the magic mathworks

The Magic Manual

Introduction

A guide for fabricators
and users to stations from the
Magic Mathworks Travelling Circus

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Main Entries

There are 11 topic sections. Each section has a number and the stations within it and the substations within each station are coded by a decimal system.

Each section begins with a synopsis, followed by an analysis. The individual stations are then described thus:

first page: caption
next few pages: apparatus diagrams and photographs, annotated with letters
last page: key to letters, specifying materials, quoting sources

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ACKNOWLEDGEMENTS, ATTRIBUTIONS & REFERENCES

Information is given as follows.

1. Author(s)
2. (publication date of 1st English edition),
3. *Title*,
4. (publisher of latest edition known to P.S. and publication date where this differs from that first quoted)/journal title:
5. chapter/journal volume &/or number.

General

The influence of these 3 works on the style and content is pervasive:

Cundy, H.M. & Rollett, A.P. (1951), *Mathematical Models* , (Oxford 1961 = Tarquin 1981).
Hilbert, D. & Cohn-Vossen, S. (English edition 1952), *Geometry and the Imagination*, (Chelsea).
Steinhaus, H. (1969), *Mathematical Snapshots*, (Oxford).

The grouping of the stations has been influenced by:

Dienes, Z.P. (1960), *Building up Mathematics*, (ESA/Hutchinson).

While not contributing ideas directly, these books are recommended for their multisensory approach to the mathematics curriculum. The age-ranges to which they are suited are given in brackets:

Griffiths, R. (1988), *Maths through Play*, (Macdonald): (4-7).
Jones, L. & Richards, R. (1990), *An Early Start to Mathematics*, (Simon & Schuster): (6-10).
Banwell, C.S., Saunders, K.D. & Tahta, D.G. (1972), *Starting Points* , (Tarquin 1986): (11-14).

Here are two compendious hands-on works. The first, whose 'paper-engineering' makes it genuinely interactive, is a school course in itself. The second complements and extends formal work:

Gardner, B. & van der Meer, R. (1994), *The Maths Pack*, (Jonathan Cape).
Vorderman, C. (1996), *How Mathematics Works*, (Dorling Kindersley).

By Section and Station

1. MULTIPLICATION

For a good introduction see:

Hale, D. & Wells, P. (1972), *Turning the Tables*, (ATM).

1.2 'Hard' version:
Ahmed, A. (1985), *Mathematics for Low Attainers*, (W.Sussex I.H.E.).

1.3.1 This station takes advantage of a fact pointed out by the author in his notes to:
Millington, J. (1989), *Tables Cubes*, (Tarquin), later used by the author himself:
Millington, J. (1995), *The Tetrahedron Game*, Mathematics in School: vol.24, no.1.

1.5 This activity adds musical interest to the traditional classroon game of 'Fizz-Buzz!'. See:
Stephenson, P. (1998), *GRAB and the Fizz-Buzz Frieze*, Mathematics in School: vol.27, no.2.

1.8 This reproduces on a larger scale one of a set of such items in a kit:
Stephenson, P. (1989), *The Factor Pack*, (The Magic Mathworks).

1.9 The Magic Masks form the core of the above.

1.11 The idea comes from a teacher who in turn had found it in a periodical. (P.S. has failed to trace the source at this date.)

1.12 Hale & Wells (op. cit.).

2. TRANSFORMATIONS

2.1 Gardner, M. (1982), *The Ambidextrous Universe*, (Pelican).

2.1.1 The name MIRA acknowledges a device, patented under that name, using the transmission-reflection principle to perform standard geometrical constructions.

2.2.2 Steinhaus, H. (op. cit.).

2.3.3 Boorman, P. (1973), *The Elastic Ruler*, Mathematics Teaching: no.64

2.4 This station was inspired by:
Dienes, Z.P. & Goulding, E.W. (1967), *Geometry through Transformations* (ESA/Hutchinson): vol.1, *The Geometry of Distortion* : ch.4, *Shadow Geometry* .

2.6 For the Renaissance study of perspective see:
Pedoe, D. (1976), *Geometry and the Liberal Arts,* (Penguin): ch.2.

2.7 For a good account of anamorphosis see:

Hickin, P. (1992), *Anamorphosis*, Mathematical Gazette: no.476.
Also visit:
www.counton.org: Explorer: Anamorphic Art.

