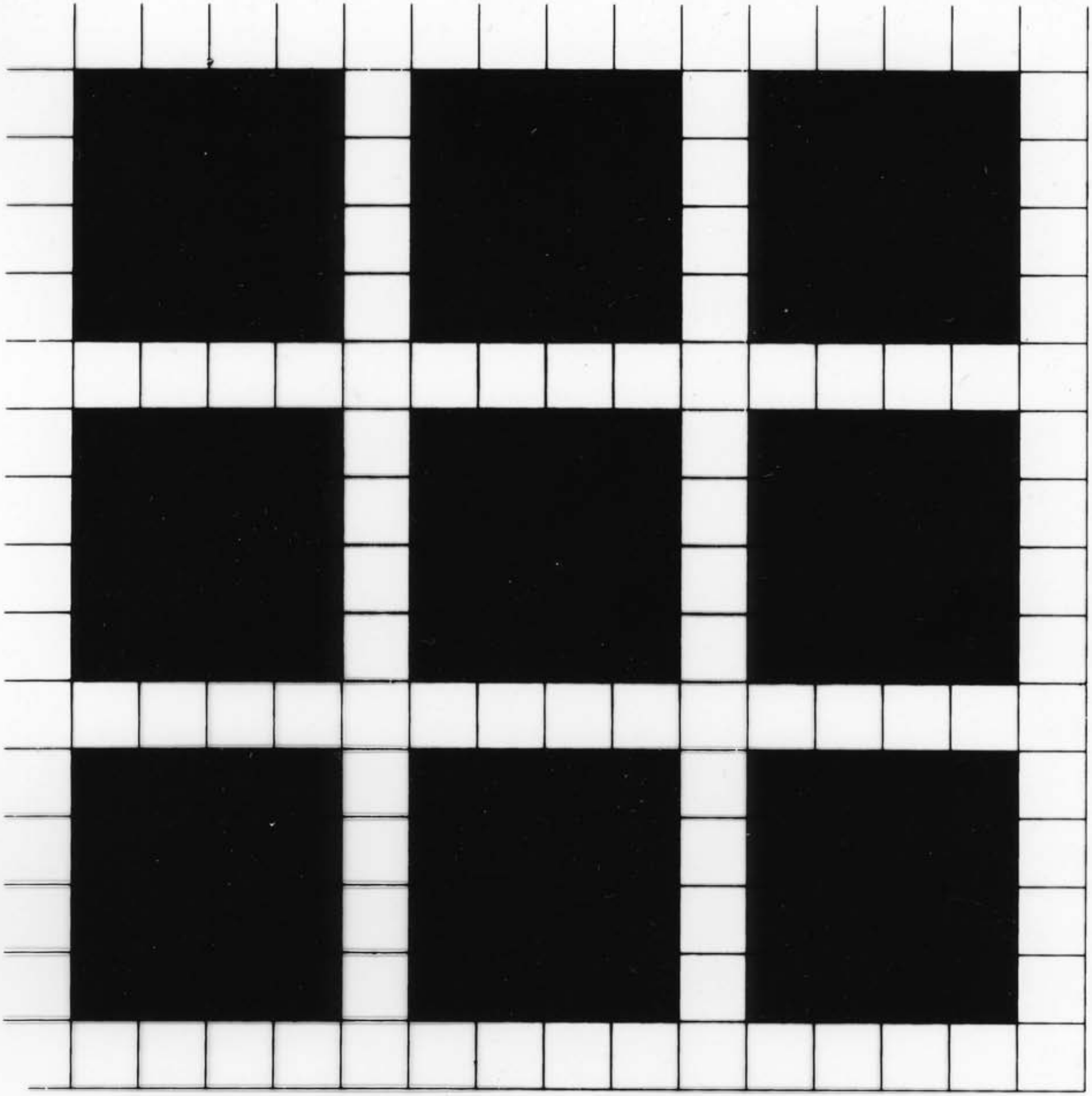




3

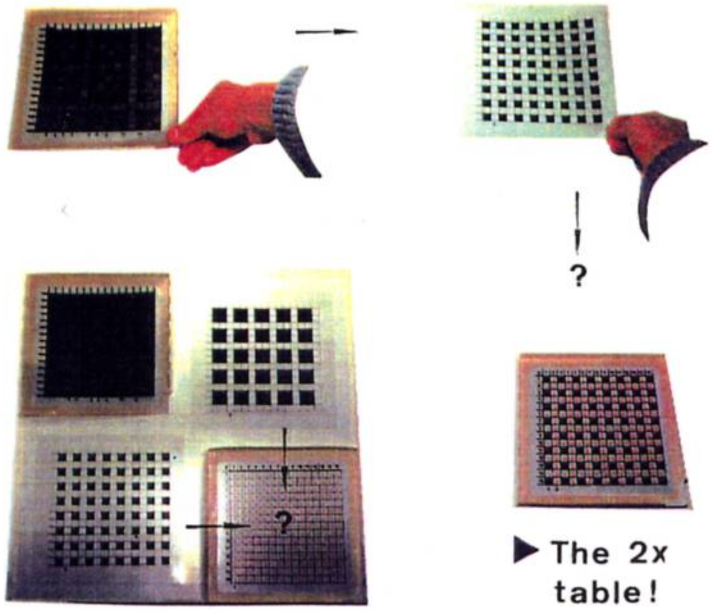


4

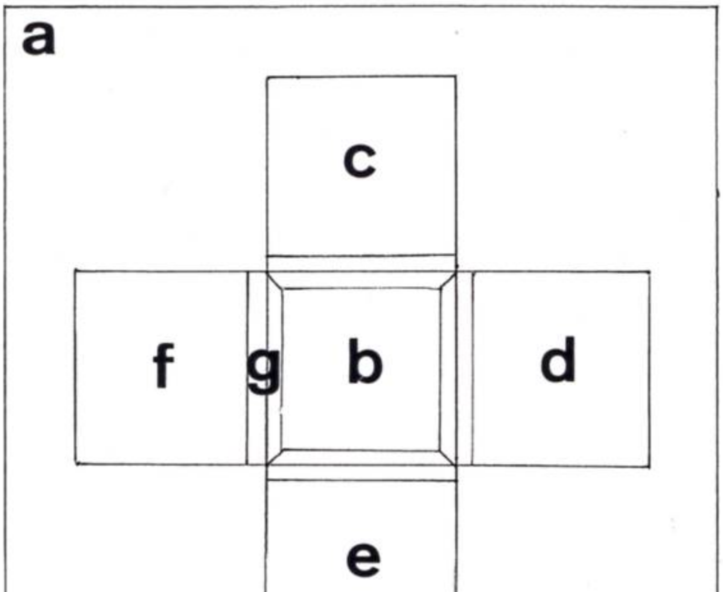


5

1




2

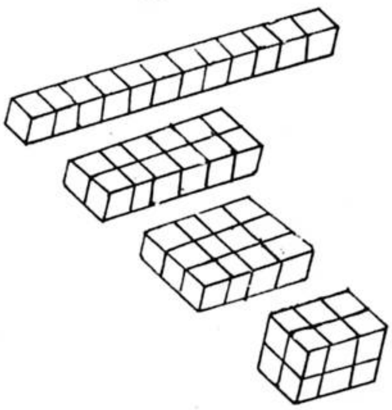


| PICTURE KEY | DESCRIPTION | TRADE NAME | U.K. SOURCE |
| --- | --- | --- | --- |
| | **Ff. enclosed:** | | |
| | multiplication square<br>(= multiple mask 1),<br>multiple masks 2, 3, 4, 5 | | The Magic Mathworks<br>(address above) |
| **1, 2** | alternative arrangements<br>**but caption refers to 2** | | |
| **a** | as **1.5 a/b**<br>but 750 mm x 750 mm | | |
| **b** | multiplication square,<br>stuck to **a** by edges | | |
| **c**<br>**d**<br>**e**<br>**f** | 2 mask<br>3 mask<br>4 mask<br>5 mask | | |
| **g** | Transpaseal strip, 30 mm wide,<br>hinges upper edges of **b**<br>to those of **c, d, e, f** as shown;<br>**c - f** so orientated that,<br>when folded on to **b**,<br>the mask numbers appear in bottom<br>right-hand corner | | |

| | NUMBER | TITLE |
|---|---|---|
| GROUP | 1 | MULTIPLICATION |
| STATION | 1.10 | CUBOIDS |
| TOPIC | | Prime v. composite numbers |

# CUBOIDS

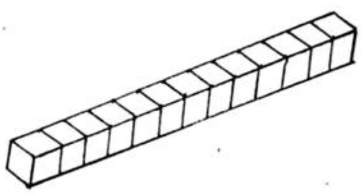

- A, give B a chosen number of cubes.