3. SYMMETRY

3.6-7 Most of these are classic dissections or derived therefrom. Many of the 2-D examples were described by H.E.Dudeney in various publications over the 1/4-century 1907-32. For an analysis of planar dissections see:
Lindgren, H. (1964), *Geometric Dissections*, (Princeton/van Nostrand).
In general, 3-D dissections can be attributed to named designers only as variants of older forms. For an account of such puzzles in wood see:
Coffin, S.T. (1991), *The Puzzling World of Polyhedral Dissections*, (Oxford).

3.6.1 was designed by P.S.

3.6.2A was published by J.Travers in 1933. (**3.6.2B** is a P.S. variant.)

3.7.4 is due to P.S.

3.9 Isenberg, C. (1978), *The Geometry of Soap Films and Soap Bubbles*, (Dover 1992).
For work with children see:
Stephenson, P. (2000), *Soap Films and Symmetry-Breaking*, Symmetry+: Part 1, spring 2000; Part 2, summer 2000.

4. PACKINGS

For a systematic treatment distinguished by many beautiful line drawings see:
Williams, R. (1972), *The Geometrical Foundation of Natural Structure: A Sourcebook of Design*, (Dover 1979).

4.1.1 See, e.g.:
Williams, R. (op. cit.): ch.2.
Steinhaus, H. (op. cit.): ch. 4.
Interesting on the basic theory of tessellations is:
Coxeter, H.S.M. (1969), *Introduction to Geometry*, (Wiley): ch.1.
The definitive work is:
Grünbaum, B. & Shephard, G. (1986), *Tilings and Patterns*, (Freeman).
This is a good workbook:
Bezuszka, S., Kenny, M. & Silvey, L. (1977), *Tessellations, the Geometry of Pattern*, (Creative Publications).

4.1.2 Idea from H. Lindgren (op. cit.) after J. Travers (v.s.)

4.1.3 Steinhaus, H. (op. cit.): ch. 1: p.177.
For work with children see:
Stephenson, P. (1994-2002), Symmetry+:
Colours and Maps, autumn 1994,
Topple-Printing for Beginners, spring 1998,
Topple-Printing, summer 1998,

More on Topple-Printing, summer 2002.

4.2   Gardner, M. (1989), *Penrose Tiles to Trapdoor Ciphers*, (W.H.Freeman): ch.1, *Penrose Tiling*.

4.3-4   Williams, R. (op. cit.): ch.5.
Steinhaus, H. (op. cit.), chs.7,8.
Thompson, d'Arcy (1917), *On Growth & Form*, (Cambridge 1971): ch.4

4.5-6   Williams, R. (op. cit.): chs.2,4.
Steinhaus, H. (op. cit.): ch.8.
(Most introductory texts on the solid state discuss the packings investigated here.)

5. DISSECTIONS

5.1.1   These exercises use the classic 7-piece tangram.

5.1.2   Longford, C.D. (1940), *Uses of a Geometric Puzzle*, Mathematical Gazette: no.260.

5.1.3   The caption panels are reproduced from:
Millington, J. (1987), *Pentominoes*, (Tarquin).

5.1.4   Alexanderson, G.L. & Seydel, K. (1978), *Kürschák's Tile*, Mathematical Gazette: no.421.

5.2.1   This puzzle was patented by the Dane Piet Hein.
Gardner, M. (1986), *Knotted Doughnuts and other Mathematical Entertainments*, (W.H.Freeman): ch.3, *Polycubes*.
This updates:
Gardner, M. (1958), *More Mathematical Puzzles and Diversions*, (Penguin 1961): ch.6, *The SOMA Cube*.
For an analysis of the puzzle, cited in the first reference, see:
Berlekamp, E.R., Conway, J.H. & Guy, R. (1982), *Winning Ways*, (Academic): pp.737-739, *The Hidden Secrets of Soma*, summarised in a diagram they christen the 'Somap', pp.802-803.
A useful workbook is:
Jackson, S. & Mulholland, V. (1991), *Polycubes and the SOMA Cube*, (Cheshire Maths Support Team).