- B, make as many cuboids★ as you can using all the cubes for each.

▶ You earn that number of points (here: 4).


- B, give A a different number.



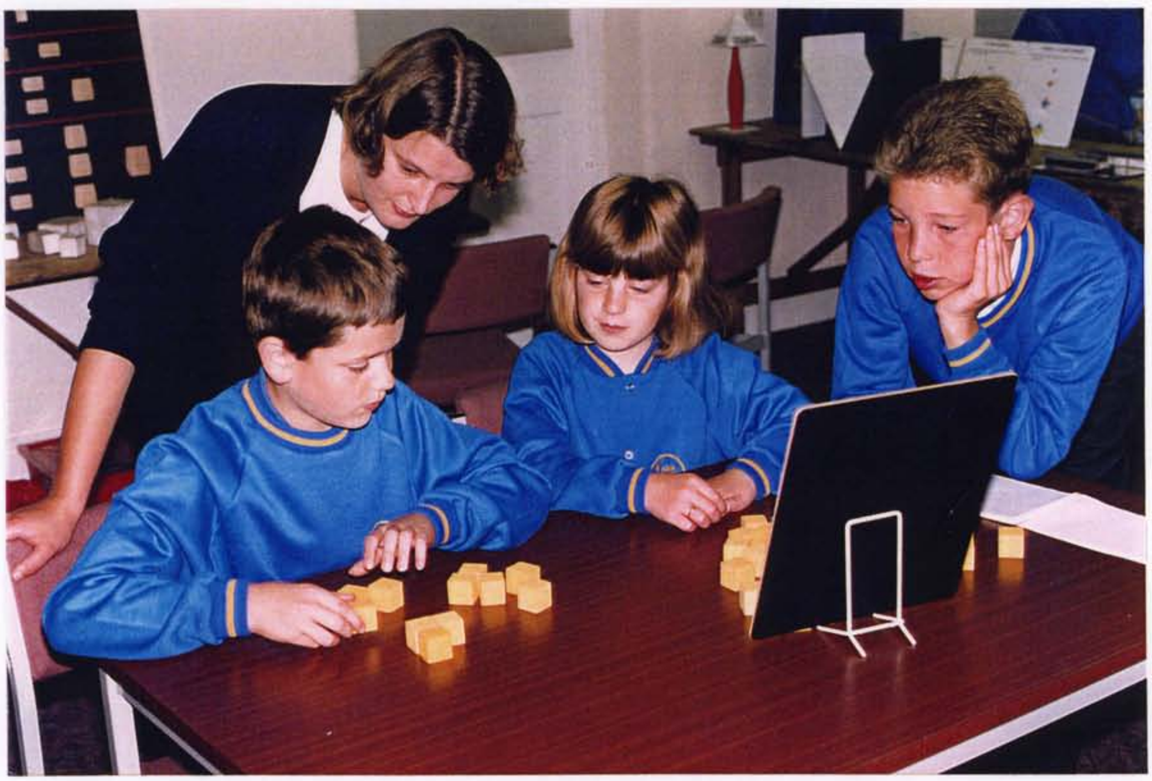
★ A cube counts.

▶ A, you score 1.

- Continue for several rounds, say 5.

▶ The better you get,
the more rounds you play
and the bigger the numbers become!


a



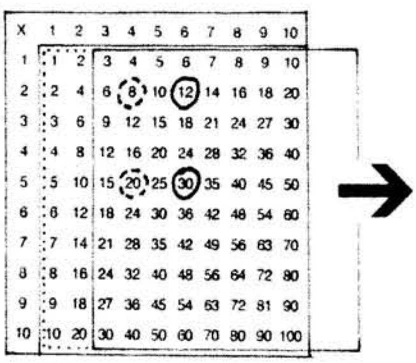
| PICTURE KEY | DESCRIPTION | TRADE NAME | U.K. SOURCE |
|---|---|---|---|
| a | expanded polyethylene cubes, 28 mm | DIME cube | Tarquin Publications<br>Stradbroke<br>Diss<br>Norfolk IP21 5JP<br>T +44 1379 384218<br>F +44 1379 384289 |

| | NUMBER | TITLE |
|---|---|---|
| GROUP | 1 | MULTIPLICATION |
| STATION | 1.11 | EQUIVALENT FRACTIONS |
| TOPIC | | The multiplication square as a source of ratios |