5.2.2   This dissection was first described in Eureka, the journal of the Cambridge Archimedean Society.

6. SOLIDS

6.1-4   Cundy, H.M. & Rollett, A.P. (op. cit.): ch.3.

6.1.3   Smart, L. & Moore, E. (1992), *Solid State Chemistry*, (Chapman & Hall): p.186, fig.5.4.

6.1.4 Cundy, H.M. & Rollett, A.P. (op. cit.): pp.142-143.
Wells, D. (1991), *The Penguin Dictionary of Curious and Interesting Geometry*, (Penguin): p.51.

6.2 For work with children see:
Stephenson, P. (1994-2001),
Colours and Maps, Symmetry+: autumn 1994.
Map-Colouring Polyhedra, Symmetry+:
Part 1, summer 2001; Part 2, winter 2001/2.

6.3.1 For one of the 3 sequences see:
Williams, R. (op. cit.): p.213, fig.6.10.

6.4 Illustrations of Tetra Pak containers and the filling process used for the 'Classic' are taken from their information pack.

6.5.1 For the technique used here see:
Lyng, M.J. (1978), *Dancing Curves*, (NCTM).
An overhead projector takes the place of the slide projector used there.
Lyng attributes the technique to Hermann von Baravalle.

6.5.2 The station was inspired by:
Ehrenfeucht, A. (1964), *The Cube Made Interesting*, (Pergamon),
in which the figures are anglyphs.
See also:
Ball, D. (1980), *Struggling with a Cube*, Mathematics Teaching: no.91.
Stephenson, P. (1985), *Searching the Cube*, Mathematics Teaching: no.113.

7. SEQUENCES

7.1-5 A workbook serving many of these stations is:
Colledge, T. (1992), *Pascal's Triangle*, (Tarquin).

7.1.2 A proprietary product.

7.1.3 A classic classroom investigation in hands-on form. See:
Cooke, C. & Anderson, I. (1978), *Counting and Configurations*, (Schools Council/Blackie): p.15,
Colledge, T. (op. cit.): p.12.

7.1.4 Colledge, T. (op. cit.): p.11.

7.1.5 Colledge, T. (op. cit.): p.13.

7.2.2 Colledge, T. (op. cit.): p.13.

7.4 Garland, T.H. (1987), *Fascinating Fibonaccis*, (Dale Seymour).

7.4.3 Cooke, C. & Anderson, I. (op. cit.): pp.15-16.

Colledge, T. (op. cit.): p.20.

7.4.4 This activity simply formats the activity suggested in the notes accompanying Geoff Giles' DIME kit sold under the name 'Pentagon Triangles' and used here.

7.4.5 For a serious attempt to investigate the phenomenon see:
Douady, S. & Couder, Y. (1991), *Phyllotaxis as a Physical Self-Organised Growth Process*, Physical Review Letters: vol.68, no.13.

7.5.1 This cellular automaton was suggested by an observation in:
Gardner, M. (1975), *Mathematical Carnival*, (Penguin 1978): ch.15, *Pascal's Triangle*.
Note also:
Colledge, T. (op. cit.): p.4.

7.5.2 Cooke, C. & Anderson, I. (op. cit.): pp.10-11.
Colledge, T. (op. cit.): p.5.

7.5.3 A familiar investigation. P.S. has failed to find a source.

7.5.4 A Y10 girl from Cheshire.

7.5.5. Colledge, T. (op. cit.): p.18.

7.6.1 The Magic Mathworks apparatus is adapted from that developed by Sheffield and Sheffield Hallam Universities for their annual 'Mathematics Bonanza'.
*The pro forma provided suggests the method of finite differences as a route to the general formula. A good workbook is:
Seymour, D. & Shedd, M. (1973), *Finite Differences*, (Dale Seymour).

7.6.2 A classic puzzle.

7.7 Lucas, E. (1891), *Récréations Mathématiques*, (Gauthier-Villars et fils).
* v.s.

7.8.1 Lucas, E. (op. cit.).
* v.s.
'USE YOUR RULER TO SOLVE THE HANOI PUZZLE ':
Gardner, M. (1959), *The Scientific American Book of Mathematical Puzzles and Diversions*, (W.H.Freeman): ch.6, *The Icosian Game and the Tower of Hanoi* : pp.61-62.