# EQUIVALENT FRACTIONS

● **Choose a column.**

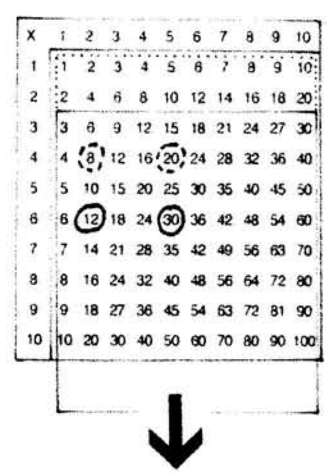
● **Ring 2 numbers on the clear board. Read them as a fraction, e.g. 8/20.**



● **Slide the board sideways.**

● **Read another fraction the rings choose, e.g. 12/30.**

■ **Are these 2 fractions the same or not?**



●■ **Slide the board downwards. What happens this time?**

●■ **Slide the board diagonally. What happens?**

● **Mark the board this way and slide it downwards:**

■ **What rule must you follow to produce 'equivalent' fractions? Why does it work?**

a

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

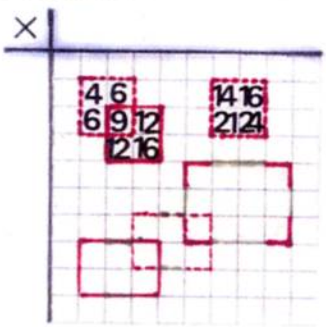
b

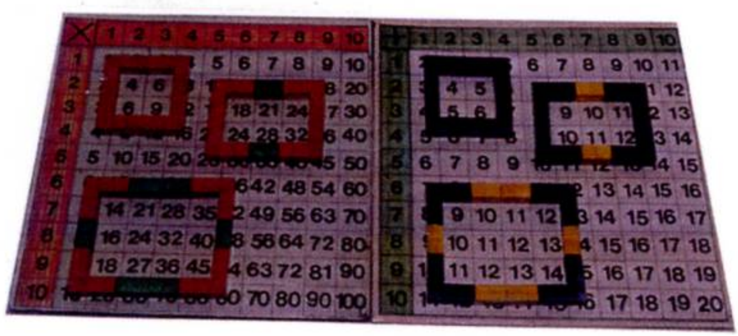
| PICTURE KEY | DESCRIPTION | TRADE NAME | U.K. SOURCE |
| --- | --- | --- | --- |
| **a** | 10 x 10 multiplication square,<br>cells 20 mm,<br>on 12 mm MDF,<br>220 mm x 220 mm,<br>faced with Glodex as **b** | | local |
| **b** | Glodex, 3 mm,<br>200 mm x 220 mm | | local |

| | NUMBER | TITLE |
|---|---|---|
| GROUP | 1 | MULTIPLICATION |
| STATION | 1.12 | TABLE WINDOWS |
| TOPIC | | A property of the multiplication square |

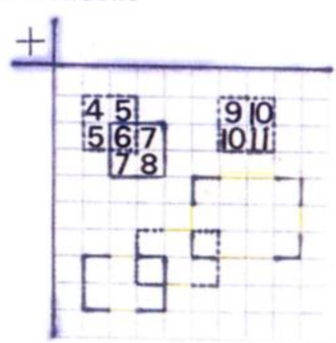
# TABLE WINDOWS

- ■ What is special about the numbers in the square window?
- ● Move it around.
- ■ Same question but try the rectangles.



- ■ What is special about the numbers in the square window?
- ● Move it around.
- ■ Same question but try the rectangles.




c
d
a
b

| PICTURE KEY | DESCRIPTION | TRADE NAME | U.K. SOURCE |
|---|---|---|---|
| a | multiplication square as 1.11 | | |
| b | addition square, 10 + 10, on base as 1.11 | | |
| c | square and rectangular frames as shown, Multilink cubes, colour-coded to draw attention to the corner cells | | NES Arnold (address above) |
| d | as **c** | | |