7.9 These two problems were posed by Leonardo of Pisa (Fibonacci) in his book *Liber Abaci* of 1202.
7.9.1 was answered by Niccolò Tartaglia (1500-1557).
7.9.2 was answered by Bachet de Méziriac (1587-1638).

8. LOCI & LINKAGES

8.1-4 The most complete survey of loci is still:
Lockwood, E.H. (1961), *A Book of Curves*, (Cambridge).

8.5-7 Linkages are covered by one volume in a series produced as part of the Mathematics for the Majority Project:
(various authors) (1970), *Machines, Mechanisms and Mathematics*, (Chatto & Windus). See more recently:
Bolt, B. (1991), *Mathematics Meets Technology*, (Cambridge).

8.6 Kempe, A.B. (1877), *How to Draw a Straight Line*, (Macmillan).

9. COMMUNICATION

9.2 This particular exercise, using a feely box designed by Geoff Giles and sold as a DIME product, was suggested by Chris Hopkins.

10. HEURISTIC

The locus classicus is:
Polya, G. (1945), *How to Solve It*, (2nd edition: Princeton = Anchor Books 1957), and the expanded version:
Polya, G. (1962), *Mathematical Discovery*, (Wiley).

10.1.1.1 A traditional game.

10.1.1.2 This 'computer' takes its name from the machine described in:
Gardner, M. (1959), *Mathematical Puzzles and Diversions*, (Penguin), who also describes the winning strategy on which it is based.

10.1.2 Gardner, M. (1986), *Knotted Doughnuts and Other Mathematical Entertainments*, (W.H.Freeman): ch.9, who christens the game but attributes it to David Gale.

10.1.3 Gardner, M. (1988), *Penrose Tiles to Trapdoor Ciphers*, (W.H.Freeman): ch.8, *Wythoff's Nim*.

10.1.6 This group expands **10.1.6.1**, borrowed from Sheffield and Sheffield Hallam Universities (v.s.).
For work for children see:
Stephenson, P. (2003), *Checks*, Symmetry+: autumn 2003,
Checks Update, Symmetry+: spring 2005.

10.2 Gardner, M. (1969), *The Unexpected Hanging and Other Mathematical Diversions*, (W.H.Freeman): ch.16.
Gardner, M. (1991), *Fractal Mirrors, Hypercards and More*, (W.H.Freeman): ch.15.
For an exhaustive method of solution see:
Schuh, F. (1968), *The Master Book of Mathematical Recreations*, (Dover): pp.344-346.
For a useful way of coding solutions see:
Clifford, F. (1991), *Permutation Groups with a Chessboard*, Mathematical Gazette: no.473, pp.330-332.

10.3.1   Burton, L., Mason, J. & Stacey, K. (1982), *Thinking Mathematically*, (Addison-Wesley): pp.74-75 & 98-99.

10.3.2   For the parity argument which forms the basis of this station see:
Golomb, S.W. (1965), *Polyominoes*, (Princeton 1994): ch.1.

10.4   This station emerged as a solution to a problem posed by John Mason in his address to the Association of Teachers of Mathematics annual course at Cheltenham in 1995, which appeared as:
Mason, J. (1995), *Shaping Up*, Mathematics Teaching: no.152.
For work for children see:
Stephenson, P. (1998), *Weaves*, Symmetry+: autumn 1998.

10.5   Gardner, M. (1966), *New Mathematical Diversions*, (Simon & Schuster): ch.14, *Euler's Spoilers: The Discovery of an Order-10 Graeco-Latin Square*.
For work for children see:
Stephenson, P. (2002), *Graeco-Latin Squares*, Symmetry+: spring 2002.

10.6   Fielker, D. (2004), *'The Missing Domino' and Other Stories*, Mathematics Teaching: no.188, p.41.
For an analysis see:
Stephenson, P. (2005), *Negative Information*, Mathematics Teaching: no.190.

11. ESTIMATION

11.4   Thompson, d'Arcy (op. cit.): ch.2.
Ling, J. (1977), *Mathematics across the Curriculum*, (Schools Council/Blackie): ch.5.

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	pen	Pilot dri-wipe, WBMA-M, black	Selectasize The Corner House 334 Brighton Road Coulsdon Surrey CR5 3ES
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